Introduction to AdS/QCD

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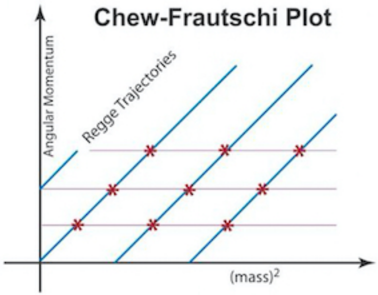
Outline

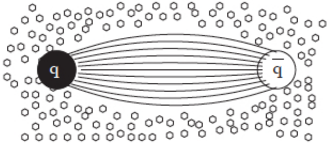
Motivation

- At strong coupling perturbative methods are inapplicable
- The lattice formulation of QCD does not work, since we have to study real-time phenomena.
- This has provided a motivation to try to understand the dynamics of QCD through the gauge/string duality
  - Wilson loops (interquark potential)
  - β-function, RG-flow
  - Thermalization

From QCD to Strings

- First classical string theory was born from hadronic phenomenology





Strings and another objects

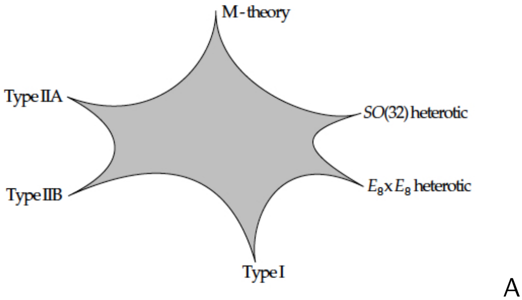


Figure: **A:**All string theories, and M-theory, as limits of one theory; **B:**A D-brane with one attached open string; Pics.from Polchinski

Dp-branes: extended objects on which open strings ends

Dp-branes carry a charge with respect to Ramond-Ramond fields, which geometrically are $p+1$-forms.

Low energy limit ($l_s \ll R$)$\Rightarrow$ SUGRA II B

Action for SUGRA IIB

$$
\begin{aligned}
S_{IIB} &= S_{NG} + S_R + S_{CS}, \\
S_{NS} &= \frac{1}{2k_{10}^2}\int d^{10}x(-G)^{1/2}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2\right), \\
S_R &= -\frac{1}{4k_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right), \\
S_{CS} &= -\frac{1}{4k_{10}^2}\int C_4\wedge H_3\wedge F_3,
\end{aligned}
$$

$$
\tilde{F}_3 = F_3 - C_0\wedge H_3, \quad \tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3
$$

The Einstein equaitons

$$
\begin{aligned}
R_{MN} &+ 2\nabla_M\nabla_N\phi + \frac{1}{4}g_{MN}A = \frac{1}{4}H_{MAB}H_N^{AB} + \frac{1}{2e^{2\phi}}F_MF_N + \frac{1}{4}e^{2\phi}\tilde{F}_{MAB}\tilde{F}_N^{AB} \\
&+ \frac{1}{44!}e^{2\phi}\tilde{F}_{MABCD}\tilde{F}_N^{ABCD}, \\
A &= e^{2\phi}\partial_M\chi\partial^M\chi + \frac{1}{3!}e^{2\phi}\tilde{F}_{ABC}A\tilde{B}C + \frac{1}{25!}e^{2\phi}\tilde{F}_{ABCDE}\tilde{F}^{ABCDE}.
\end{aligned}
$$

+ EOM for dilaton + Bianchi identities

D3-branes

The D3-brane metric

$$ds^2 = (1 + R^2/r^4)^{-1/2}(dx^i)^2 + (1 + R^4/r^4)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

$x^i, i = 1, \ldots, 4$ – coordinates along the world-volume of the $D3$-branes, and Ω_5 and r are spherical coordinates.

Near-horizon limit

$$ds^2 = R^2 \frac{(dx^i)^2 + dy^2}{y^2} + R^2\Omega_5^2, \quad y = \frac{R^2}{r}.$$

- We obtain: the $AdS_5 \times S^5$ space with boundary located at $y = 0$.
- N D-branes on top of each other – the gauge symmetry is enlarged to $U(N)$.

Outline

Back to QCD

Back to QCD: the AdS/CFT conjecture

The AdS/CFT correspondence Maldacena'98

The AdS/CFT conjecture claims exact equivalence between the theory in the bulk, which is a low energy approximation to $D = 10$ IIB string theory on $AdS_5 \times S^5$ the theory defined on the boundary, which is $\mathcal{N} = 4$ supersymmetric Yang-Mills with gauge group $SU(N_C)$ at large N_C.

- The strong coupling regime of one theory reflects the weak coupling regime of the other one

Gubser,Klebanov,Polyakov, Witten'98

$$Z_4[\phi_0(x)] = \int \mathcal{D}\exp\{iS_4 + \int_{x^4} \phi_0\mathcal{O}\}, \quad S_4 = \int d^4x\mathcal{L}$$

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$$Z_5[\phi_0(x)] = \int_{\phi(x,\varepsilon)=\phi_0(x)} \mathcal{D}[\phi]e^{iS_5[\phi]}$$

$$Z_4 = Z_5$$

Generating functional[4d sources $\phi_0(x)$] = Effective action[fields $\phi_0(x)$]

Deviation from conformal symmetry

Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures $T > 300$ MeV and the equation of state $\sim E = 3P$ (a traceless conformal energy-momentum tensor).

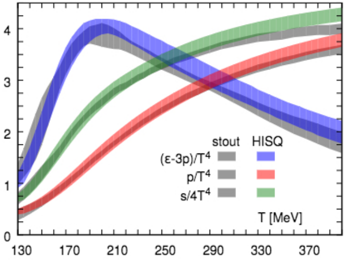


Figure: The comparison of the HISQ/tree and stout results for the trace anomaly, the pressure, and the entropy density

Pic. from Bazavov et al.'14

Finite temperature

$T \neq 0$, Witten'98

Finite temperature Yang-Mills theory in 4d dual to a $AdS_5 \times S^5$ set up with a black hole(Schawrzschild-$AdS_5 \times S^5$)

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + (dx^i)^2 + \frac{dz^2}{f(z)}\right], \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^4.$$

- The temperature of the the Yang-Mills theory is identified with the Hawking temperature of the black hole.

Holographic dictionary-I

boundary: field theory	bulk: gravity
energy momentum tensor T^{ab}	metric field g_{ab}
global internal symmetry current J^a	Maxwell field A_a
order parameter/scalar operator $\mathcal{O}_b$	scalar field ϕ
fermionic operator $\mathcal{O}_f$	Dirac field ψ
conformal dimension of the operator	mass of the field
global space symmetry	global isometry
temperature	Hawking temperature
chemical potential/charge density	boundary values
phase transition	instability of black holes

Outline

AdS/QCD model: CONSTRUCTING

Top-down approach: Sakai-Sugimoto model

Top-down approach: low-energy approximation of string theory (supergravity model) trying to find a gravitational background with features similar to QCD

Sakai-Sugimoto: $D4 - D8 - \bar{D}8$ 5d theory with chiral quarks

	0	1	2	3	4	5	6	7	8	9
D4	X	X	X	X	X					
$D8 - \bar{D}8$	X	X	X	X		X	X	X	X	X

- a D4 brane with one direction wrapped on a circle
- quarks are included by including separated $D8$ and $\bar{D}8$ branes
- the key feature is chiral symmetry breaking

$$\begin{aligned} ds^2 &= \left(\frac{u}{R}\right)^{3/2}(dx_4^2 + f(u)d\tau^2) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \\ f(u) &\equiv 1 - \left(\frac{u_{KK}}{u}\right)^3 . \end{aligned}$$

Top-down approach: Sakai-Sugimoto model

- Mesons correspond to fluctuations of the $D8$ brane solutions in the $D4$ background

The Dirac-Born-Infeld (DBI) action for the embedding of $D8$

$$S_{DBI} = Vol(S^4)\int d^4x \int z\frac{2}{3}g_s u_{kk}^5\left(\frac{R}{u_{KK}}\right)^{3/2}(1+z^2)^{2/3}$$
$$\times\sqrt{1+\frac{9}{4u_{KK}^2}(\frac{u_{KK}}{R})^3 z^2(1+z^2)^{1/3}\tau'(z)^2},$$

where $1+z^2=\frac{u_{KK}}{u}$, τ is periodic, $\delta\tau=\frac{4\pi}{3}\frac{R^{3/2}}{u_{KK}^{1/2}}$

Bottom-up approach: the hard wall AdS/QCD model

Effective $5D$ theory (gravity + matter); hadrons $\equiv$ the normalizable modes of the 5d fields.

The hard wall AdS/QCD model Erlich et al. PRL'05

The theory in the bulk

$$S = \int d^5x\sqrt{g}Tr\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right\},$$

with $D_\mu X = \partial_\mu X - iA_{L_\mu}X + iXA_{R\mu}$, $A_{L,R} = A^a_{L,R}t^a$, $m_5^2 = 3$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, defined on the AdS background

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

The geometry is cutted by two "branes" $z=0$: "UV-brane",
$z = z_{max}$:"IR-brane" (Polchinski&Strassler'02)
scaling $x \to \lambda x$, $z \to \lambda z$: if x is the length scale to which physics is examined, low values of x correspond to low values of z (UV brane)
large distances correspond to large z (IR) ($z_{max} \sim 1/\Lambda_{QCD}$)

Bottom-up approach: the hard wall AdS/QCD model

4d: $\mathcal{O}(x)$	5d:$\phi(x,z)$	p	$\triangle_{\mathcal{O}}$	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

The expectation value of X:

$$X_0(z) = 1/2Mz + 1/2\Sigma z^3,$$

defined as the classical solutions of the field equations with BC in the UV

$$(2/\epsilon)X(\epsilon) = M$$

M is the quark mass matrix (Explicit chiral breaking), Σ is chiral condensate, M, Σ are fixed from experimental data (free parameters).

- Confinement – good :)
- Realizes the chiral symmetry breaking – good:)
- The resulting mass spectra for the exited mesons are contrary to the experimental data,a good Lagrangian – a bad wall :(

Bottom-up approach: the soft wall AdS/QCD model

- Effective $5D$ gravity theory (+ matter), break conformal inv. of CFT: introducton of QCD Λ_{QCD}

The soft wall AdS/QCD model, Karch et al. PRD'06

The meson action

$$S = \int d^5 \sqrt{g} e^{-\Phi(z)} Tr \left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5} \left(F_L^2 + F_R^2 \right) \right],$$

with $D_\mu X = \partial_\mu X - iA_{L\mu} X + iXA_{R\mu}$, $A^M_{L,R} = A^{Ma}_{L,R} t^a$, $Tr[t^a t^b] = \delta^{ab}/2$, $g_5 = \frac{12\pi^2}{N_c}$, N_c– a number of colors.

$$ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad A(z) = -\ln(z/L)$$

Confinement: soft cutoff of AdS space – running dilaton $e^{-\Phi(z)}$, $\Phi(z) = \kappa^2 z^2$.

The bulk fields X are decomposed into the scalar and pseudoscalar mesons, the chiral fields A_L and A_R – the vector and axial-vector mesons.

Bottom up approach: the soft wall AdS/QCD model

ρ-meson vecor field $V = \frac{A_L + A_R}{2}$ The classical EOM:

$$\partial_M \left(\sqrt{q} e^{-\Phi} \left[\partial^M V^N - \partial^N V^M \right] \right) = 0.$$

$V_\mu(x,z) = \epsilon_\mu e^{iqx} \psi(x)$, $\Rightarrow -\psi'' + V(z)\psi = m_n^2 \psi(z)$ Soft-wall AdS/QCD model

- describes the linear confinement :)
- desired mass spectra for the excited vector mesons $m_n^2 \sim J(L)$ (Regge behavior) :)
- can't consistently realize the chiral symmetry breaking (but seems to be sovable)

Improved HQCD model

Improved holographic QCD model — Gursoy, Kiritis Nitti

$$S_5 = -\frac{1}{16\pi G_5}\int dx^5 \sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{4}{3}(\partial\Phi)^2 + V(\Phi_s)\right].$$

The metric is given by

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2).$$

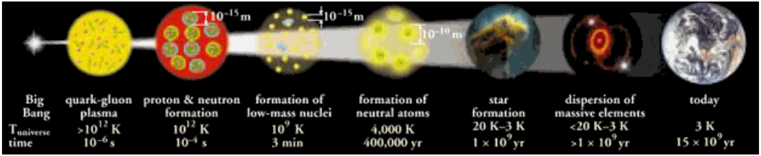
$g_{\mu\nu}$ is dual to $T_{\mu\nu}$, ϕ is dual to $tr[F^2]$

- The potential can be tuned to reproduce the beta-function
  - For asymptotically AdS UV $V = V_0 + v_1\lambda + v_2\lambda^2 + \ldots$ $(\lambda \to 0)$
  - For confinement in the IR $V \sim \lambda^{4/3}\sqrt{\lambda}$ $(\lambda \to \infty)$
- Confinement — finite-T transition between thermal gas and BH
- Reproduces an asymptotically-linear glueball spectrum
- With an appropriate tuning of two parameters in V the model describes well both $T = 0$ properties (spectra) as well as thermodynamics

Outline

The quark-gluon plasma (2005)

- A new state of matter produced in Heavy-Ion collisions at **RHIC** and **LHC**
- Deconfined quarks, antiquarks, and gluons at high temperature (nuclear matter $\rightarrow$ deconfined phase).
- QGP does not behave like a weakly coupled gas of quarks and gluons, but a strongly coupled fluid.
- Highly anisotropic at the early stage of its evolution



The quark-gluon plasma (2005)

$$\tau_{therm}(0.1 fm/c) < \tau_{hydro} < \tau_{hard}(10 fm/c) < \tau_f(20 fm/c)$$

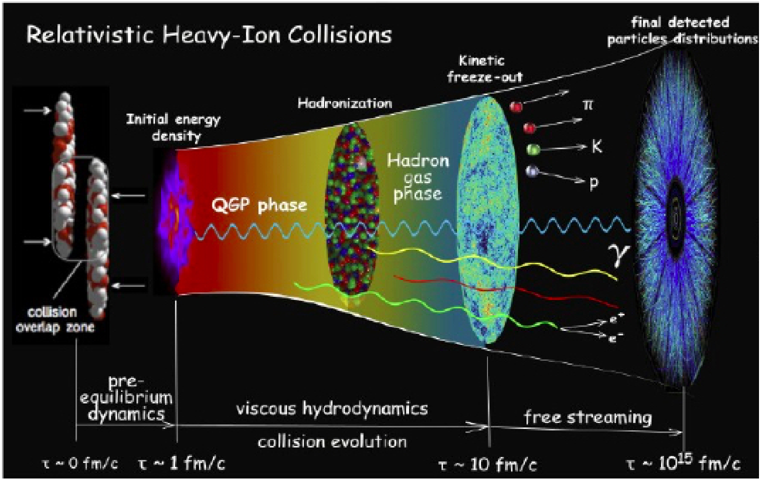


Figure: Picture from: P.Sorensen, C.Shen

The quark-gluon plasma (2005)

- Multiptlicity
- Thermalization
- Observables (Correlators, Wilson loops, etc)

We have to "mimic" the heavy ions collision

Models:

- shock waves collision in AdS
- infalling shell (Vaidya solutions)

Holographic dictionary- II

- $4d$ Multiplicity in HIC $=$ BH entropy in AdS_5 Gubster et al.'08

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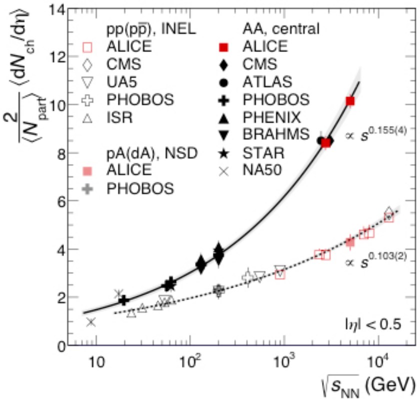
- Wilson loops $=$ 2-dimensional minimal surfaces
- Entenglament entropy $=$ 3-dimensional minimal surfaces

- $D=4$ Multiplicity is proportional to entropy of $D=5$ BH Gubser'08

Experiment:
$S_{data} = \mathbf{s}_{NN}^{0.155}$
ALICE collaboration'15

Modified AdS:
$S_{data} = \mathbf{s}_{NN}^{0.12}$
Kiritis & Taliotis'11

Modified AdS+ ghosts:
$S_{data} = \mathbf{s}_{NN}^{0.16}$
Aref'eva et al.'14



ALICE collaboration'15

Broken scaling
$S_{data} = \mathbf{s}_{NN}^{0.16}$
Aref'eva & A.G.'14

Anisotropic duals

The AdS/CFT correspondence:

The Field Theory	The Gravitational Background
• the conformal group $SO(D,2)$	• the group of isometries
of a D-dimensional CFT	of AdS_{D+1}
$(t,x_i) \to (\lambda t, \lambda x_i)$, $i=1,..,d-1$	$ds^2 = r^2\left(-dt^2 + d\vec{x}^2_{d-1}\right) + \frac{dr^2}{r^2}$

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Generalizations?

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Generalizations?

Lifshitz scaling: $t \to \lambda^{\nu} t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r,$

where ν is the Lifshitz dynamical exponent

Lifshitz metric: $ds^2 = -r^{2\nu} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}_{d-1}^2$

Kachru, Liu, Millgan '08

Lifshitz-like spacetimes for holography

- Lifshitz-like metrics

$$ds^2 = r^{2\nu}\left(-dt^2+dx^2\right)+r^2dy_1^2+r^2dy_2^2+\frac{dr^2}{r^2},$$

$(t, x, y, r) \rightarrow (\lambda^{\nu} t, \lambda^{\nu} x, \lambda y_1, \lambda y_2, \frac{r}{\lambda})$, M. Taylor'08, Pal'09.

The 5d Lifshitz-like metrics, $z = \dfrac{1}{r^{\nu}}$

$$\text{Type} - (\mathbf{1},\mathbf{2}) \quad ds^2 = L^2\left[\frac{(-dt^2+dx^2)}{z^2}+\frac{(dy_1^2+dy_2^2)}{z^{2/\nu}}+\frac{dz^2}{z^2}\right].$$

$$\text{Type} - (\mathbf{2},\mathbf{1}) \quad ds^2 = L^2\left(\frac{(-dt^2+dx_1^2+dx_2^2)}{z^2}+\frac{dy^2}{z^{2/\nu}}+\frac{dz^2}{z^2}\right).$$

Type IIB SUGRA, $D3 - D7$-branes: Anisotropic QGP

$\mathcal{M} = M_5 \times X_5$: M_5 is a $5d$ Lifshitz-like metric, X_5 is an Einstein manifold.

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D3-D7 system, Azeyanagi et al.'09, Mateos&Trancanelli'09

$$ds^2 = \tilde{R}^2\left[\rho^2\left(-dt^2+dx^2+dy^2\right)+\rho^{4/3}dw^2+\frac{d\rho^2}{\rho^2}\right]+R^2 ds^2_{X_5}.$$

$$\nu = 3/2, \quad r \equiv \rho^{2/3}, \quad (t,x,y,w,\rho) \to \left(\lambda t, \lambda x, \lambda y, \lambda^{2/3} w, \frac{\rho}{\lambda}\right)$$

$\mathcal{M}_5 \times X_5$	t	x	y	r	w	s_1	s_2	s_3	s_4	s_5
D3	$\times$	$\times$	$\times$		$\times$					
D7	$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$

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AdS/CFT: $D3$-probes in $D3$-background $AdS_5\times S^5\Rightarrow\mathcal{N}=4$ SYM.

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$\mathcal{M}_5\times X_5$	t	x	y	r	w	s_1	s_2	s_3	s_4	s_5
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D7	$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$

AdS/CFT: $D3$-probes in $D3$-background $AdS_5\times S^5\Rightarrow\mathcal{N}=4$ SYM.
$D7$-probes in $D3$-background $Lif_{IR}/AdS_{5,UV}\times X_5\Rightarrow$ deformed SYM.
Jet quenching, drag force, potentials... see Giataganas et al.'12

Possible models

$$S=\frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left(R[g]+\Lambda-\frac{1}{2}\partial_M\phi\partial^M\phi-\frac{1}{4}e^{\lambda\phi}F^2_{(2)}\right),$$

Λ is negative cosmological constant.

The Einstein equations

$$R_{mn}=-\frac{\Lambda}{3}g_{mn}+\frac{1}{2}(\partial_m\phi)(\partial_n\phi)+\frac{1}{4}e^{\lambda\phi}\left(2F_{mp}F^p_n\right)-\frac{1}{12}e^{\lambda\phi}F^2g_{mn}.$$

The scalar field equation

$$\Box\phi=\frac{1}{4}\lambda e^{\lambda\phi}F^2,\quad \text{with}\quad \Box\phi=\frac{1}{\sqrt{|g|}}\partial_m(g^{mn}\sqrt{|g|}\partial_n\phi).$$

The gauge field

$$D_m\left(e^{\lambda\phi}F^{mn}\right)=0.$$

Gravity duals: black brane and infalling shell

The Lifshitz-like black brane

$$ds^2 = e^{2\nu r}\left(-f(r)dt^2 + dx^2\right) + e^{2r}\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{f(r)},$$

where $f(r) = 1 - me^{-(2\nu+2)r}$. Aref'eva,AG, Gourgoulhon'16.

$$\nu = 4,\ \lambda = \pm\frac{2}{\sqrt{3}},\ \Lambda = 90,\ \mu q^2 = 240.$$

The Vaidya solution in Lifshitz background

$$ds^2 = -z^{-2}f(z)dv^2 - 2z^{-2}dvdz + z^{-2}dx^2 + z^{-2/\nu}(dy_1^2 + dy_2^2),$$

$f = 1 - m(v)z^{2/\nu+2}, v < 0$ – inside the shell, $v > 0$ – outside,

$$dv = dt + \frac{dz}{f(z)}, \quad z = \frac{1}{r^\nu}.$$

Holographic Thermalization

Def.

Thermalization time at scale l is the time at which the tip of the geodesic with endpoints $(-l/2)$ and $(l/2)$ grazes the middle of the shell.

The Lagrangian of the pointlike probe

$$\mathcal{L} = \sqrt{-\frac{f(z)}{z^2}\frac{dv}{d\tau}\frac{dv}{d\tau} - \frac{2}{z^2}\frac{dv}{d\tau}\frac{dz}{d\tau} + \frac{1}{z^2}\frac{dx}{d\tau}\frac{dx}{d\tau} + \frac{1}{z^{2/\nu}}\left(\sum \frac{dy_i}{d\tau}\frac{dy_i}{d\tau}\right)}$$

$\tau = x$ or $\tau = y_i$, $i = 1, 2$.

Holographic Thermalization

The thermalization time t_{therm} at scale l along x

$$\ell = 2z_* \int_0^1 \frac{w dw}{\sqrt{f(z_* w)(1-w^2)}}, \quad t_{\text{therm}} = z_* \int_0^1 \frac{dw}{f(z_* w)},$$

where $w = \dfrac{z}{z_*}$, z_* is the turning point.

The thermalization time t_{therm} at scale l along y

$$\ell = 2\, z_*^{1/\nu} \int_0^1 \frac{w^{-1+2/\nu} dw}{\sqrt{f(wz_*)\left(1-w^{2/\nu}\right)}},, \quad t_{\text{therm}} = z_* \int_0^1 \frac{dw}{f(z_* w)}.$$

Holographic Thermalization

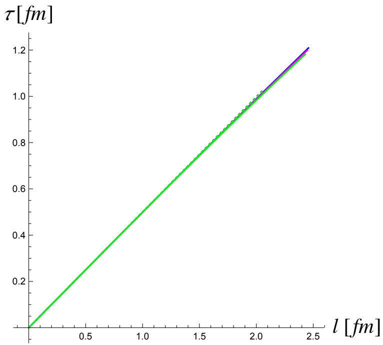


Figure: Thermalization along the longitudinal direction with $m = 0.5$ and $m = 0.1$. All lines coincide.

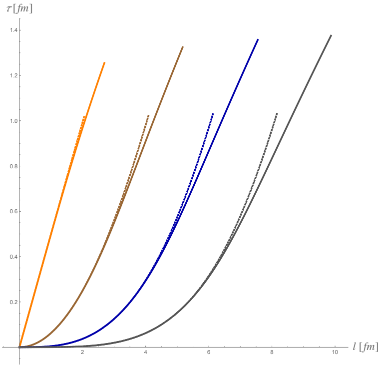


Figure: Thermalization along the transversal direction, $\nu = 1$ (orange), $\nu = 2$ (brown), $\nu = 3$ (blue) and $\nu = 4$ (gray).

Holographic Wilson Loops

- The expectation value of WL in the fundamental representation calculated on the gravity side: Maldacena et.al.'98

$$W[C] = \langle \mathrm{Tr}_F \, e^{i \oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]},$$

where C in a contour on the boundary, F – the fundamental representation, S is the minimal action of the string hanging from the contour C in the bulk. The Nambu-Goto action is

$$S_{string} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}, \quad (1)$$

with the induced metric of the world-sheet $h_{\alpha\beta}$ given by

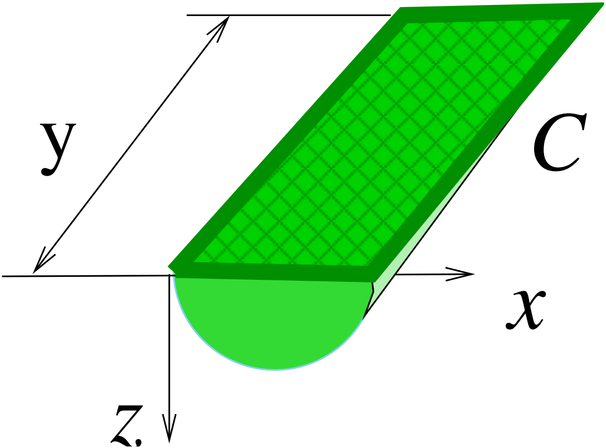
$$h_{\alpha\beta} = g_{MN} \partial_\alpha X^M \partial_\beta X^N, \quad \alpha, \beta = 1, 2, \quad (2)$$

g_{MN} is the background metric, $X^M = X^M(\sigma^1, \sigma^2)$ specify the string, σ^1, σ^2 parametrize the worldsheet.

- The potential of the interquark interaction

$$W(T, X) = \langle \mathrm{Tr} \, e^{i \oint_{T \times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}.$$

Holographic spatial Wilson loops



Holographic Wilson Loops

A similar operator to probe QCD is the spatial rectangular Wilson loop of size $X \times Y$ (for large Y)

$$W(X,Y) = \langle \mathrm{T}re^{i\oint_{X\times Y} dx_\mu A_\mu} \rangle = e^{-\mathcal{V}(X)Y}$$

defines the so called pseudopotential $\mathcal{V}$:

$$\mathcal{V}(X) = \frac{S_{string}}{Y}.$$

The spatial Wilson loops obey the area law at all temperature, i.e.

$$\mathcal{V}(X) \sim \sigma_s X,$$

where σ_s defines the spatial string tension

$$\sigma_s = \lim_{X\to\infty} \frac{\mathcal{V}(X)}{X}.$$

Spatial WL in Lifshitz-like backgrounds

Rectangular WL in the spatial planes xy_1(or xy_2) and y_1y_2. Possible configurations:

- a rectangular loop in the xy_1 (or xy_2) plane with a short side of the length ℓ in the longitudinal x direction and a long side of the length L_{y_1} along the transversal y_1 direction

$$x \in [0, \ell < L_x], \quad y_1 \in [0, L_{y_1}];$$

- a rectangular loop in the xy_1 plane with a short side of the length ℓ in the transversal y_1 direction and a long side of the length L_x along the longitudinal x direction:

$$x \in [0, L_x], \quad y_1 \in [0, \ell < L_{y_1}];$$

- a rectangular loop in the transversal y_1y_2 plane with a short side of the length ℓ in one of transversal directions (say y_1) and a long side of the length L_{y_2} along the other transversal direction y_2

$$y_1 \in [0, \ell < L_{y_1}], \quad y_1 \in [0, L_{y_2}].$$

Static WL. Case 1: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(x)$, $v = v(x)$.

The renormalized Nambu-Goto action

$$S_{x,y_{1(\infty)},ren} = \frac{L_{y_1}}{2\pi\alpha'}\frac{1}{z_*^{1/\nu}}\int\limits_0^1 \frac{dw}{w^{1+1/\nu}}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{2+2/\nu}\right)}}-1\right]-\frac{\nu}{z_*^{1/\nu}},$$

where $w = z/z_*$.The length scale is

$$\frac{\ell}{2} = 2z_* \int\limits_{z_0/z_*}^1 \frac{w^{1+1/\nu}\,dw}{f(z_* w)(1-w^{2+2/\nu})}.$$

Then pseudopotential $\mathcal{V}_{x,y_{1(\infty)}} = \dfrac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}$.

For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{x,y_{1(\infty)}}(\ell,\nu) \underset{\ell\sim 0}{\sim} -\frac{\mathcal{C}_1(\nu)}{\ell^{1/\nu}}.$$

For large ℓ

$$\mathcal{V}_{x,y_{1(\infty)}}(\ell,\nu) \underset{\ell\to\infty}{\sim} \sigma_{s,1}(\nu)\,\ell.$$

Static WL. Case 1: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(x)$, $v = v(x)$.

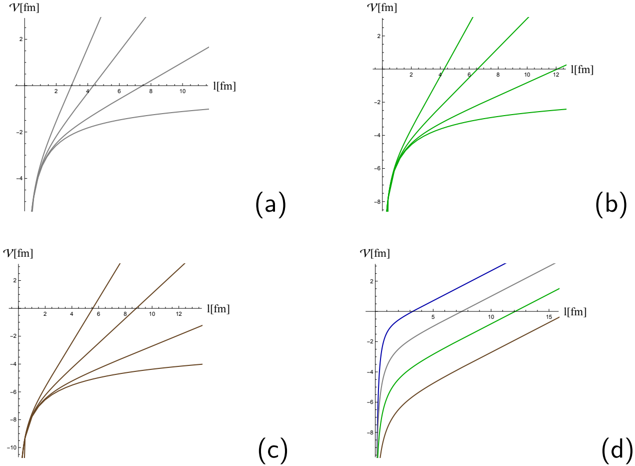


Figure: $\mathcal{V}_{x,y_{1(\infty)}}$ as a function of ℓ, $\nu = 2, 3, 4$((a),(b),(c)). The temperature $T = 30, 100, 150, 200$ MeV (from down to top) for all. In (d): $\mathcal{V}_{x,y_{1(\infty)}}$ for $\nu = 1, 2, 3, 4$ (from top to down) at $T = 100$ MeV.

Static WL. Case 2: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(y_1)$, $v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1,\, x_{(\infty)},\, ren} = \frac{L_x}{2\pi\alpha'}\frac{1}{z_*}\int\limits_{z_0/z_*}^{1}\frac{dw}{w^2}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{2+2/\nu}\right)}}-1\right]-\frac{1}{z_*}.$$

The length scale is

$$\ell = 2z_*^{1/\nu}\int\limits_0^1 \frac{w^{2/\nu}dw}{f(z_* w)\left(1-w^{2+2/\nu}\right)}$$

The pseudopotential $\mathcal{V}_{y_1,\, x_{(\infty)}} = \dfrac{S_{y_1,\, x_{(\infty)},\, ren}}{L_x}$.

For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{y_1,\, x_{(\infty)}} \underset{\ell\sim 0}{\sim} -\frac{\mathcal{C}_2(\nu)}{\ell^{\nu}}.$$

For large ℓ

$$\mathcal{V}_{y_1,\, x_{(\infty)}}(\ell,\nu) \underset{\ell\to\infty}{\sim} \sigma_{s,2}(\nu)\,\ell.$$

Static WL.Case 2: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(y_1)$, $v = v(y_1)$

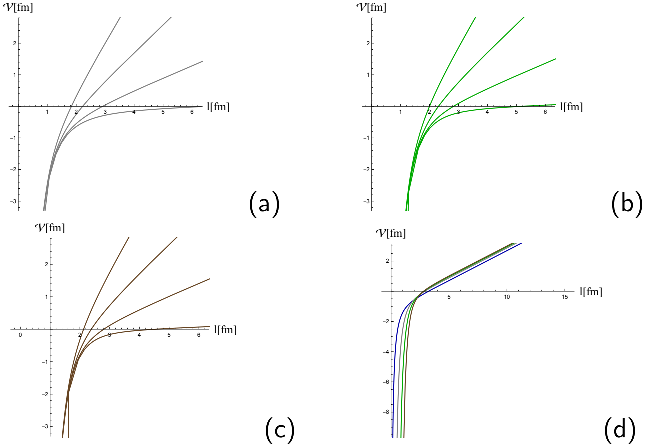


Figure: $\mathcal{V}_{y_1, x_{(\infty)}}$ as a function of ℓ for $\nu = 2, 3, 4$ ((a),(b),(c)). $T = 30, 100, 150, 200$ MeV from down to top, respectively, for all. In (d) $\mathcal{V}$ for $\nu = 1, 2, 3, 4$ (from left to right, respectively) at $T = 100$ MeV.

Static WL. Case 3: $\sigma^1 = y_1$, $\sigma^2 = y_2$, $z = z(y_1)$, $v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1 y_{2(\infty)},ren} = \frac{L_{y_2}}{2\pi\alpha'}\frac{1}{z_*^{1/\nu}}\int_{\frac{z_0}{z_*}}^{1}\frac{dw}{w^{1+1/\nu}}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{4/\nu}\right)}}-1\right]-\frac{\nu}{z_*^{1/\nu}}.$$

The length scale is

$$\ell = z_*^{1/\nu}\int\frac{dw}{w^{1-3/\nu}\sqrt{f(z_* w)\left(1-w^{4/\nu}\right)}}.$$

The pseudopotential $\mathcal{V}_{y_1,\, y_{2(\infty)}} = \dfrac{S_{y_1,\, y_{2(\infty)}}}{L_{y_2}}$.

For small ℓ –

$$\mathcal{V}_{y_1,\, y_{2(\infty)}} \underset{\ell\to 0}{\sim} -\frac{\mathcal{C}_3(\nu)}{\ell}.$$

For large ℓ

$$\mathcal{V}_{y_1,\, y_{2(\infty)}}(\ell,\nu) \underset{\ell\to\infty}{\sim} \sigma_{s,3}(\nu)\,\ell.$$

Static WL.Case 3: $\sigma^1 = y_1$, $\sigma^2 = y_2$, $z = z(y_1)$, $v = v(y_1)$

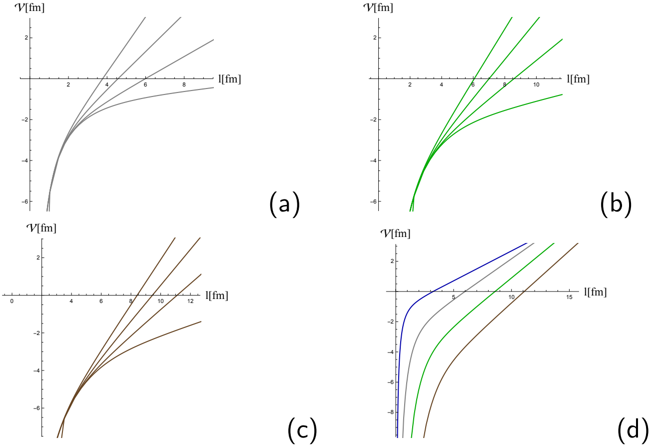


Figure: $\mathcal{V}_{y_1, y_{2(\infty)}}$ as a function of ℓ for $\nu = 2, 3, 4$ ((a),(b),(c), respectively). We take $T = 30, 100, 150, 200$ MeV from down to top, for (a),(b) and (c). In (d) $\mathcal{V}_{y_1, y_{2(\infty)}}$ for $\nu = 1, 2, 3, 4$ (from left to right) at $T = 100$ MeV.

Static WL. Spatial string tension

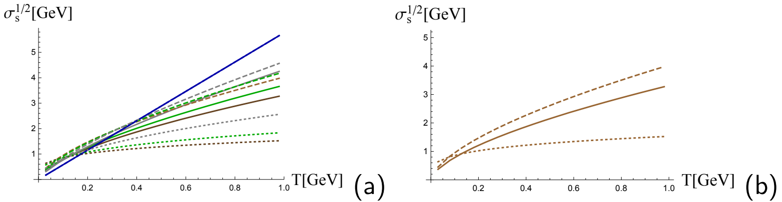


Figure: The dependence of the spatial string tension $\sqrt{\sigma_s}$ on orientation and temperature. The solid lines corresponds to the rectangular Wilson loop with a short extent in the x-direction, while the dashed lines correspond to a short extent in the y-direction. The dotted lines correspond to the rectangular Wilson loop in the transversal $y_1 y_2$ plane. (a) Blue line corresponds to $\nu = 1$, gray lines correspond to $\nu = 2$, green lines correspond to $\nu = 3$ and the brown ones correspond to $\nu = 4$. (b) The spatial string tension $\sqrt{\sigma_s}$ for different orientations for $\nu = 4$.

Alanen et al.'09,A. Dumitru et al.'13-14

WL in time-dependent backgrounds.Case 1

$$S_{x,y_{1(\infty)}}=\frac{L_y}{2\pi\alpha'}\int\frac{dx}{z^{1+1/\nu}}\sqrt{1-f(z,v)v'^{\,2}-v'z'},\quad \prime\equiv\frac{d}{dx}.$$

The corresponding equations of motion are

$$\begin{aligned}
v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2+\frac{(\nu+1)}{\nu z}(1-fv'^2-2v'z'),\\
z'' &= -\frac{\nu+1}{\nu}\frac{f}{z}+\frac{\nu+1}{\nu}\frac{f^2v'^2}{z}-\frac{1}{2}\frac{\partial f}{\partial v}v'^2-\frac{1}{2}fv'^2\frac{\partial f}{\partial z}-v'z'\frac{\partial f}{\partial z},\\
&+ 2\frac{(\nu+1)}{\nu z}fv'z'.
\end{aligned}$$

The boundary conditions $z(\pm\ell)=0$, $v(\pm\ell)=t$. The initial conditions $z(0)=z_*$, $v(0)=v_*$, $z'(0)=0, v'(0)=0$. The pseudopotential is

$$\mathcal{V}_{x,y_{1(\infty)}}=\frac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}$$

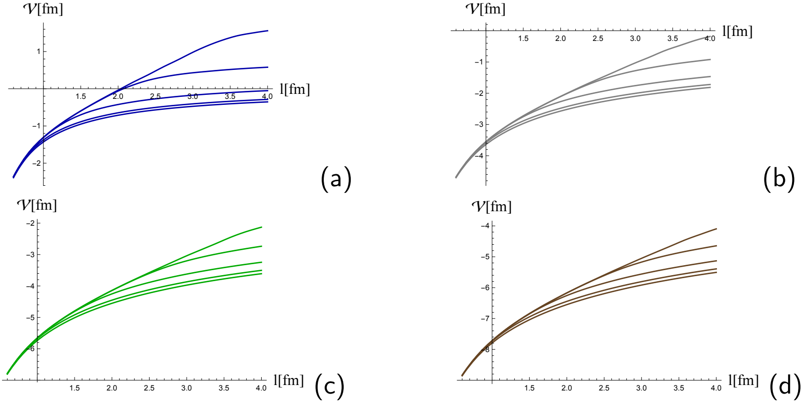


Figure: $\mathcal{V}_{x,y_{1(\infty)}}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to time $t = 0.1, 0.5, 0.9, 1.4, 2$ (from down to top, respectively).

$$\delta\mathcal{V}_1(x,t)=\mathcal{V}_{x,y_{1(\infty)}}(x,t)-\mathcal{V}_{x,y_{1(\infty)}}(x,t_f).$$

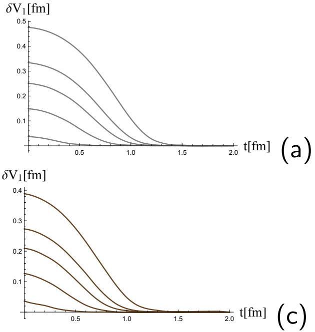


Figure: The time dependence of $-\delta\mathcal{V}_1(x,t)$, for different values of the length ℓ, $\nu=2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell=0.7,1.2,1.5,1.7,2$ (from down to top, respectively). In (d) we have shown $-\delta\mathcal{V}_1(x,t)$ as a function of t at $\ell=2$ for $\nu=1,2,3,4$ (from top to down).

WL in time-dependent backgrounds.Case 2

$$S_{y_1,\,x_{(\infty)}} = \frac{L_x}{2\pi\alpha'}\int dy_1 \frac{1}{z^2}\sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(z,v)(v')^2 - 2v'z'\right)}, \quad {}' \equiv \frac{d}{dy_1}.$$

The corresponding equations of motion are

$$\begin{aligned}
v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{\nu+1}{\nu z}\left(z^{2-2/\nu} - \frac{2\nu}{(1+\nu)}fv'^2 - 2v'z'\right), \\
z'' &= -\frac{\nu+1}{\nu}fz^{1-2/\nu} + \frac{2(\nu-1)z'^2}{\nu} + \frac{2}{\nu}\frac{f^2v'^2}{z} - \frac{1}{2\nu}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2\nu}f\frac{\partial f}{\partial z}v'^2 \\
&- z'v'\frac{\partial f}{\partial z} + \frac{4}{z}fz'v'.
\end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1,\,x_{(\infty)}} = \frac{S_{y_1,\,x_{(\infty)},ren}}{L_{y_1}}.$$

WL in time-dependent backgrounds.Case 2

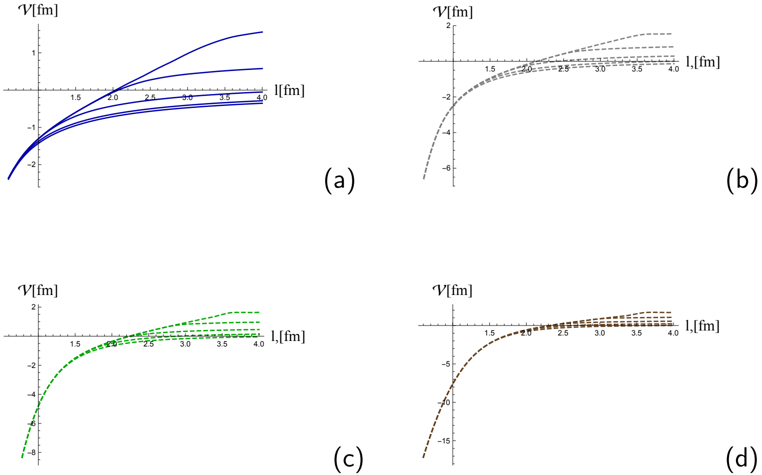


Figure: $\mathcal{V}_{y_1 x_\infty}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top.

WL in time-dependent backgrounds.Case 2

$$\delta\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)=\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)-\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t_f).$$

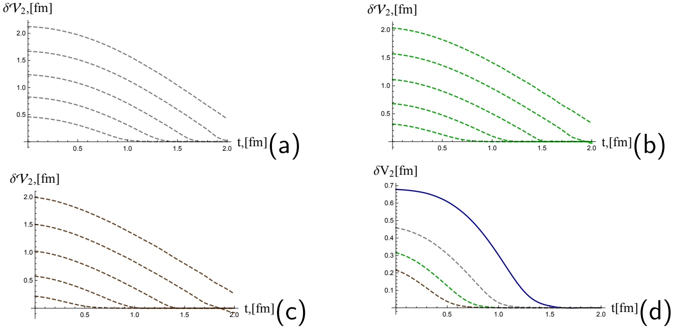


Figure: The time dependence of $-\delta\mathcal{V}_{y_1,\,x_{(\infty)}}(x,t)$ for different values of the length ℓ, $\nu=2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell=2,2.5,3,3.5,4$ (from down to top, respectively). In (d) $-\delta\mathcal{V}_2(x,t)$ as a function of t at $\ell=2$ for $\nu=1,2,3,4$ (from top to down, respectively).

WL in time-dependent backgrounds.Case 3

$$S_{y_1,\,y_{2,(\infty)}}=\frac{L_{y_2}}{2\pi\alpha'}\int dy_1\,\frac{1}{z^{1+1/\nu}}\sqrt{\left(\frac{1}{z^{2/\nu-2}}-f(v')^2-2v'z'\right)}.$$

The corresponding equations of motion are

$$\begin{aligned}
v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2+\frac{2}{z\nu}\left(z^{2-2/\nu}-\frac{\nu+1}{2}fv'^2-2v'z'\right),\\
z'' &= -\frac{2}{\nu}fz^{1-2/\nu}+2\frac{\nu-1}{\nu}\frac{z'^2}{z}+\frac{\nu+1}{\nu z}f^2v'^2-\frac{1}{2}\frac{\partial f}{\partial v}v'^2-\frac{1}{2}f\frac{\partial f}{\partial z}v'^2\\
&- z'v'\frac{\partial f}{\partial z}+\frac{2(\nu+1)}{\nu z}fv'z'. \qquad (3)
\end{aligned}$$

The boundary conditions $z(\pm\ell)=0$, $v(\pm\ell)=t$. The initial conditions $z(0)=z_*$, $v(0)=v_*$, $z'(0)=0, v'(0)=0$. The pseudopotential is

$$\mathcal{V}_{y_1,\,y_{2,(\infty)}}(t,\ell)=\frac{S_{y_1,\,y_{2,(\infty)},ren}}{L_{y_2}}.$$

WL in time-dependent backgrounds.Case 3

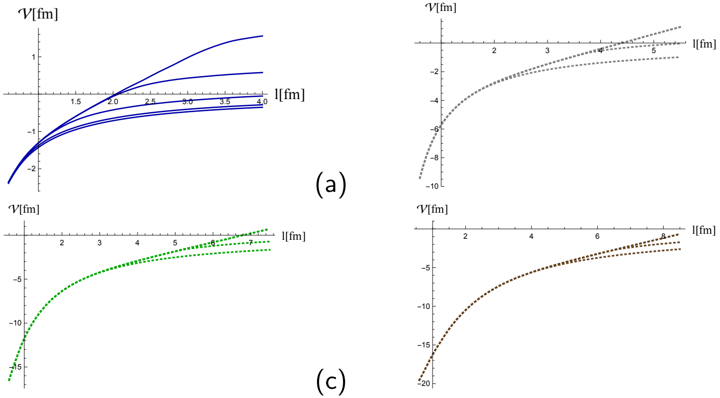


Figure: $\mathcal{V}_{y_1,\, y_{2,(\infty)}}(l,t)$ as a function of the length ℓ at fixed values of t, $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d)). (a): we take $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top, respectively; for plots (b),(c),(d): $t = 0.4, 1.5, 2.5, 3.34, 4$ from down to top, respectively.

WL in time-dependent backgrounds.Case 3

$$\delta\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t)=\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t)-\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t_f).$$

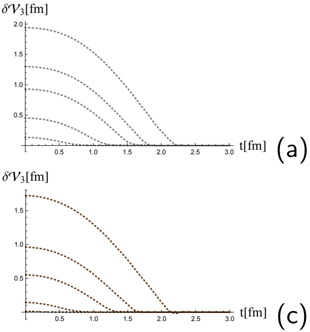


Figure: $-\delta\mathcal{V}_{y_1,\,y_{2,(\infty)}}(x,t)$ on t for different ℓ, $\nu=2,3,4$ ((a),(b),(c)). (a): $l=2.2,3,3.85,4.4,5.2$ from top to down; (b): $l=3,4.1,5.2,6,7.1$ from top to down; (c): $l=3.4,4.6,5.9,6.8,8$ from top to down. In (d): $-\delta\mathcal{V}_3(x,t)$ as a function of t at $\ell=3$ for $\nu=1,2,3,4$ (from top to down, respectively).

Thank you for your attention!