

Hadron Structure, Hadronic Matter, and Lattice QCD

Phases of QCD, topology and axions - II

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I Symmetries and phases
of QCD in the
Temperature, N_f space

II Results on the phase diagram

III Topology - broken phase

IV Topology - hot QCD & axions

II Results on the phase diagram

II.1 The magnetic EoS
at finite temperature.

II.2 The conformal phase

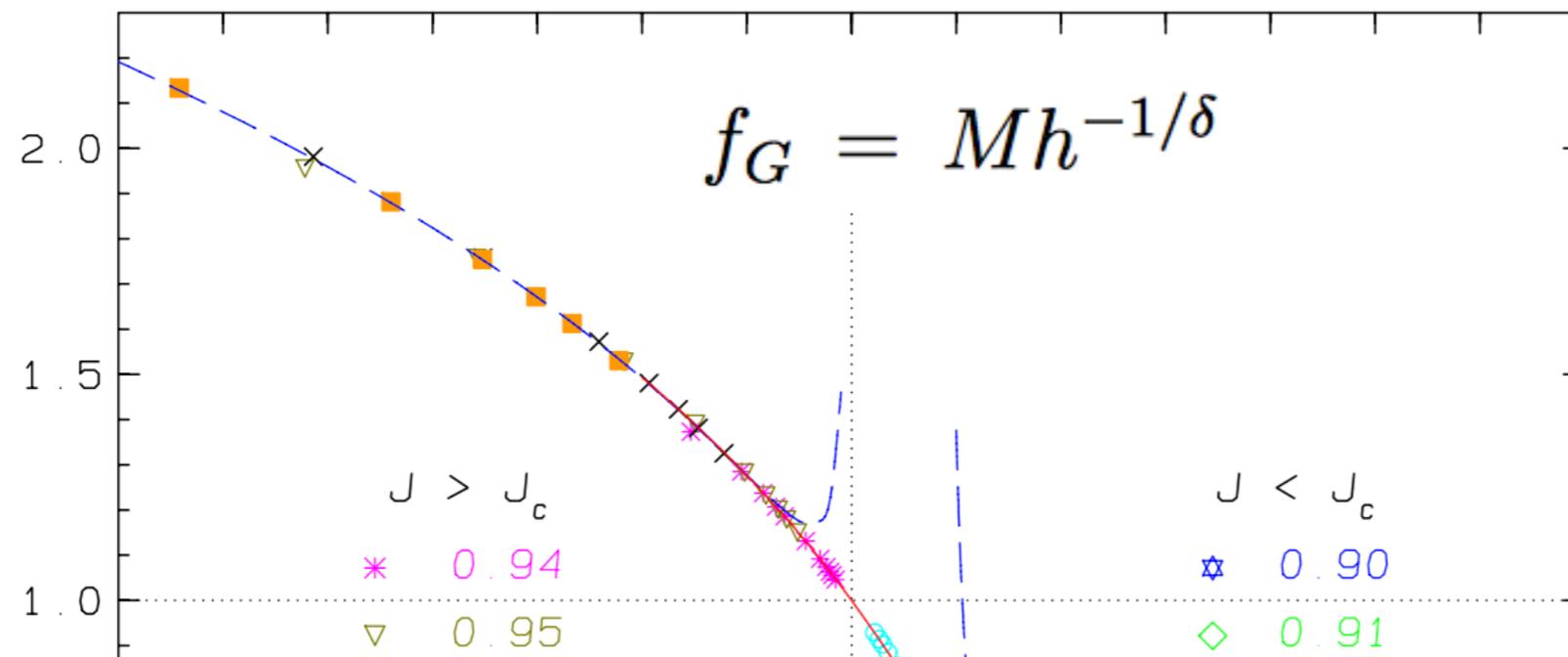
II.3 The preconformal phase

II.1 Magnetic EoS in QCD at finite temperature

3d O(4) model

$$\beta \mathcal{H} = -J \sum_{\langle \vec{x}, \vec{y} \rangle} \vec{\phi}_{\vec{x}} \cdot \vec{\phi}_{\vec{y}} - \vec{H} \cdot \sum_{\vec{x}} \vec{\phi}_{\vec{x}}$$

$$M = h^{1/\delta} f_G(z)$$



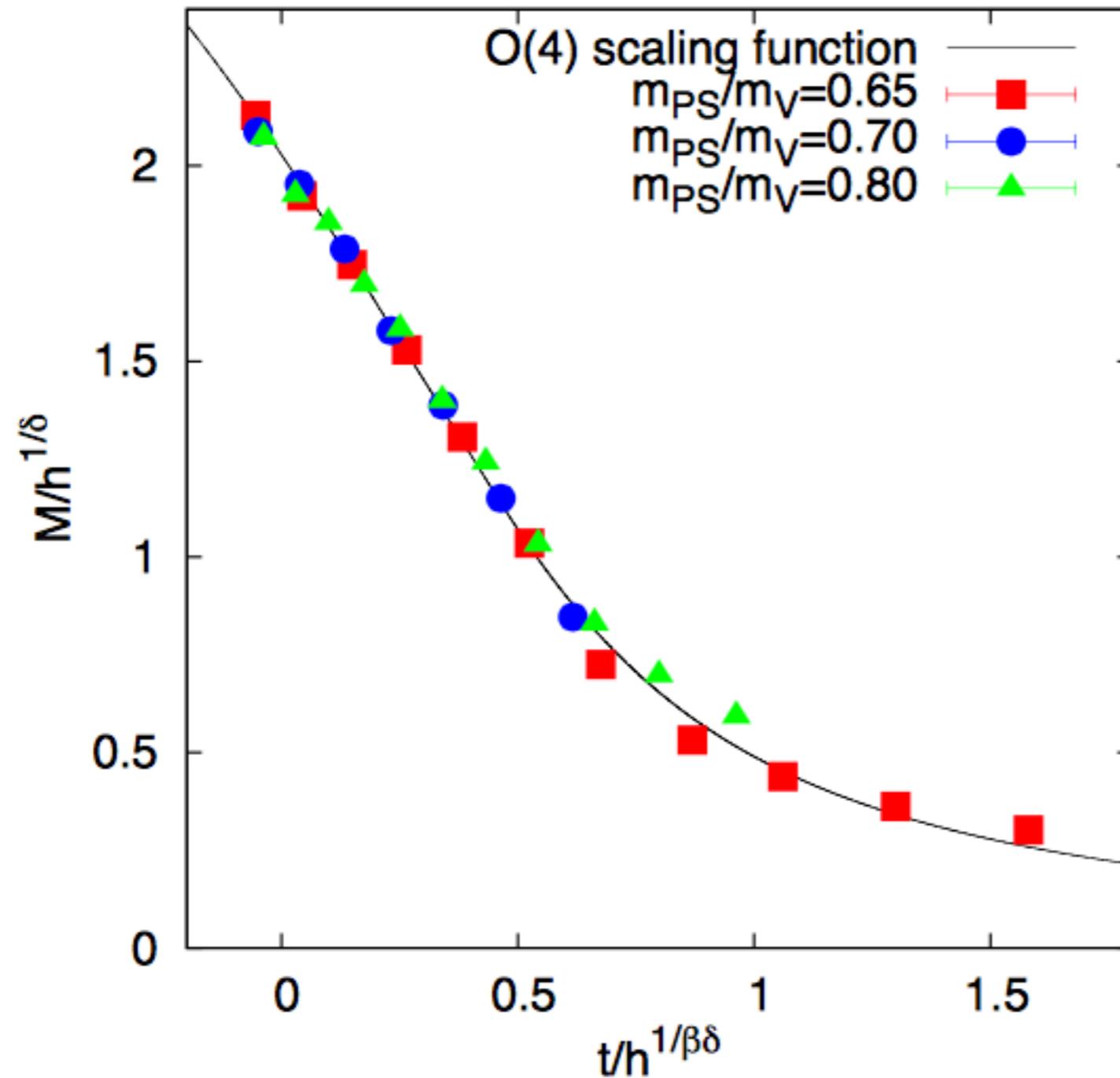
Building & parametrizing the scaling function $f(z)$

Karsch Engels 2012

$$z = \bar{t}/h^{1/\Delta} \quad \Delta \equiv \beta\delta$$

O(4) scaling analysis in two-flavor QCD at finite temperature and density with improved Wilson quarks

Umeda et al. 2017



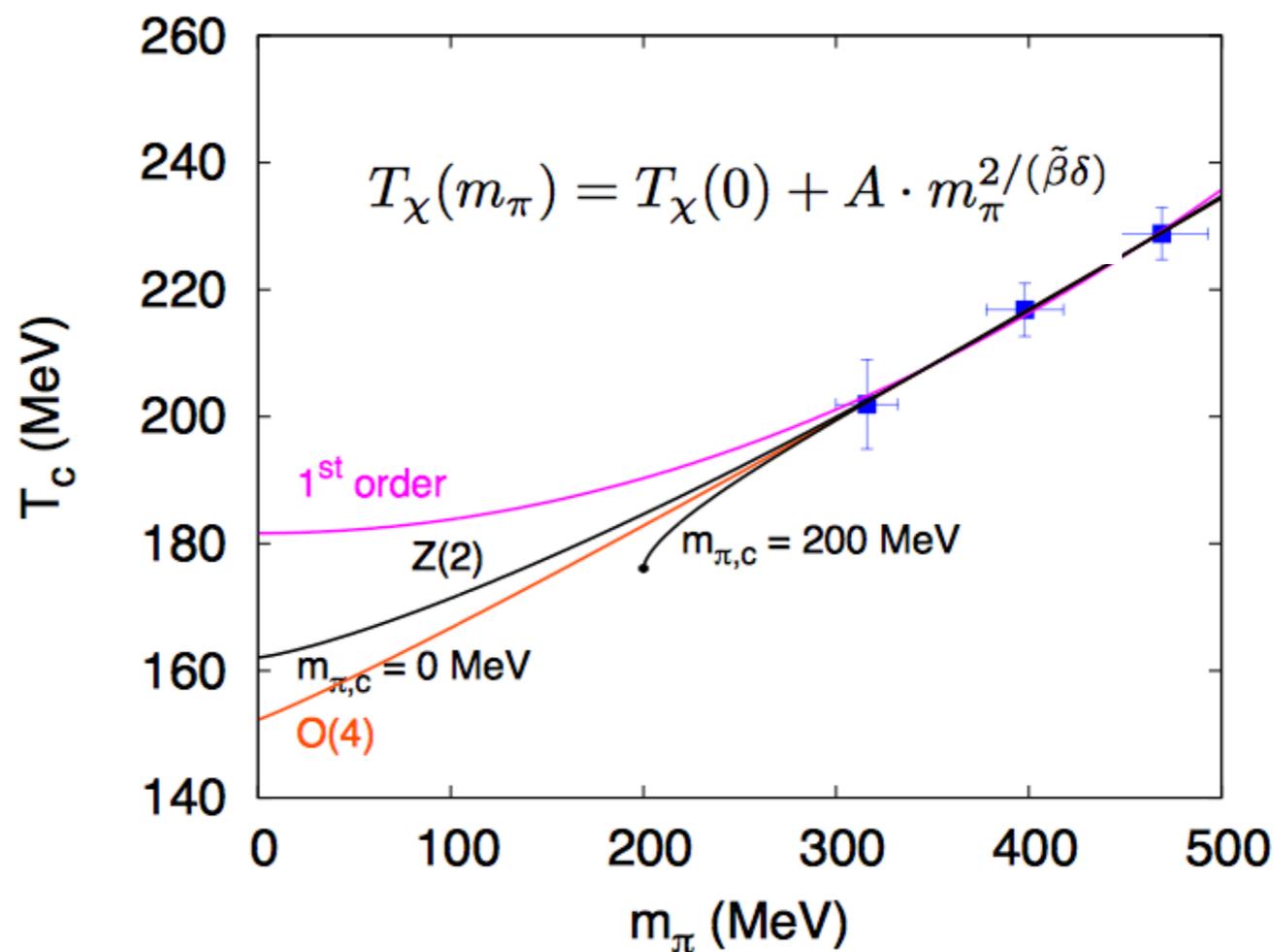
Apparently nice O(4) scaling for the order parameter

Twisted mass
at finite temperature
collaboration,
 $N_f=2$

Hard to discriminate
between different
univ. classes

$$M/h^{1/\delta} = f(t/h^{1/\beta\delta})$$

gives
scaling
of
 $T_\chi(m_\pi)$



Chiral extrapolation for $T_\chi(m_\pi)$ for various scenarios

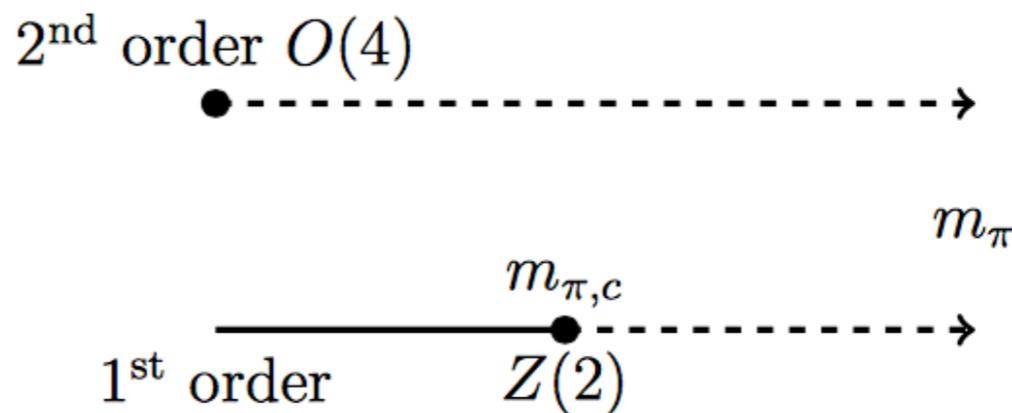
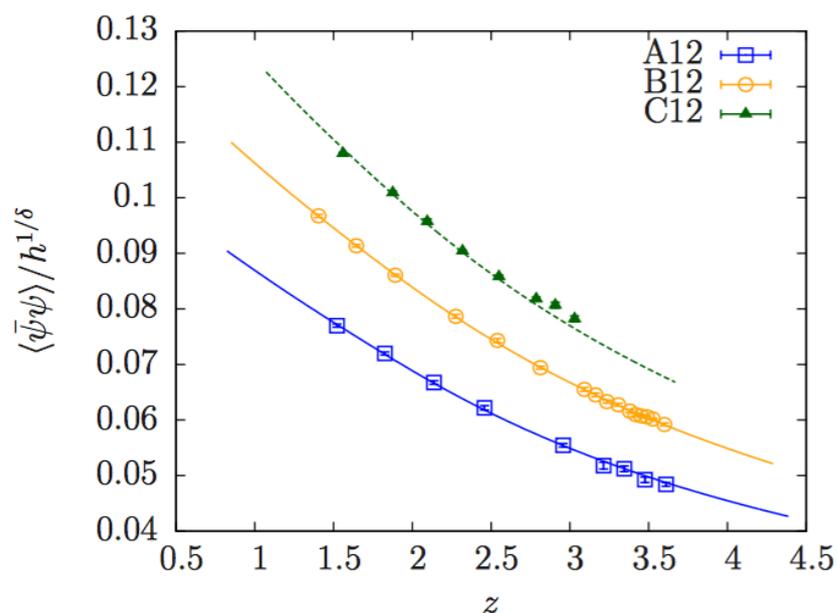
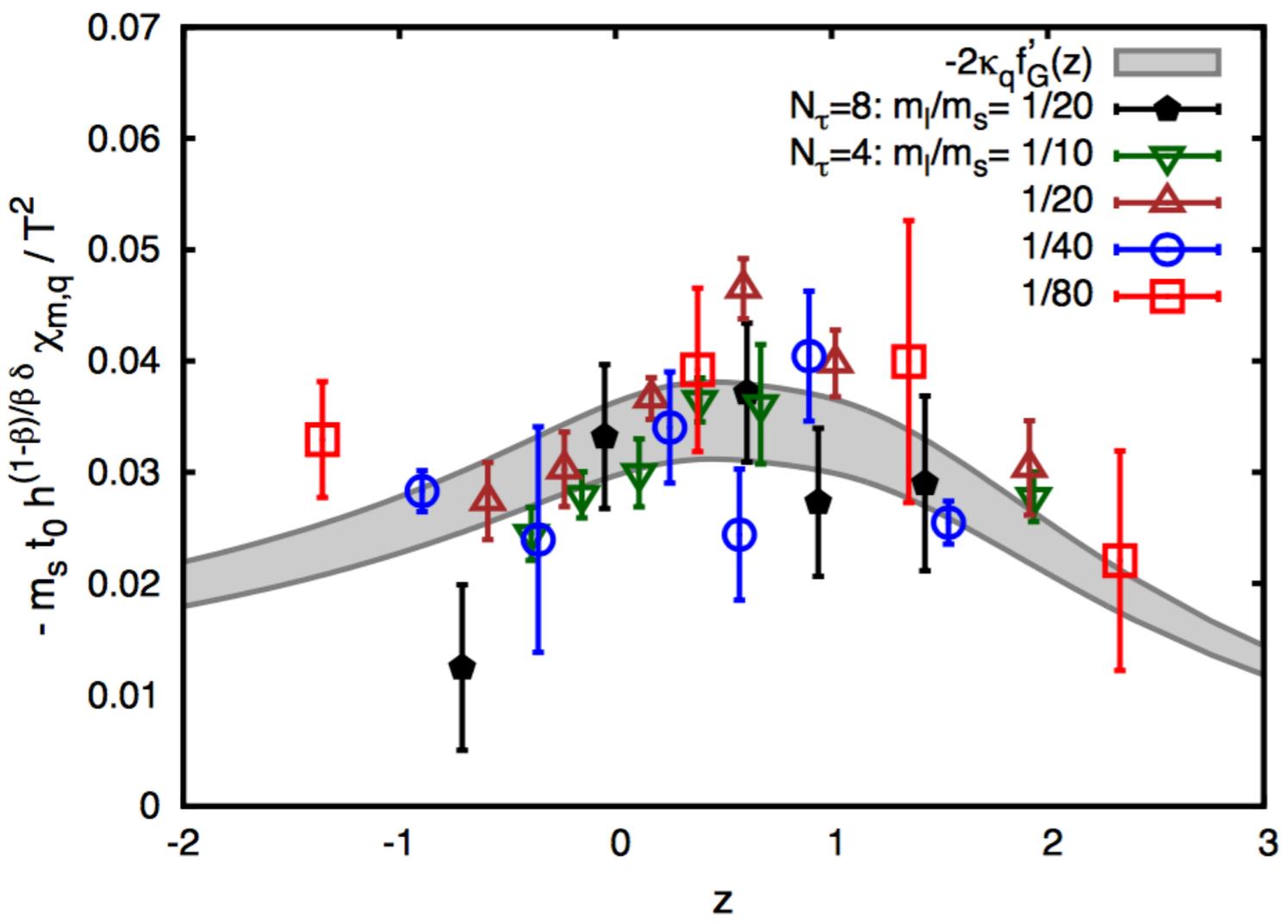


Illustration of possible scenarios for the $N_f = 2$
chiral limit.

Phase boundary for the chiral transition in (2+1)-flavor QCD at small values of the chemical potential

O. Kaczmarek et al. 2011



The EoS extended at finite μ

$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)$$

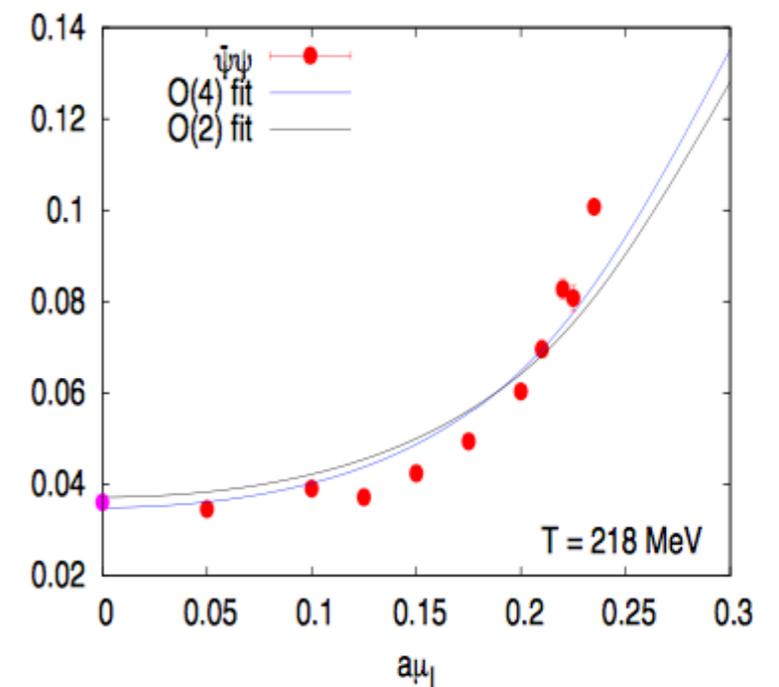
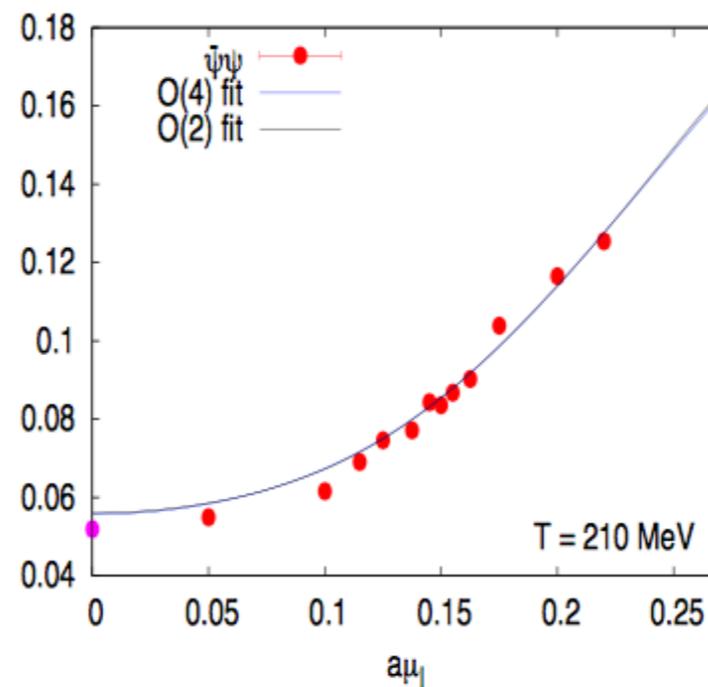
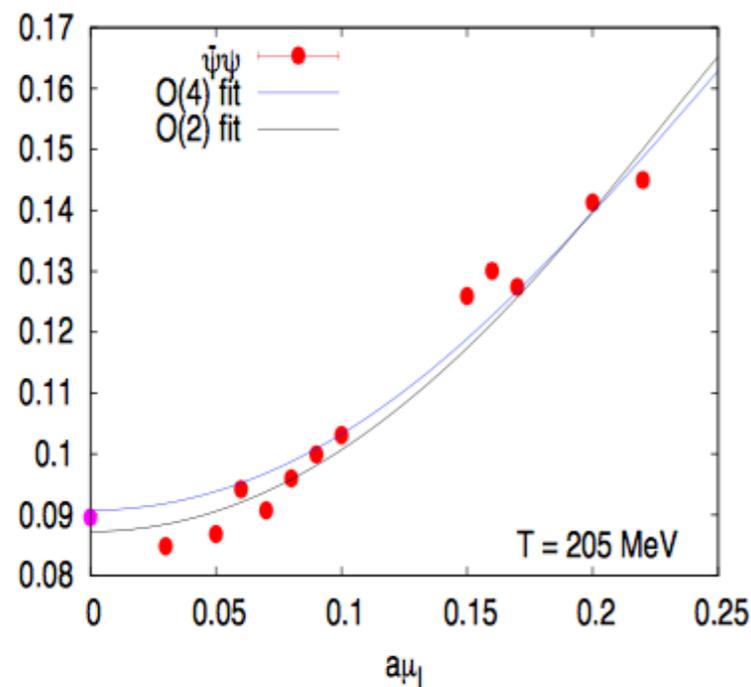
determines slope of the critical line

$$\frac{\chi_{m,q}}{T} = \frac{2\kappa_q T}{t_0 m_s} h^{-(1-\beta)/\beta\delta} f'_G(z)$$

Making the most of Taylor expansion and imaginary μ

Same strategy may be used
at imaginary μ

Laermann, Meyer, MpL 2013



within largish error, either O(2) and O(4)
universality classes nicely describe the data
and allow an estimate of the slope.

II.2 Establishing the conformal window



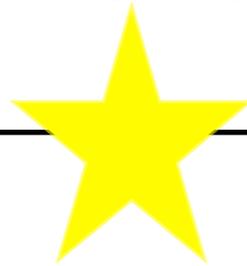
Similarities and differences between a conformal PT and a 2nd order one

Conformal transition

IR IR IR IR IR.....



(X power-law)



Conformal scaling

*Analogies
in the broken phase*

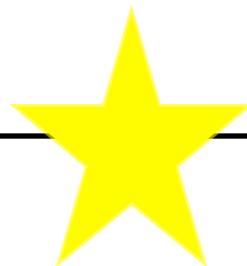
*Differences
in the symmetric phase*

EoS

IR

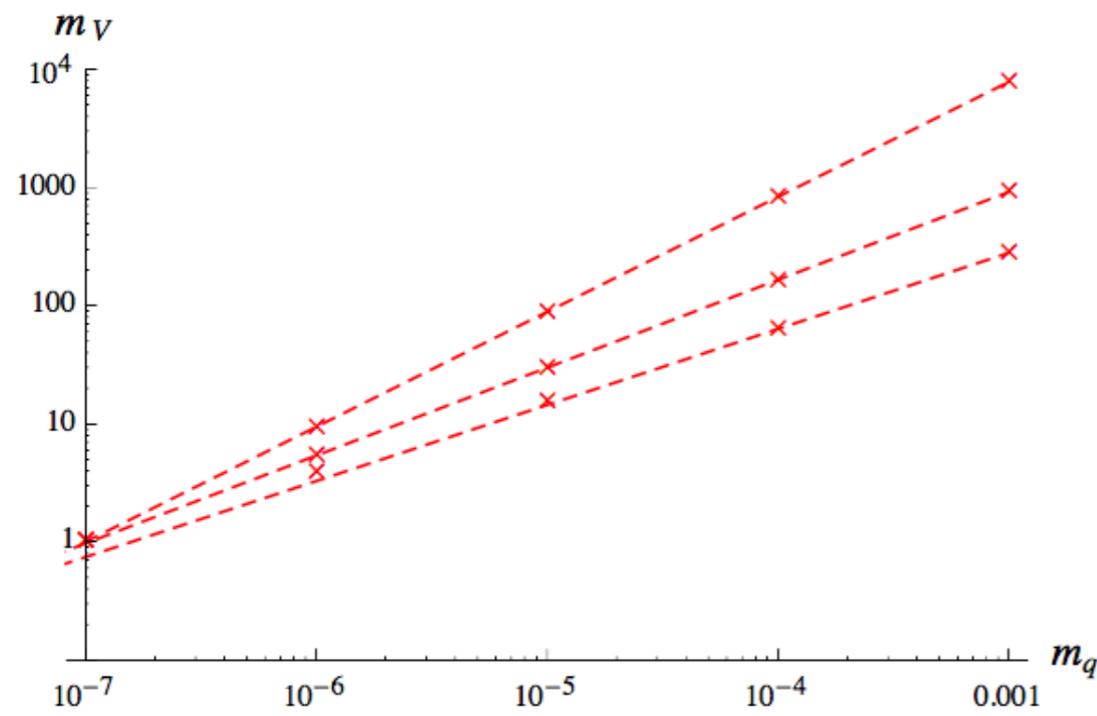


2nd order transition



Griffith's analyticity

Conformal scaling



$$M_H = c_H m^{1/y_h}$$

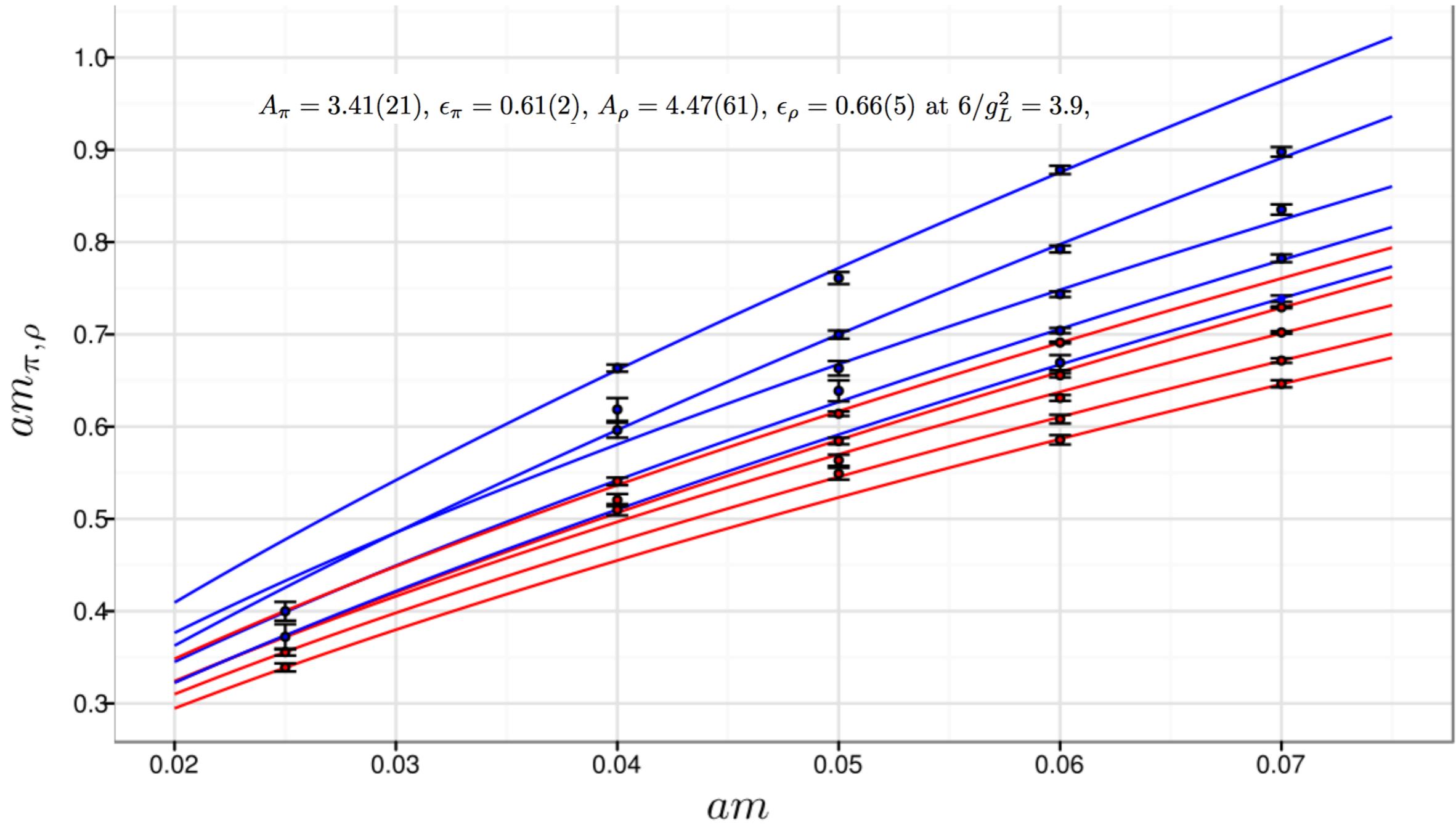


Power law Scaling with anomalous dimension

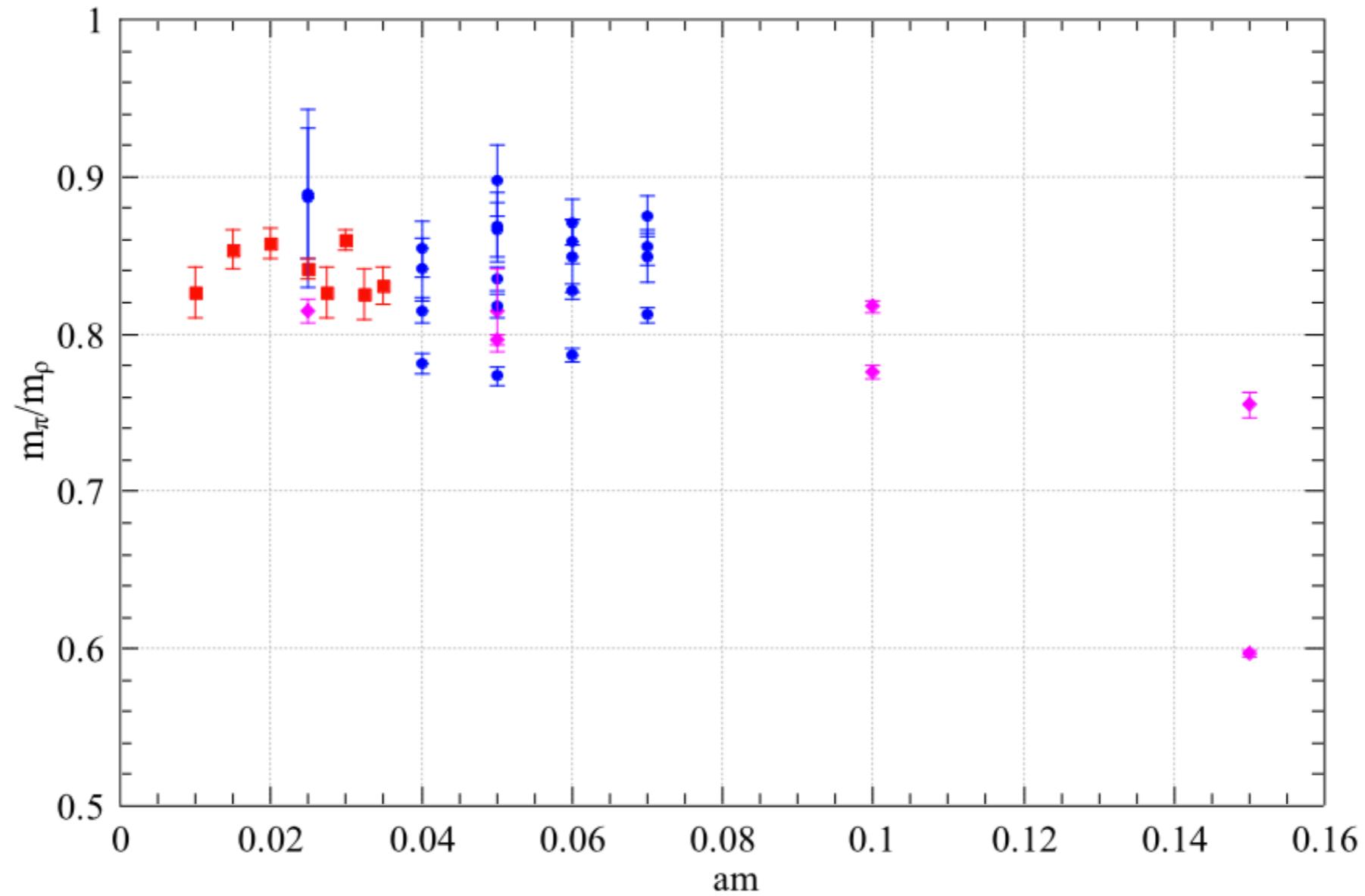
Alho Evans Tuominen 2014

$$m_{\pi,\rho} = A_{\pi,\rho} m^{\epsilon_{\pi,\rho}}$$

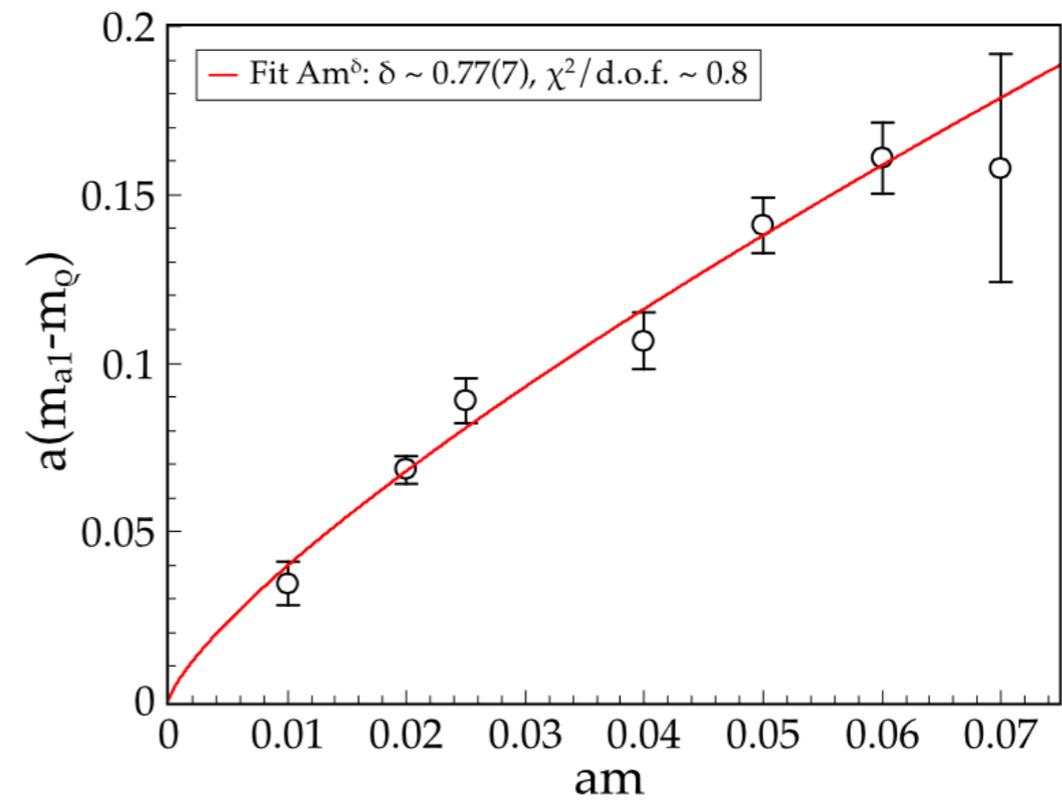
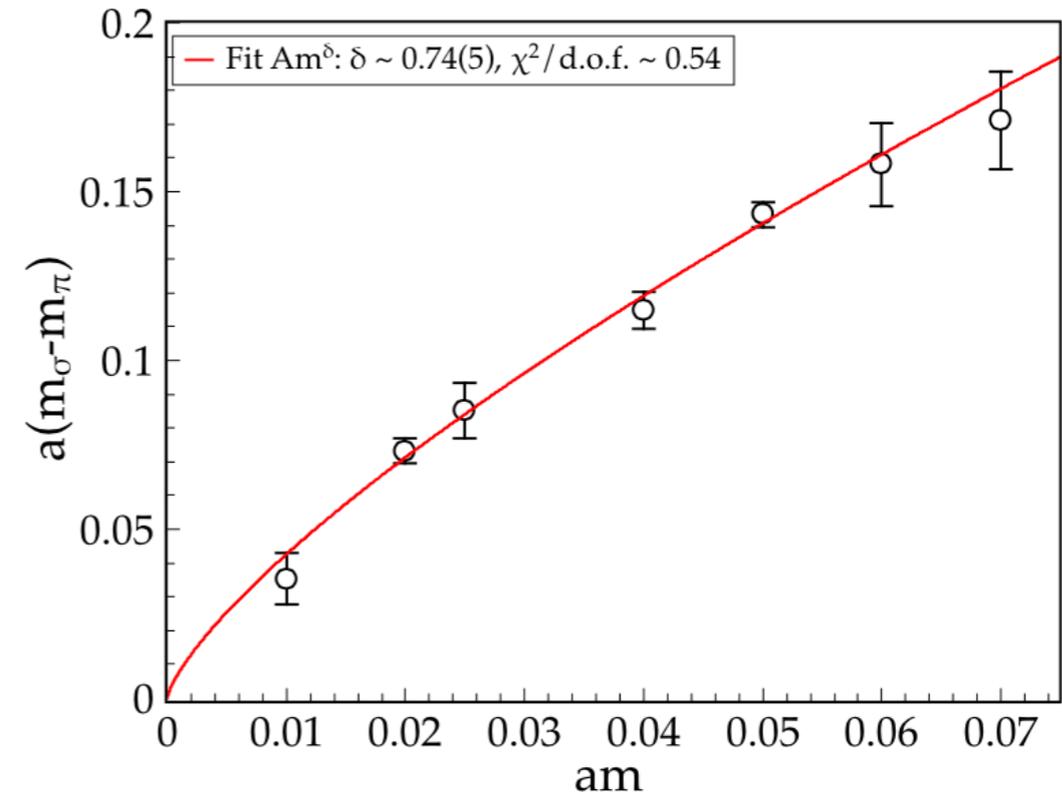
$A_\pi = 3.41(21)$, $\epsilon_\pi = 0.61(2)$, $A_\rho = 4.47(61)$, $\epsilon_\rho = 0.66(5)$ at $6/g_L^2 = 3.9$,



$m_{\pi,\rho} = A_{\pi,\rho} m^{\epsilon_{\pi,\rho}}$: mass ratios m-independent in the chiral limit



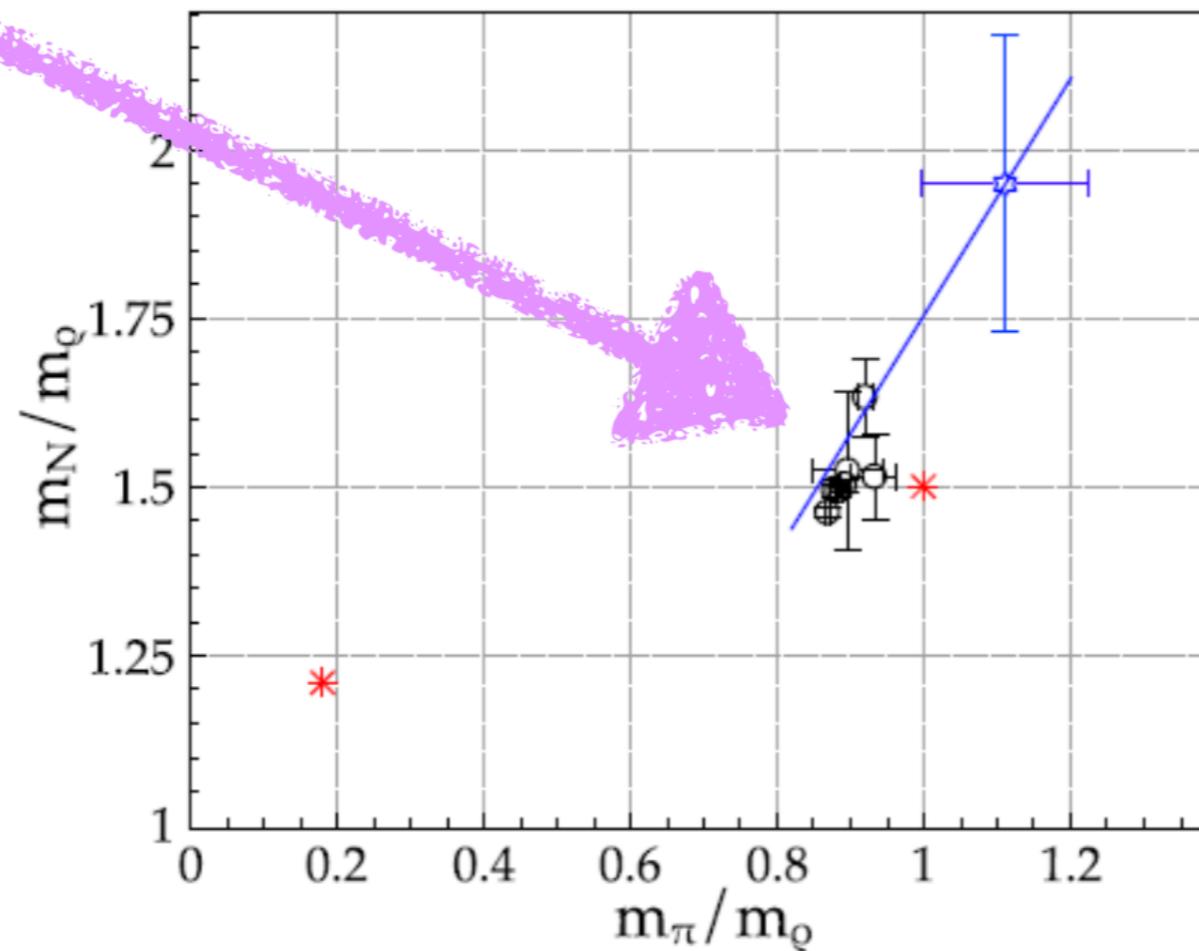
Degeneracy of the
chiral partners
towards the chiral limit



Ratios in the conformal window at a glance: the Edinburgh plot

Nf=12

Exact conformality: point like



Data show some deviation from exact conformality

da Silva, Miura, Pallante, MpL

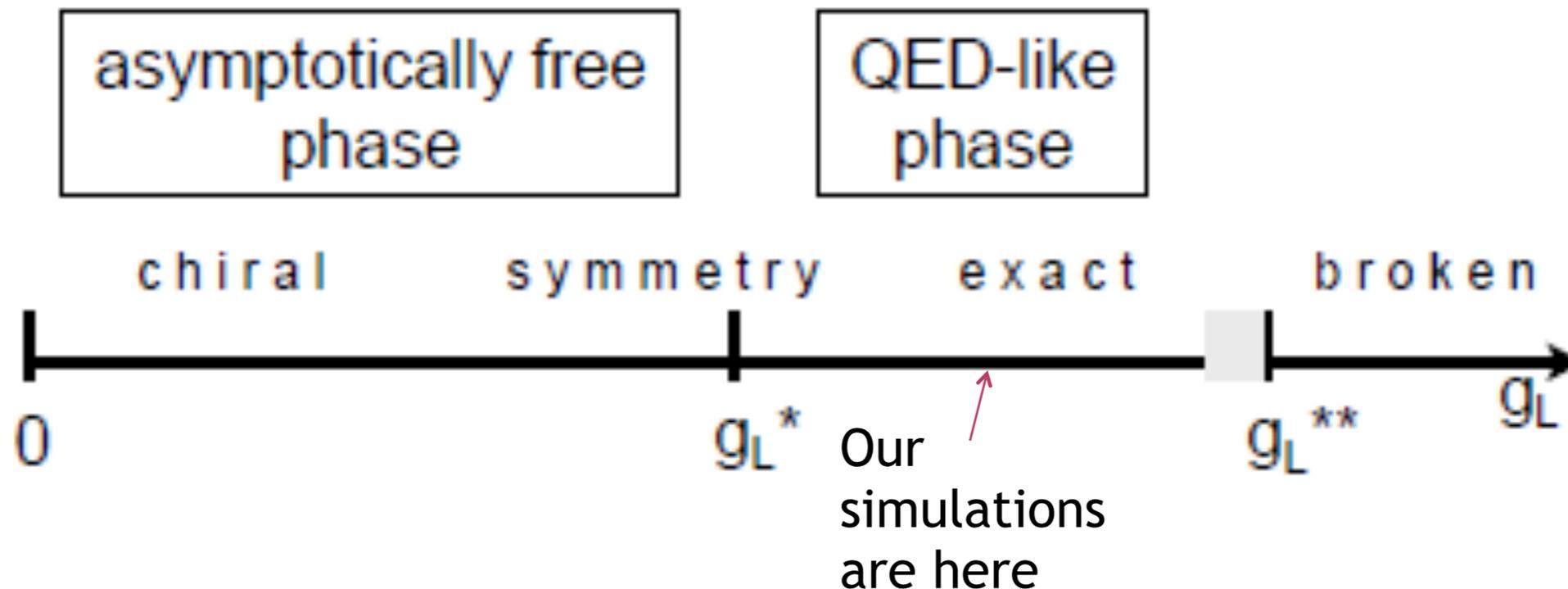
Lattice corrections to conformal scaling

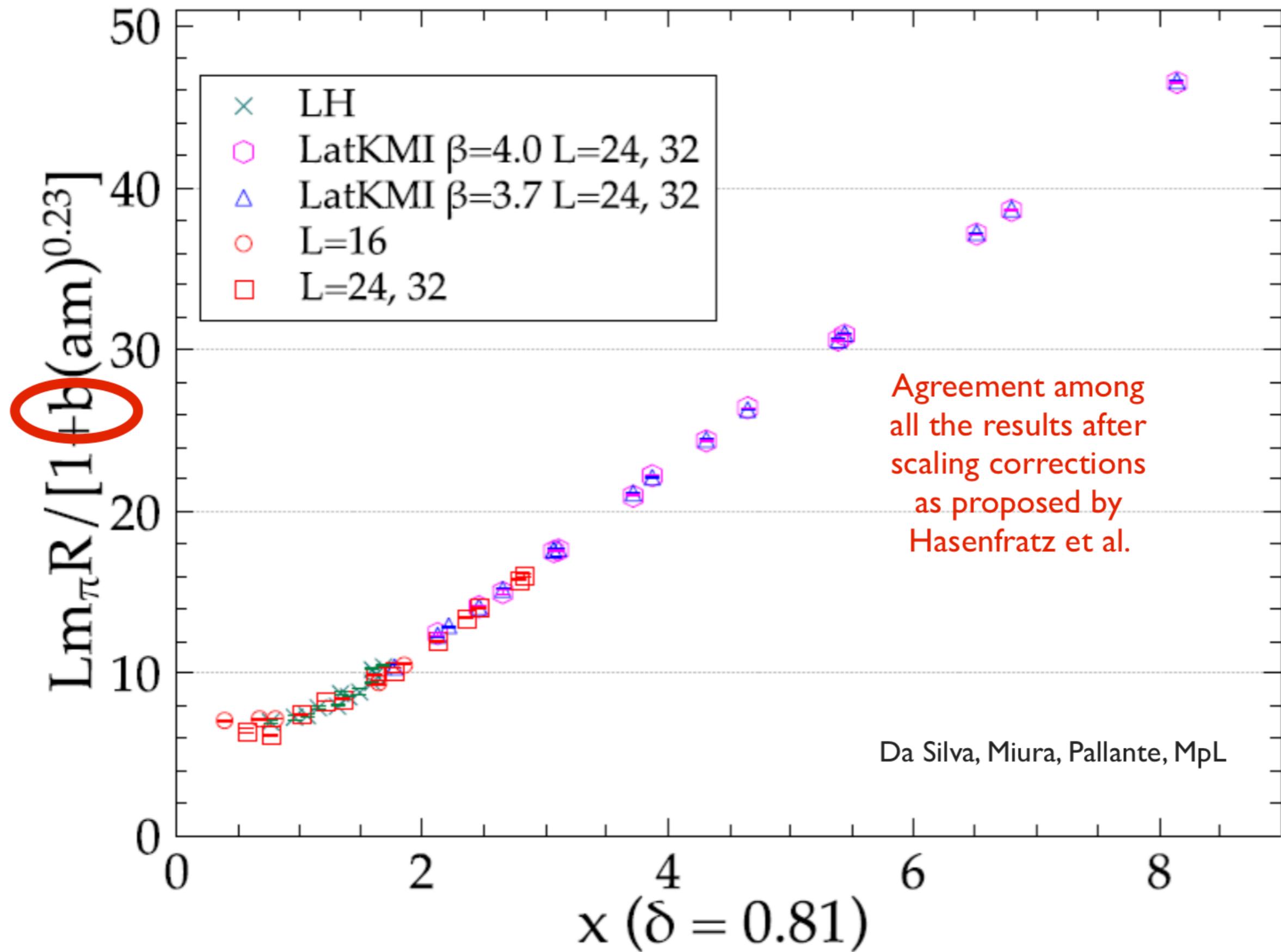
1: Size $M_H = L^{-1} f_H(x), \quad x \equiv Lm^{1/y_m}$

2: Coupling $M_H = L^{-1} f_H(x, g_0 m^\omega)$

Del Debbio, Zwicky;
Hasenfratz et al;
MpL, da Silva, Miura, Pallante

$$LM_H = F_H(x) \left\{ 1 + g_0 m^\omega G_H(x) + \mathcal{O}(g_0^2 m^{2\omega}) \right\}$$

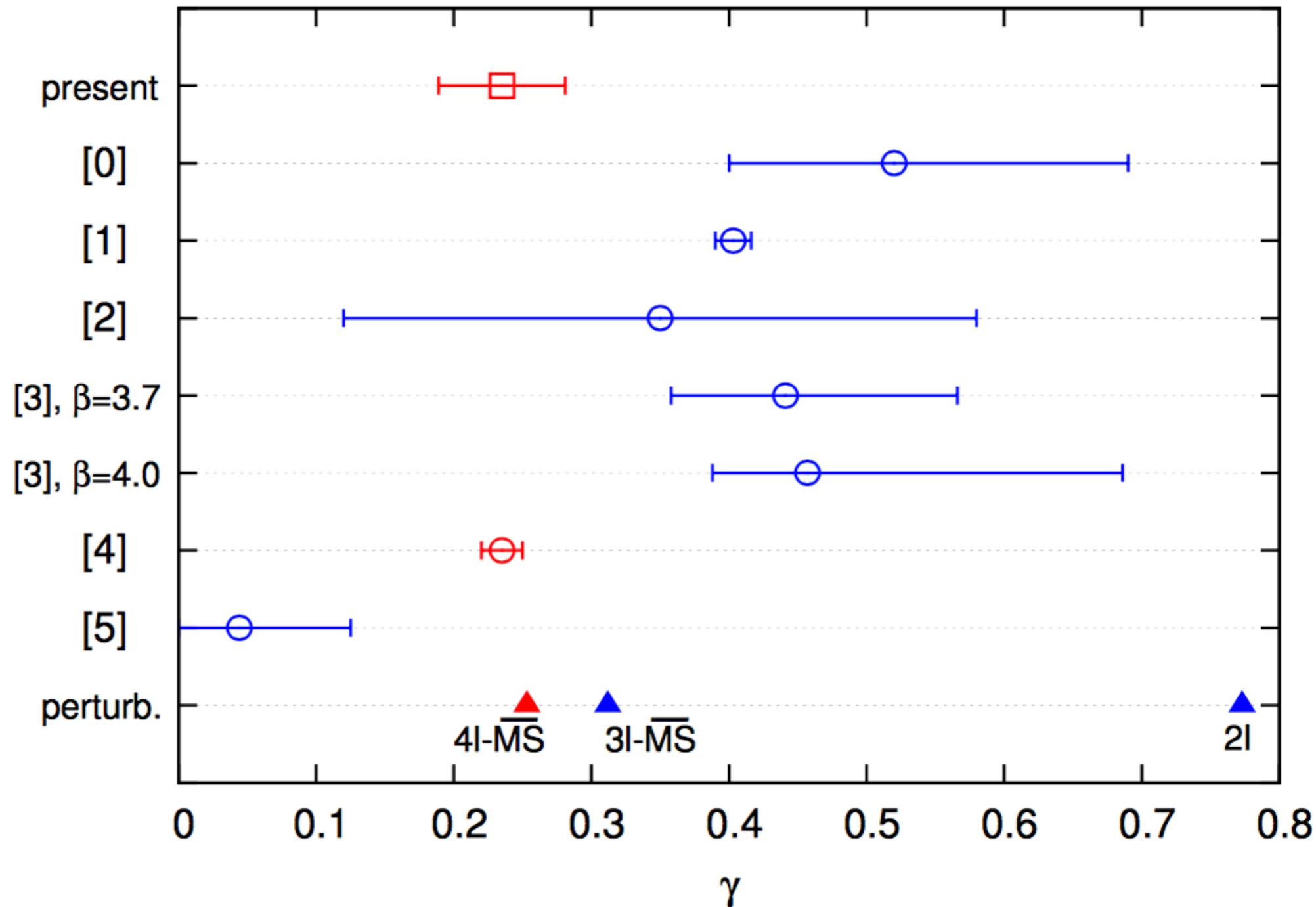




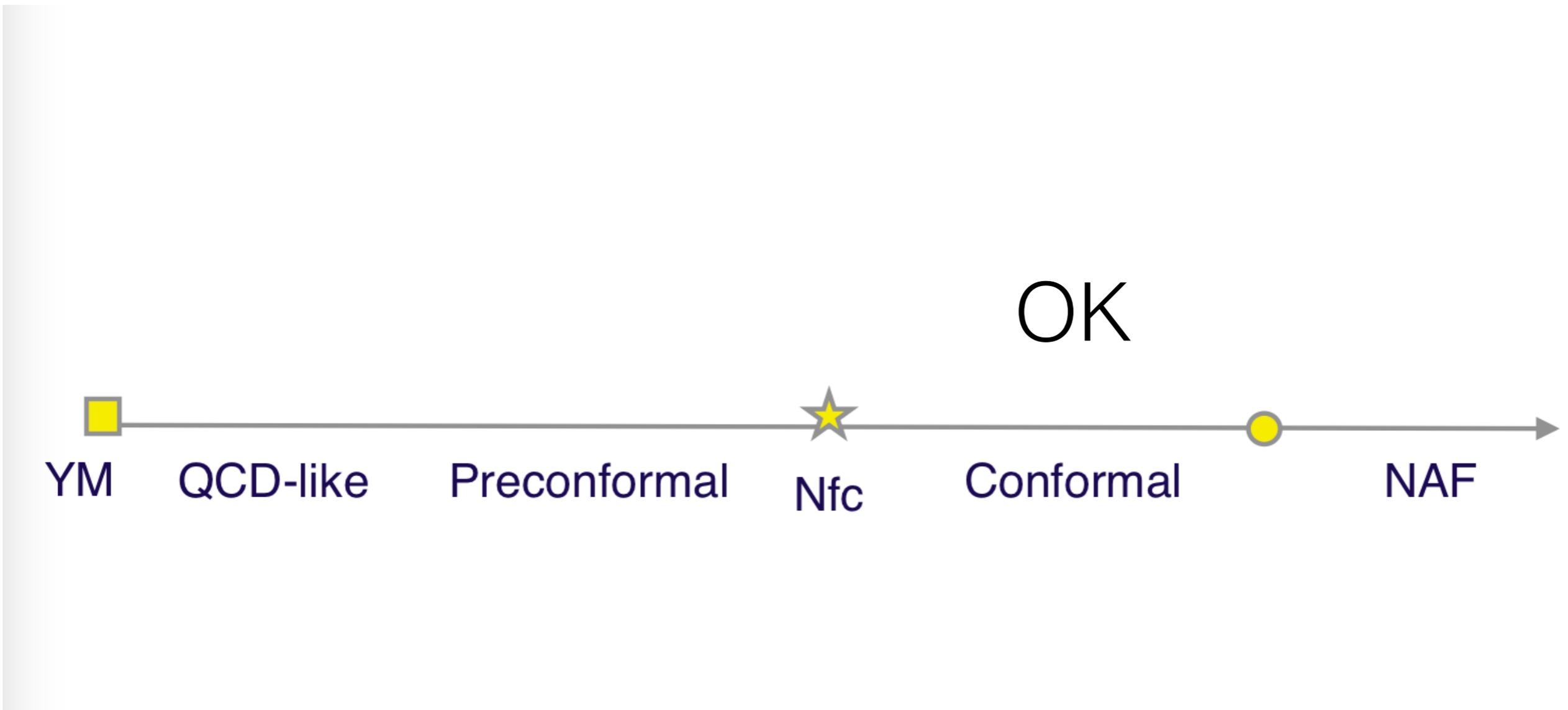
Compilation of results for the anomalous dimension, $N_f=12$

Blue: raw
Red: corrected
(scaling violations taken into account)

- [0] DPL, 2009
- [1] LSD coll. 2011
- [2] De Grand et al. 2011
- [3] KMI coll. 2012
- [4] Cheng et al. 2014
- [5] Itou 2013



MpL, Miura, Nunes da Silva, Pallante 2014



YM

QCD-like

Preconformal

Nfc

Conformal

NAF

OK

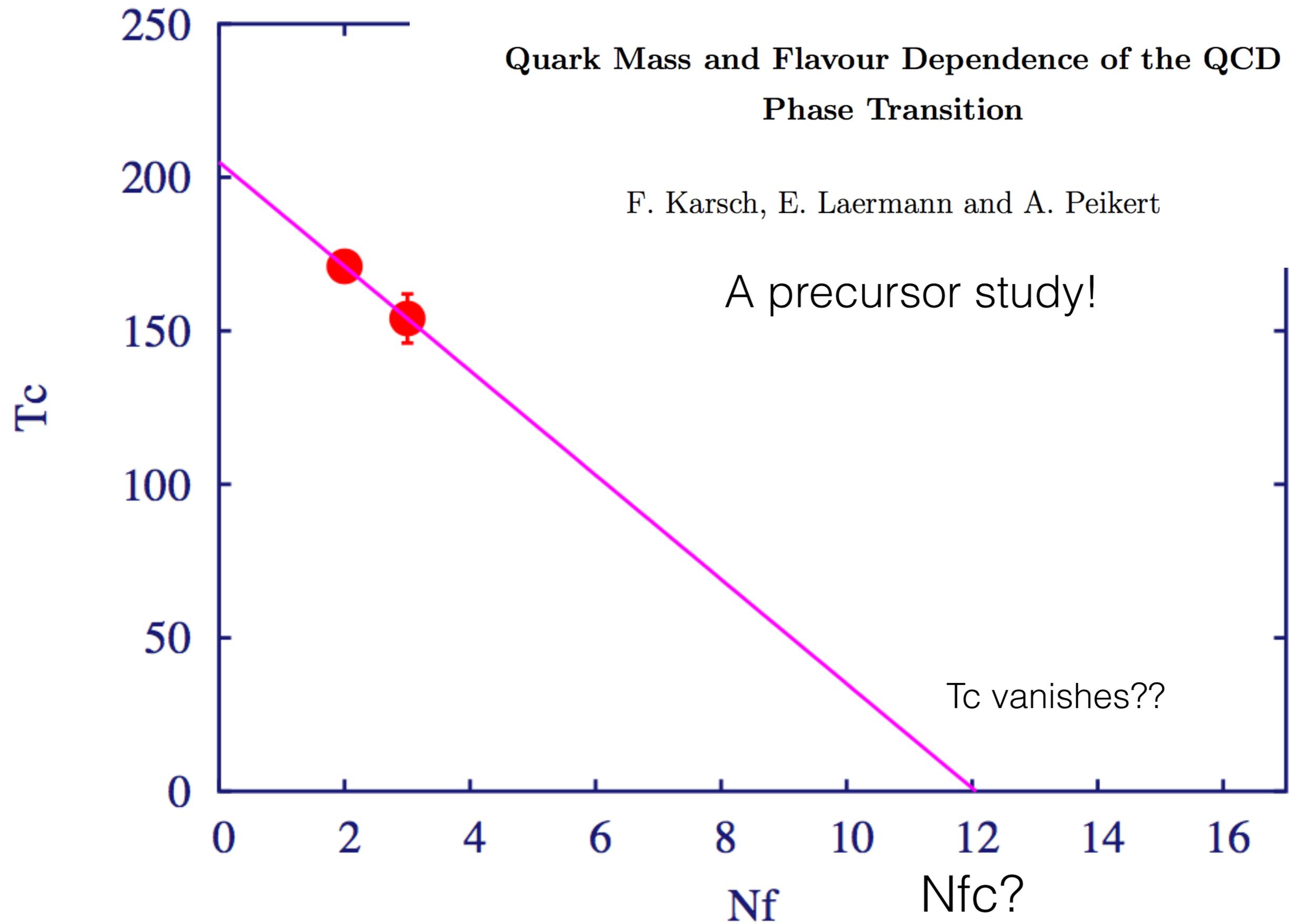
II.3 The preconformal window



Quark Mass and Flavour Dependence of the QCD Phase Transition

F. Karsch, E. Laermann and A. Peikert

A precursor study!



Standard picture of scale separation

$$\Lambda_{\text{IR}}/\Lambda_{\text{UV}} = \mathcal{O}(1).$$

Λ_{UV}

In the conformal phase IR scales vanish but UV ones survive

Λ_{IR}

N_{fc}

$$\frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

$$x = N_f/N_c$$

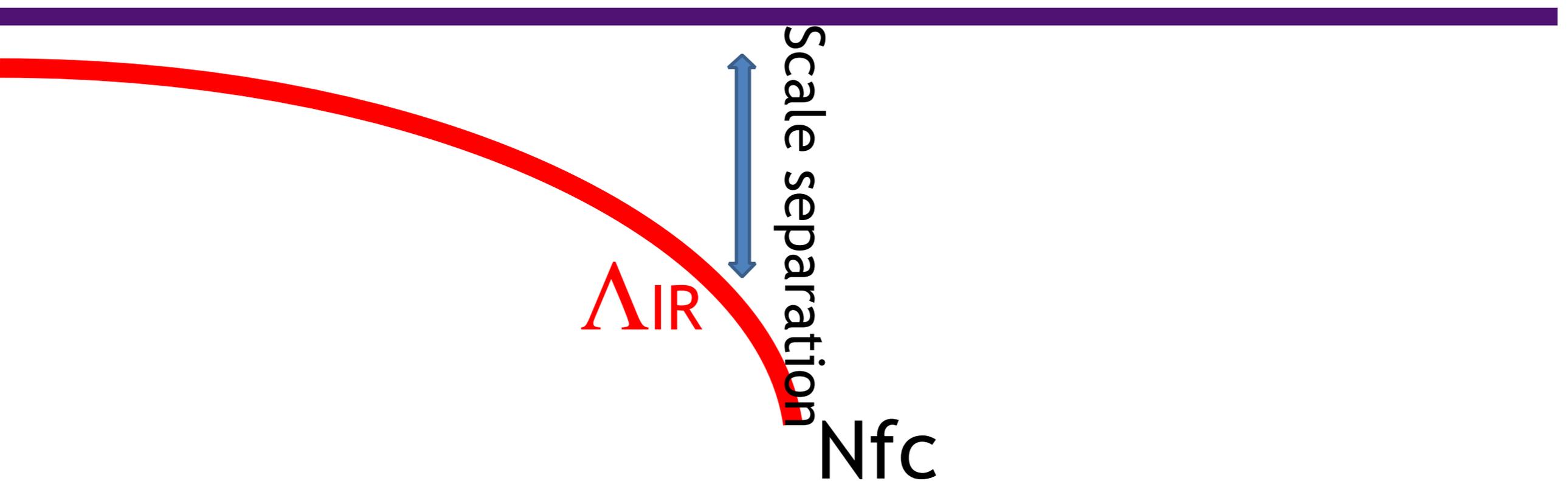
The coupling walks for

$$\Lambda_{\text{UV}}^{-1} \ll r \ll \Lambda_{\text{IR}}^{-1}$$

Standard picture of scale separation

$$\Lambda_{\text{IR}}/\Lambda_{\text{UV}} = \mathcal{O}(1).$$

Λ_{UV}



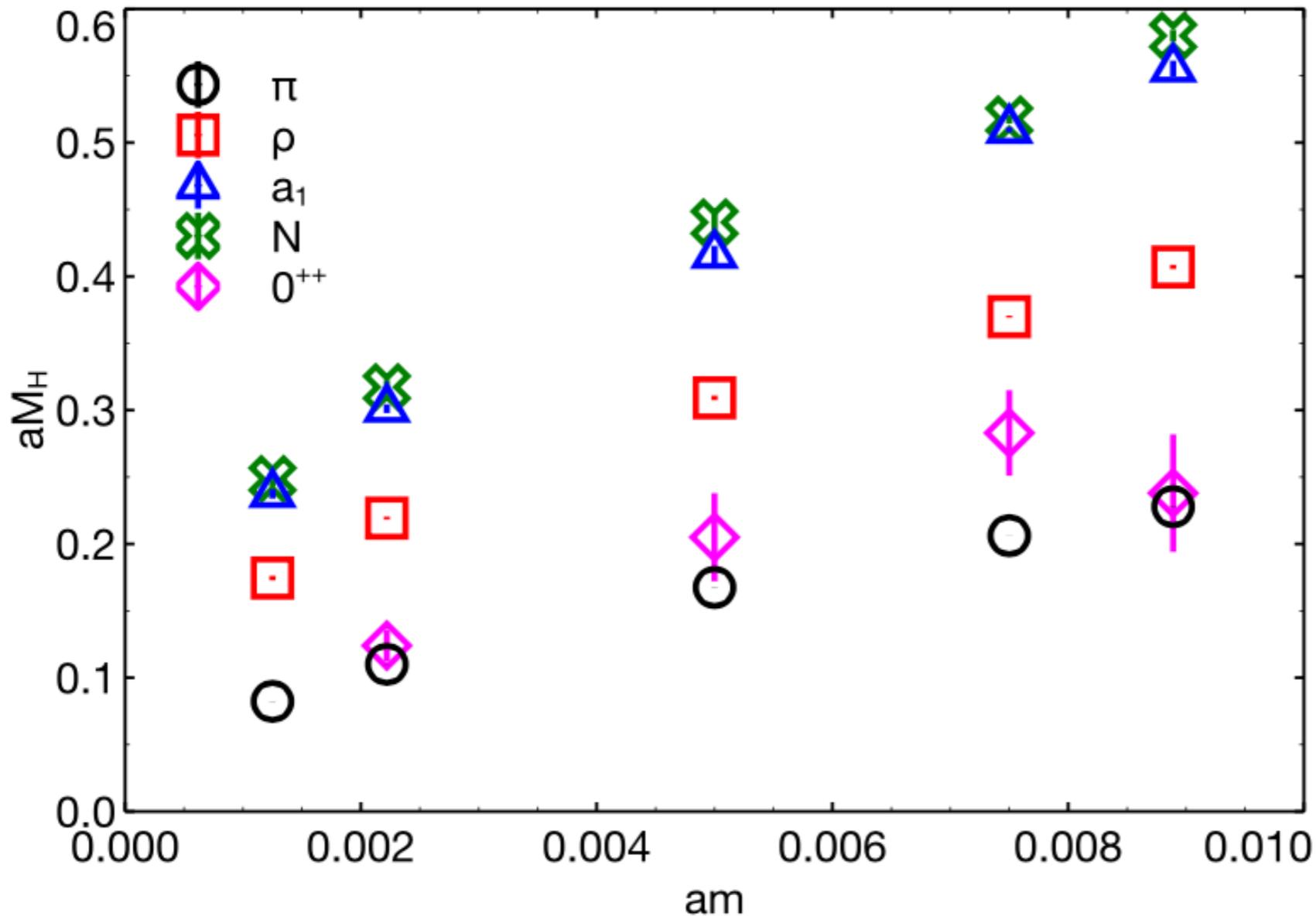
$$\frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

Strongly interacting dynamics and the search for new physics at the LHC

T. Appelquist,¹ R. C. Brower,^{2,3} G. T. Fleming,^{1,3} A. Hasenfratz,^{4,3} X. Y. Jin,⁵ J. Kiskis,⁶ E. T. Neil,^{4,7,3}
J. C. Osborn,^{5,3} C. Rebbi,² E. Rinaldi,^{8,3} D. Schaich,^{9,3,10} P. Vranas,⁸ E. Weinberg,¹¹ and O. Witzel^{12,3}

(Lattice Strong Dynamics (LSD) Collaboration)

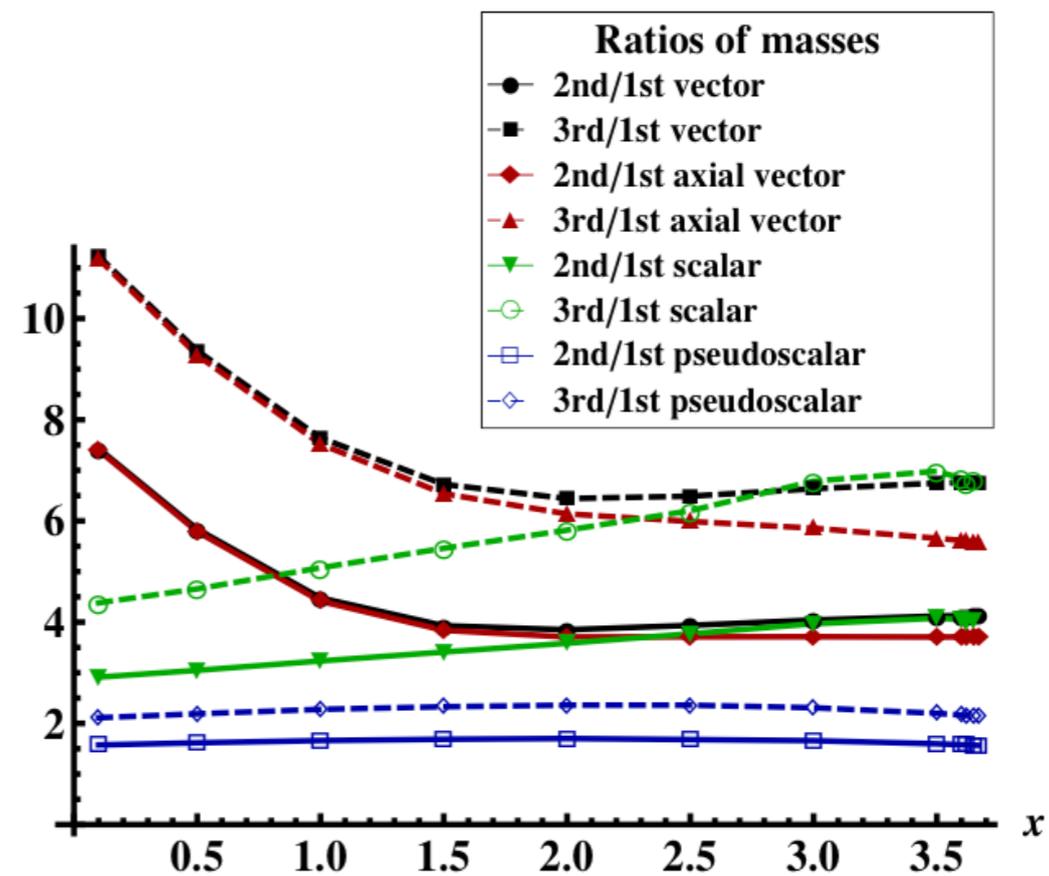
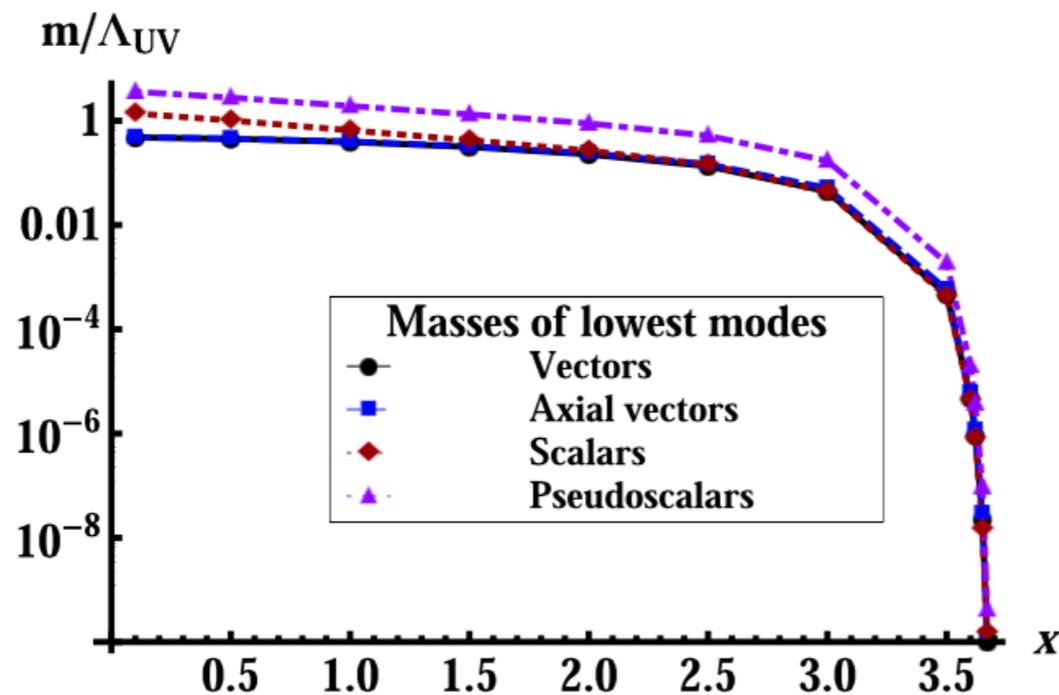
Beyond scale separation:



hierarchy of scales
in the spectrum:
light scalar
needed for
phenomenology

(Essential)
singularity in the
chiral limit and mass ratios:
example from holographic
V-QCD

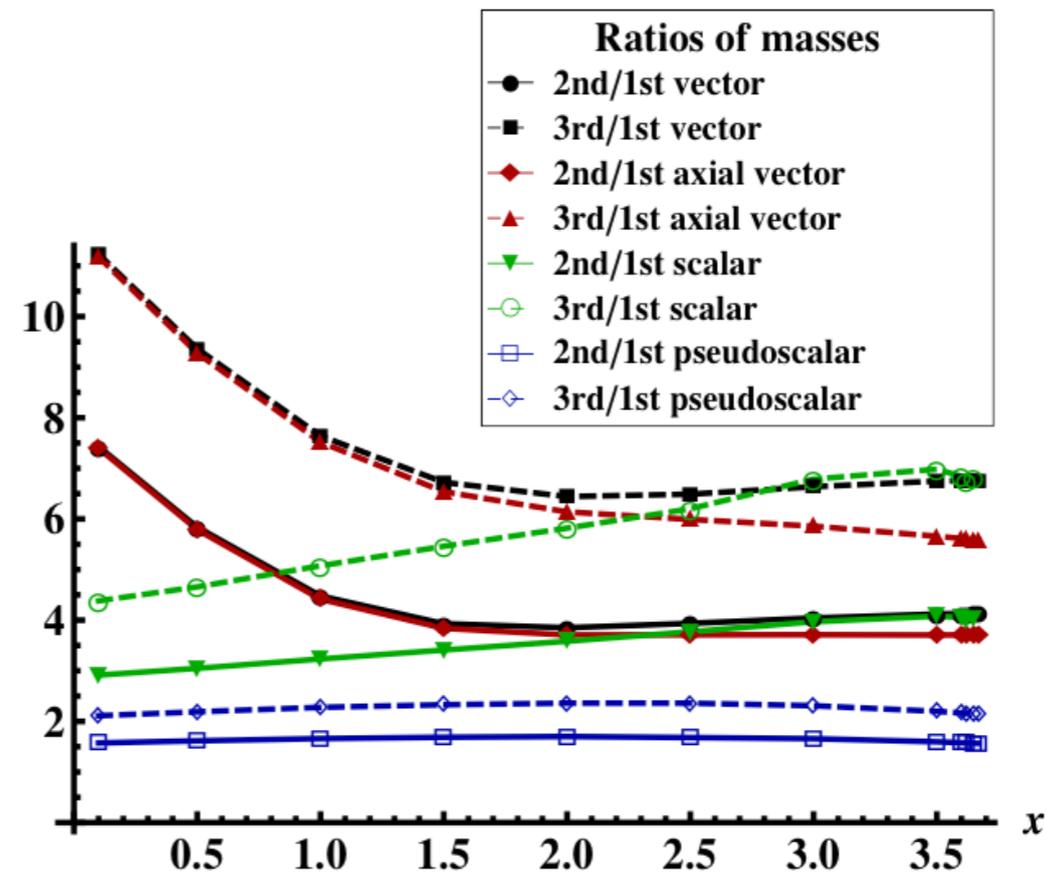
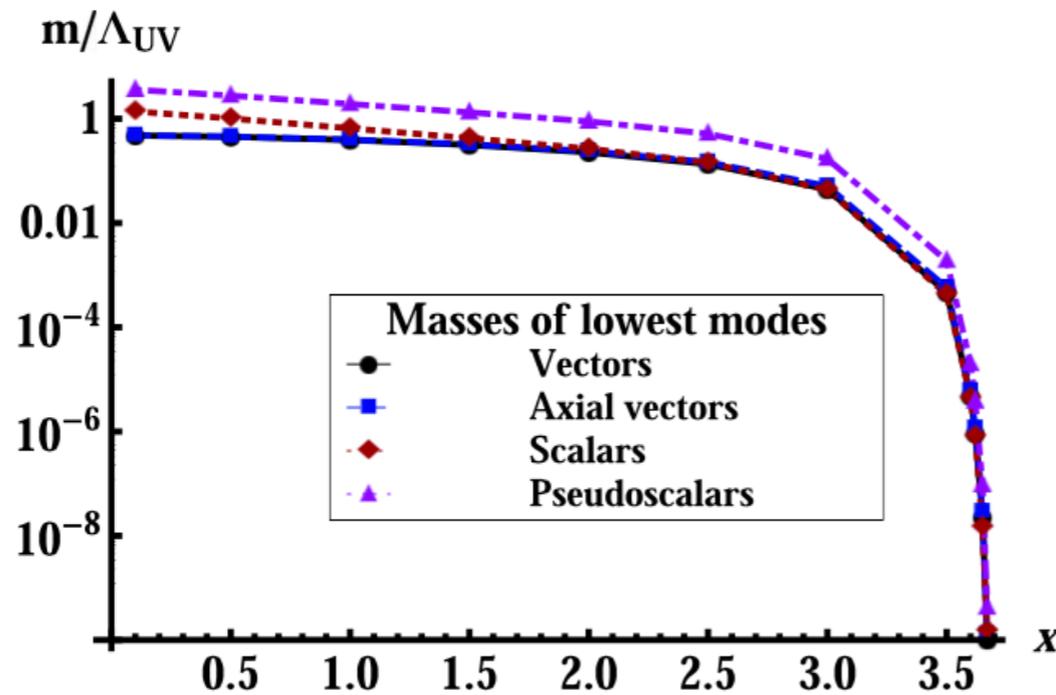
$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \exp\left(\frac{\hat{K}(n+1)}{\sqrt{x_c - x}}\right)$$



(Essential)
singularity in the
chiral limit and mass ratios:
example from holographic
V-QCD

$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \exp\left(\frac{\hat{K}(n+1)}{\sqrt{x_c - x}}\right)$$

Not unique



$\overline{\Lambda_{\text{IR}}}$ not unique:

Power-law corrections to essential singularity

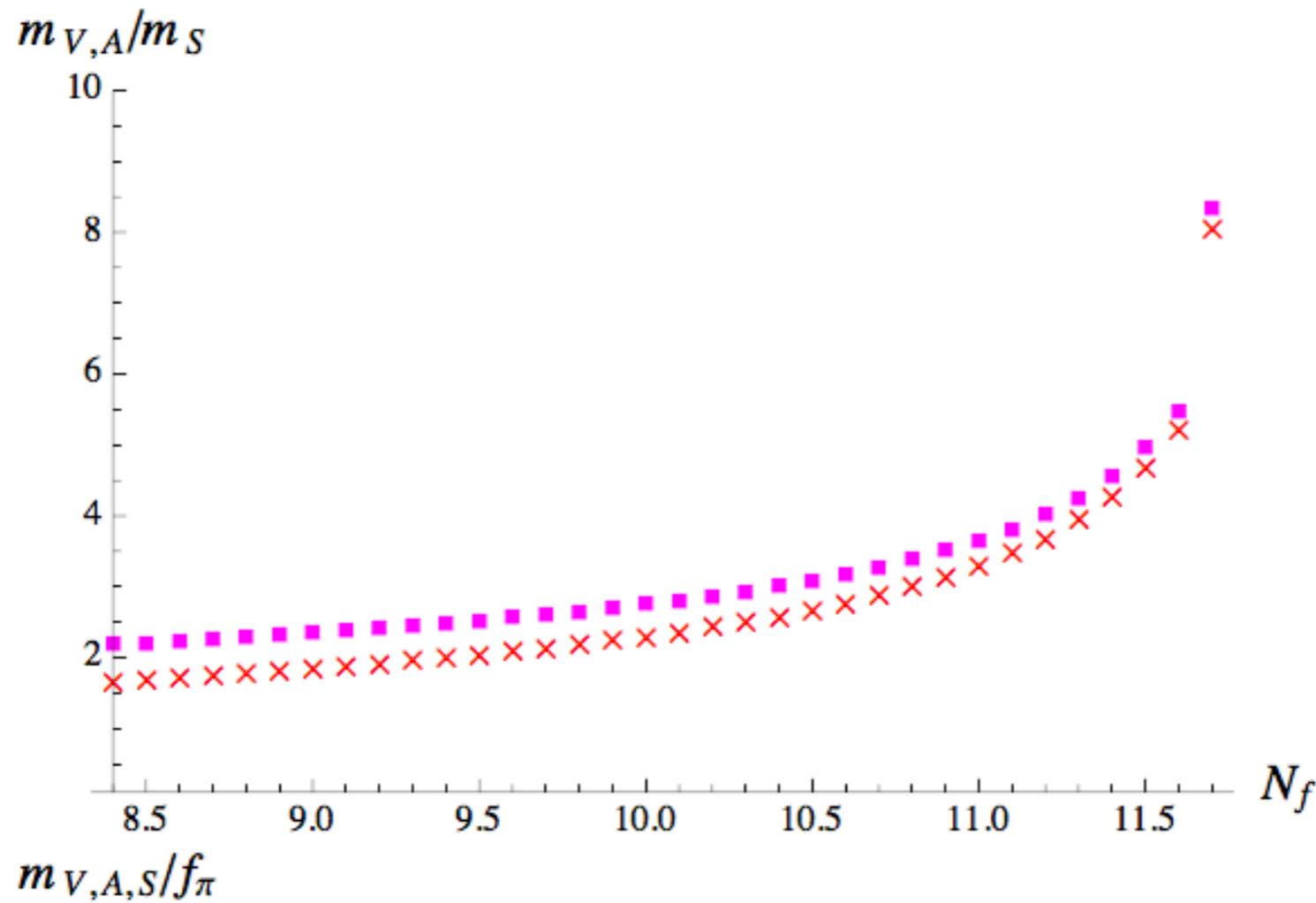
Gies et al. 2013

Alho, Evans, Tuominen 2013

$$O_i = A_i (N_f^c - N_f)^{p_i} \langle \bar{q}q \rangle^{1/3}$$

Power-law X Miranski scaling

May account for hierarchy of scales



Mass deformed theory I: EoS approach for IR quantities

$$y = f(x)$$

$$y = m / \langle \bar{\psi}\psi \rangle^\delta \quad \delta = \frac{6-\eta}{2-\eta}$$

Second order transition:

$$x = (N_f^c - N_f) / \langle \bar{\psi}\psi \rangle^{\frac{1}{\beta}} \quad \langle \bar{\psi}\psi \rangle = (N_f^c - N_f)^\beta$$

Essential singularity:

Nogawa, Hasegawa, Nemoto, 2012

$$x = e^{\sqrt{(N_f^c - N_f)}} / \langle \bar{\psi}\psi \rangle \quad \langle \bar{\psi}\psi \rangle = e^{\sqrt{(N_f^c - N_f)}}$$

Continuity of $f(x)$ plus asymptotic forms for $m \rightarrow 0$ and $N_f \rightarrow N_f^c$ imply

$$\langle \bar{\psi}\psi \rangle \propto e^{\sqrt{(N_f^c - N_f)}} \text{ for } m \text{ } \textit{smallish} \text{ and } (N_f^c - N_f) \text{ } \textit{largish}$$

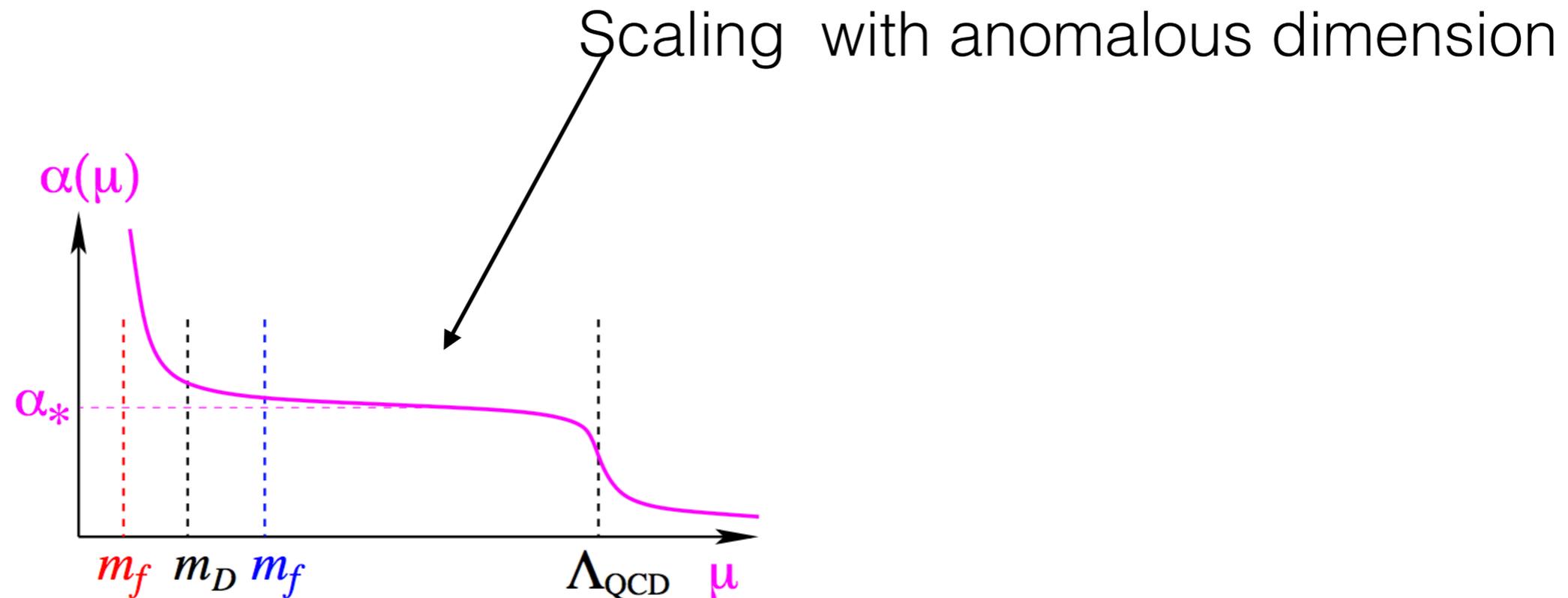
$$\langle \bar{\psi}\psi \rangle \propto m^{1/\delta} \text{ for } m \text{ } \textit{largish} \text{ and } (N_f^c - N_f) \text{ } \textit{smallish}$$

Anomalous dimension appears naturally below N_f^c

Scaling limited by Goldstone singularities in the chiral limit (Wallace Zia)

Mass deformed theory II: KMI discussion

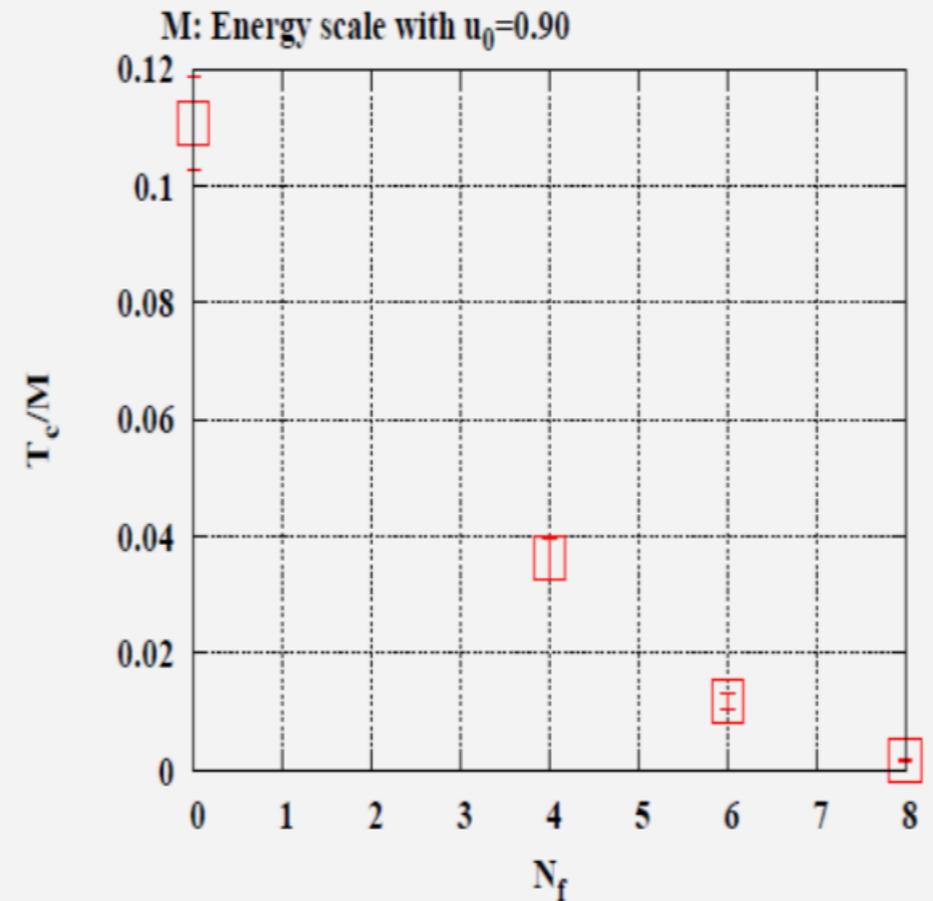
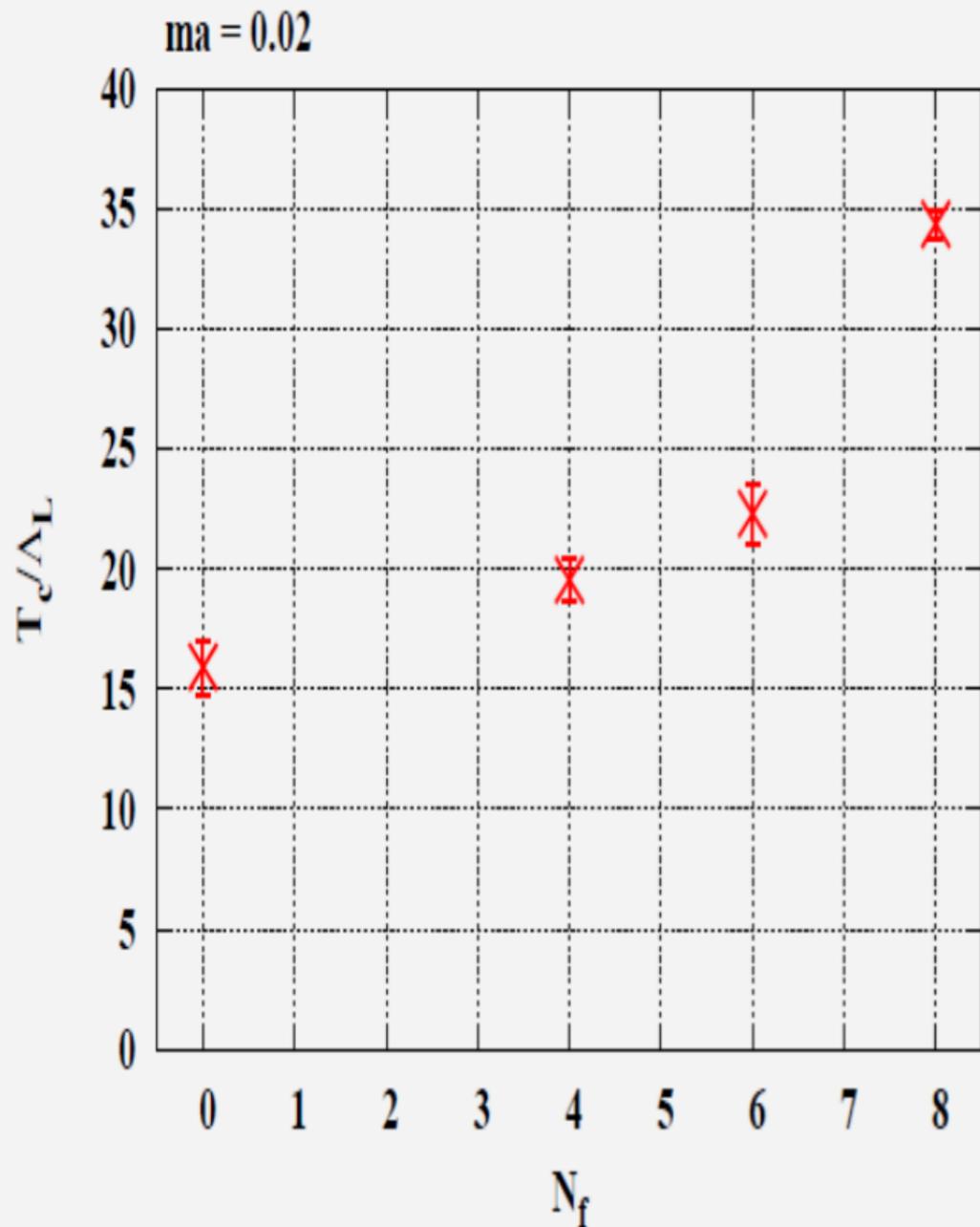
Mutatis mutandis,
Eos approach reproduces KMI scenario:



KMI 2013

Search for scale hierarchy -

Kohtaroh Miura, MpL, Tiago Nunes da Silva, E Pallante

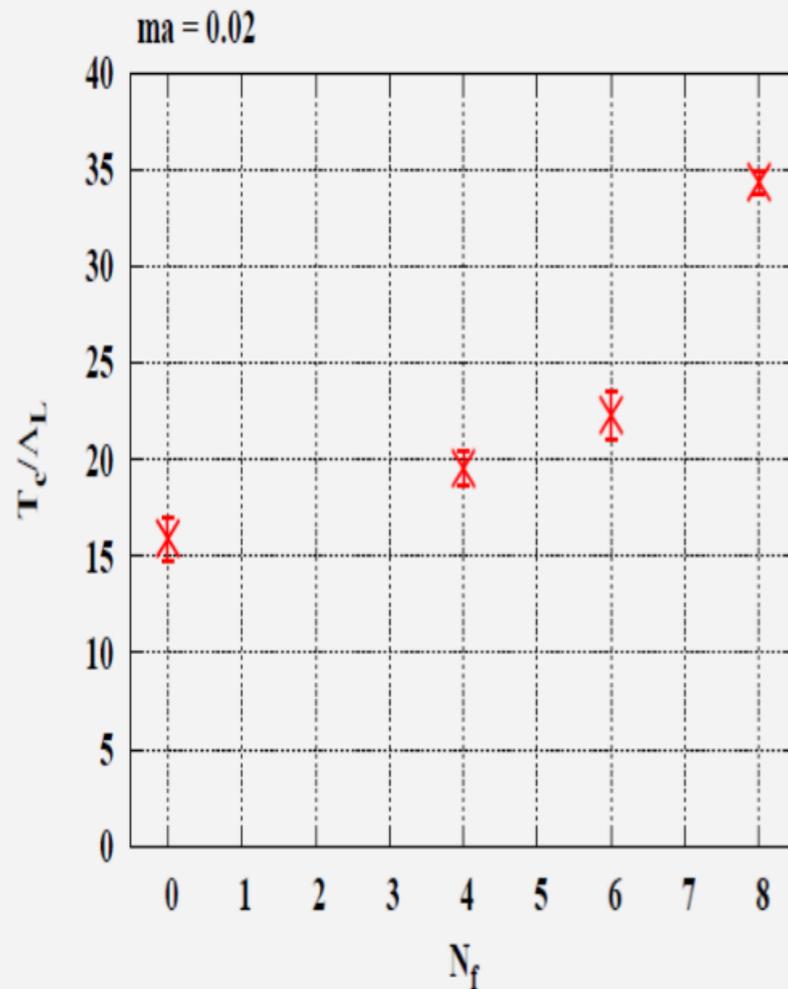


$$\frac{T_c}{M} = \frac{1}{N_t} \exp \left[\int_{g_{\text{ref}}}^{g_c} \frac{dg}{B(g)} \right].$$

Towards a quantitative comparison with holography

K. Miura, MpL, E. Pallante, in progress

$$\frac{2\pi T_c}{M_{KK}} = 1 - \frac{1}{126\pi^3} \lambda^2 \frac{N_f}{N_c} \left(1 + \frac{12\pi^{3/2}}{\Gamma(-\frac{2}{3}) \Gamma(\frac{1}{6})} \right)$$



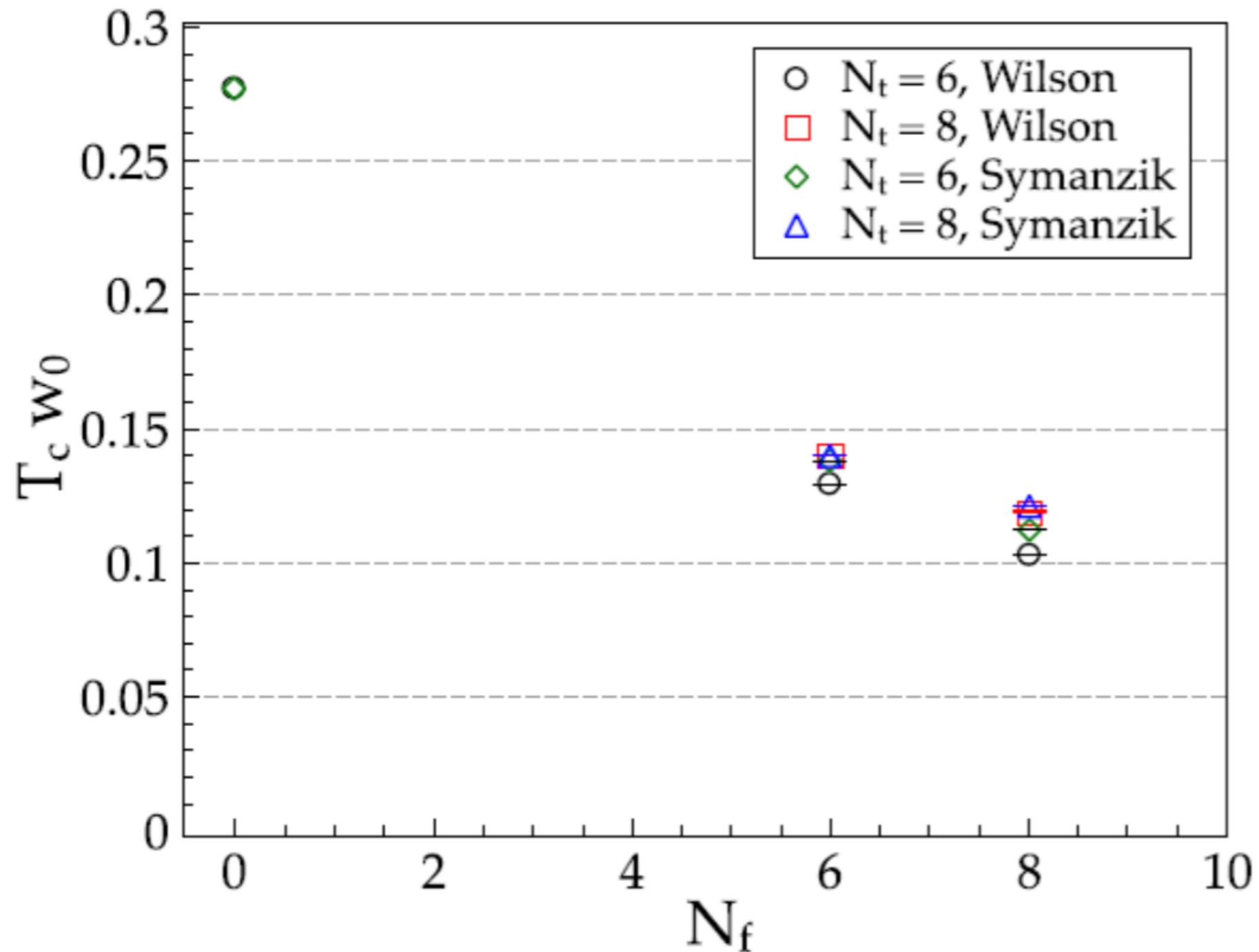
Bigazzi and Cotrone, JHEP 2015



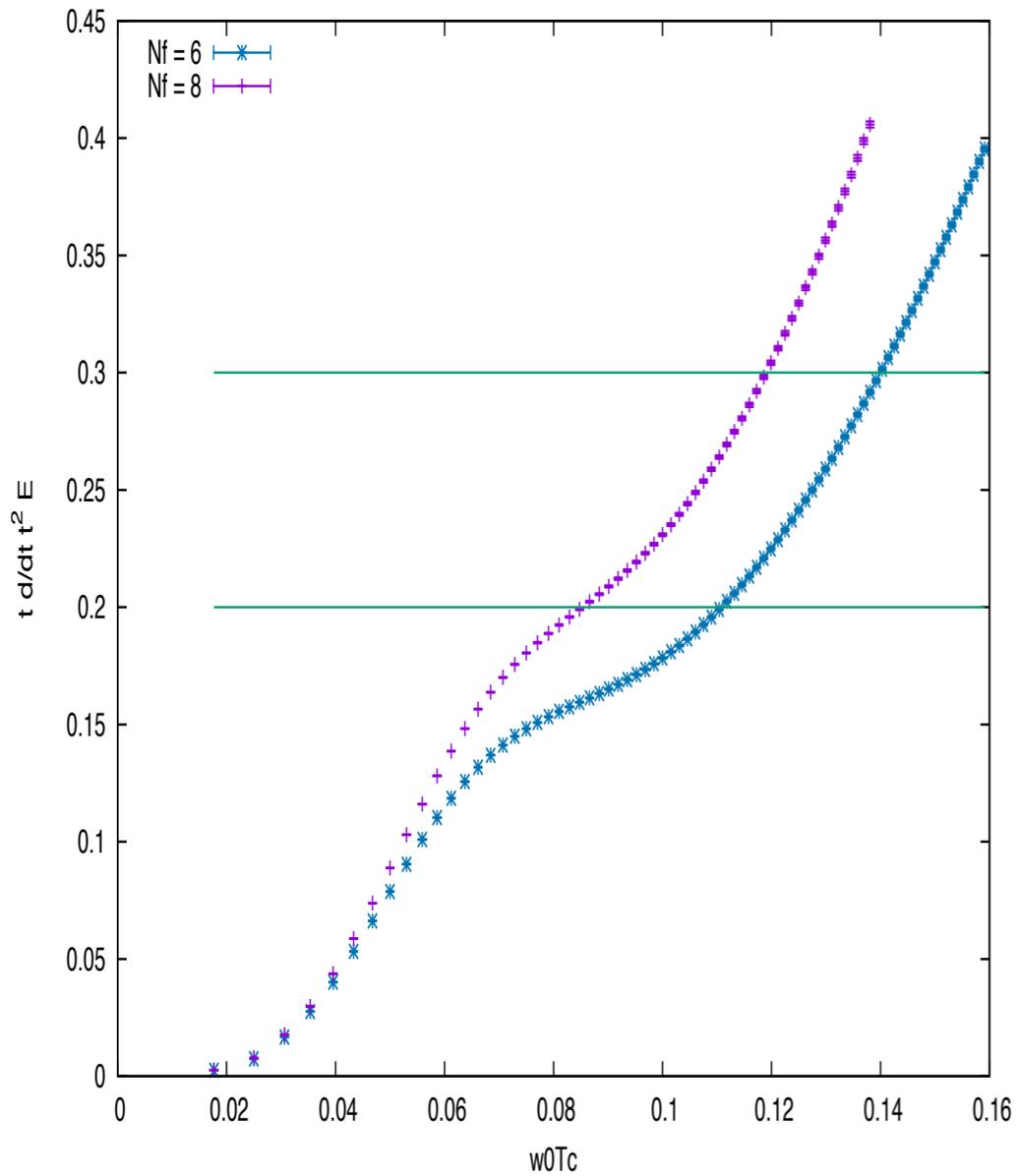
$$\left(1 + \frac{12\pi^{3/2}}{\Gamma(-\frac{2}{3}) \Gamma(\frac{1}{6})} \right) \approx -1.987$$

T increases with N_f on the scales used in these two studies

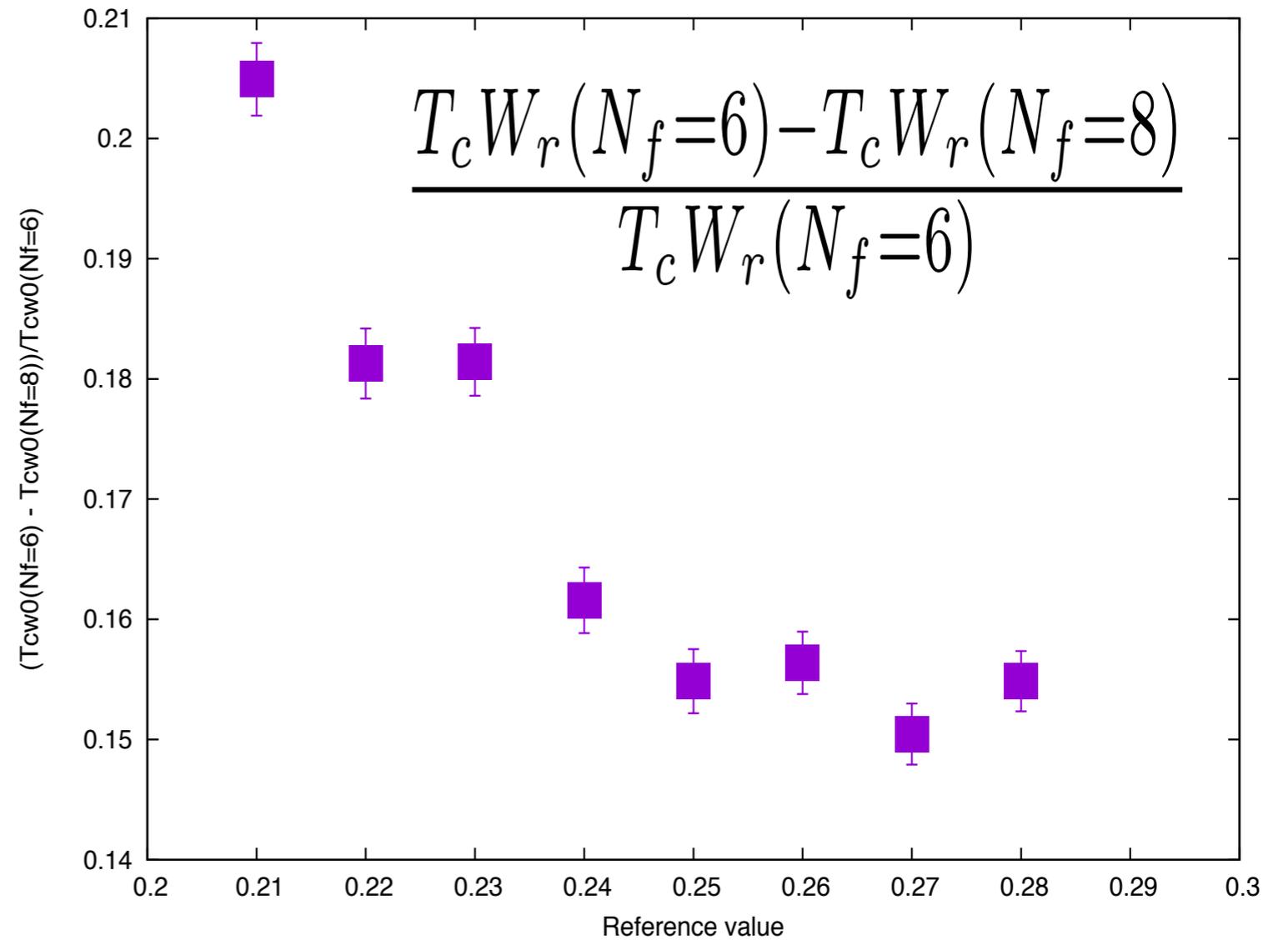
Tc on the 1/w0 scale



Moving the scale with Wilson flow



Qualitatively as expected,
limited by lattice artifacts

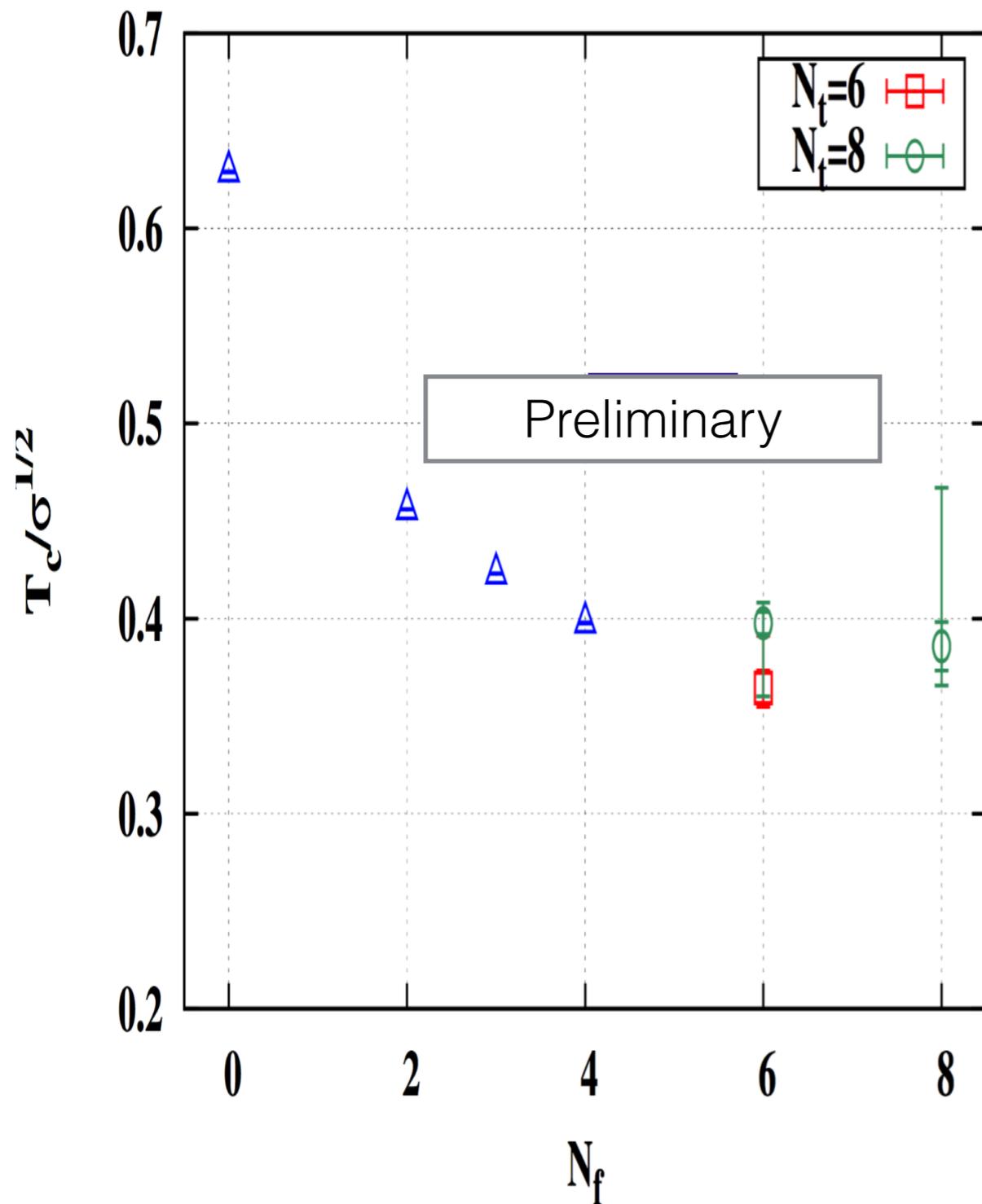


UV

Tc and the string tension

KM, MpL, EP, in progress

Mild decrease, possibly constant as $N_f \rightarrow N_f^c$

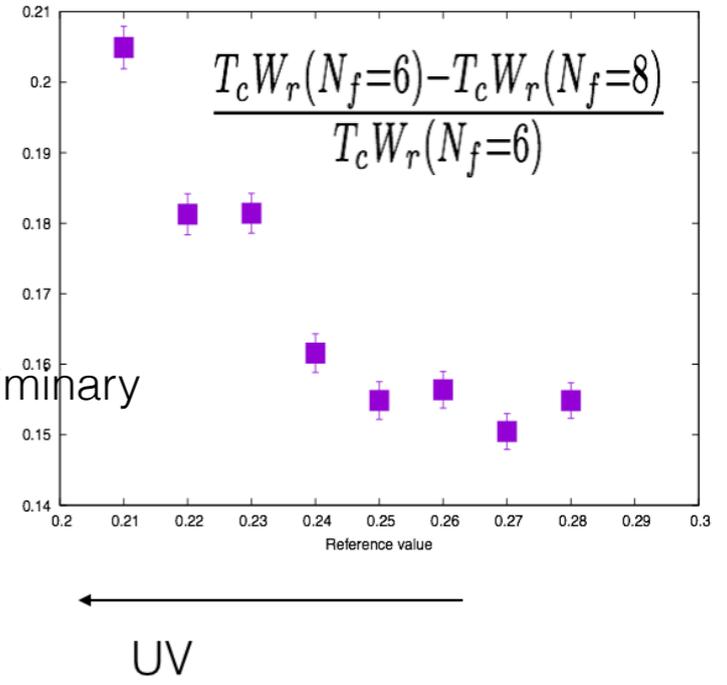
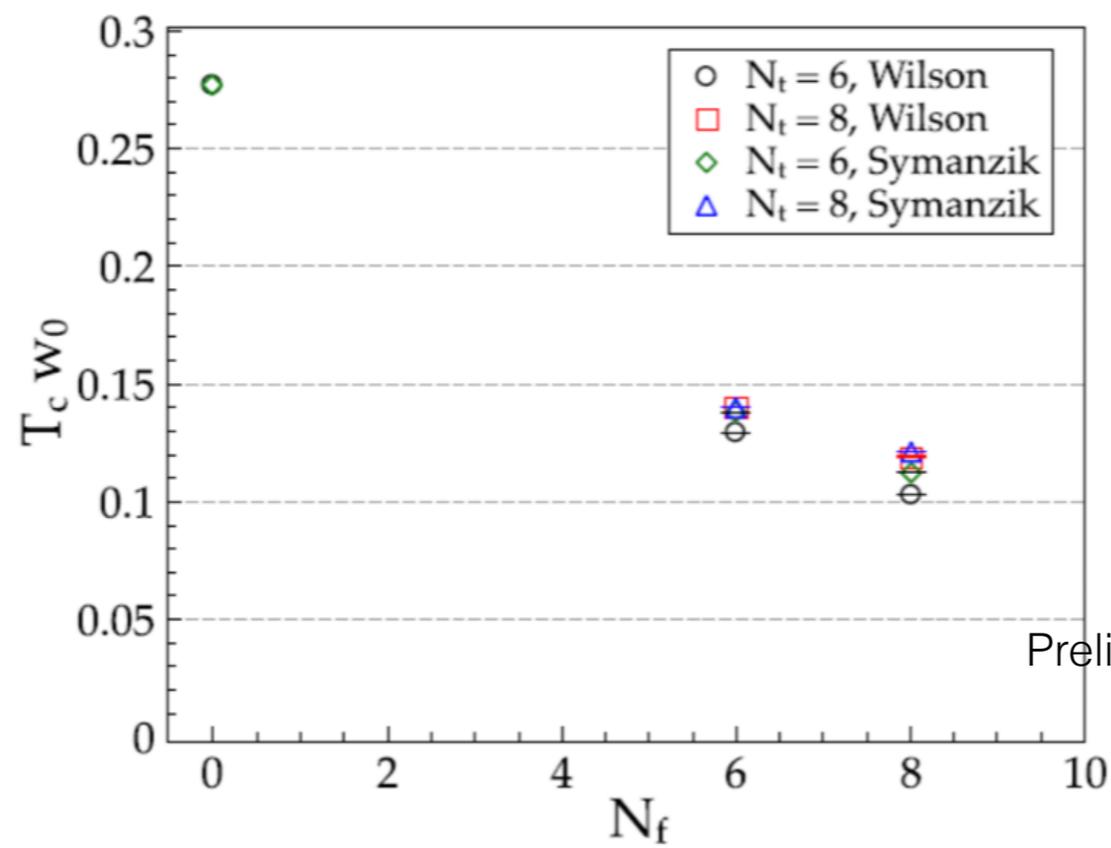
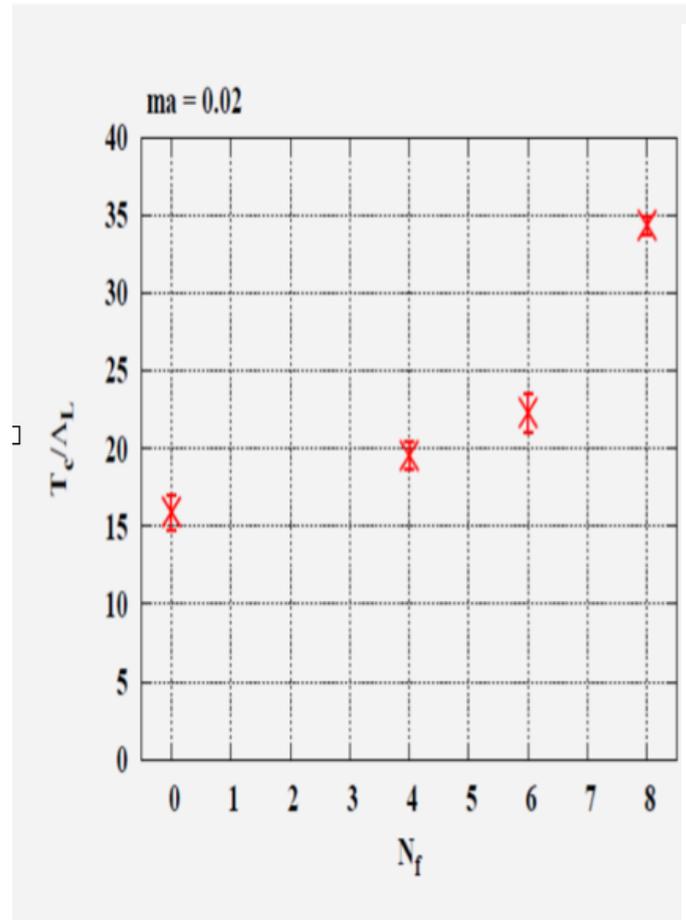
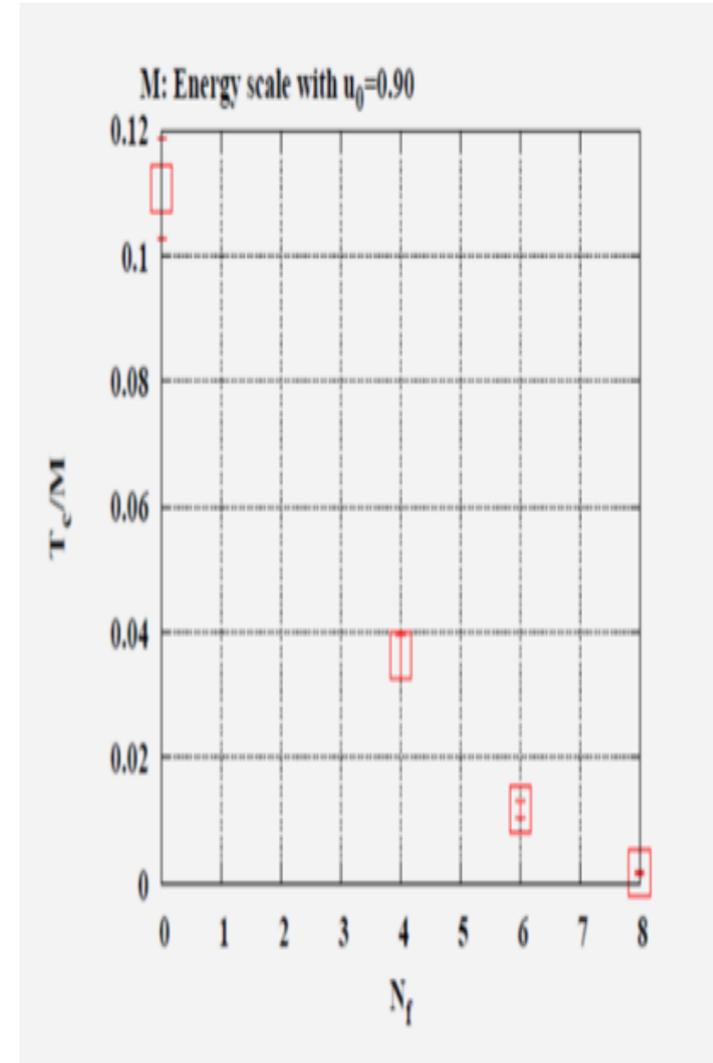
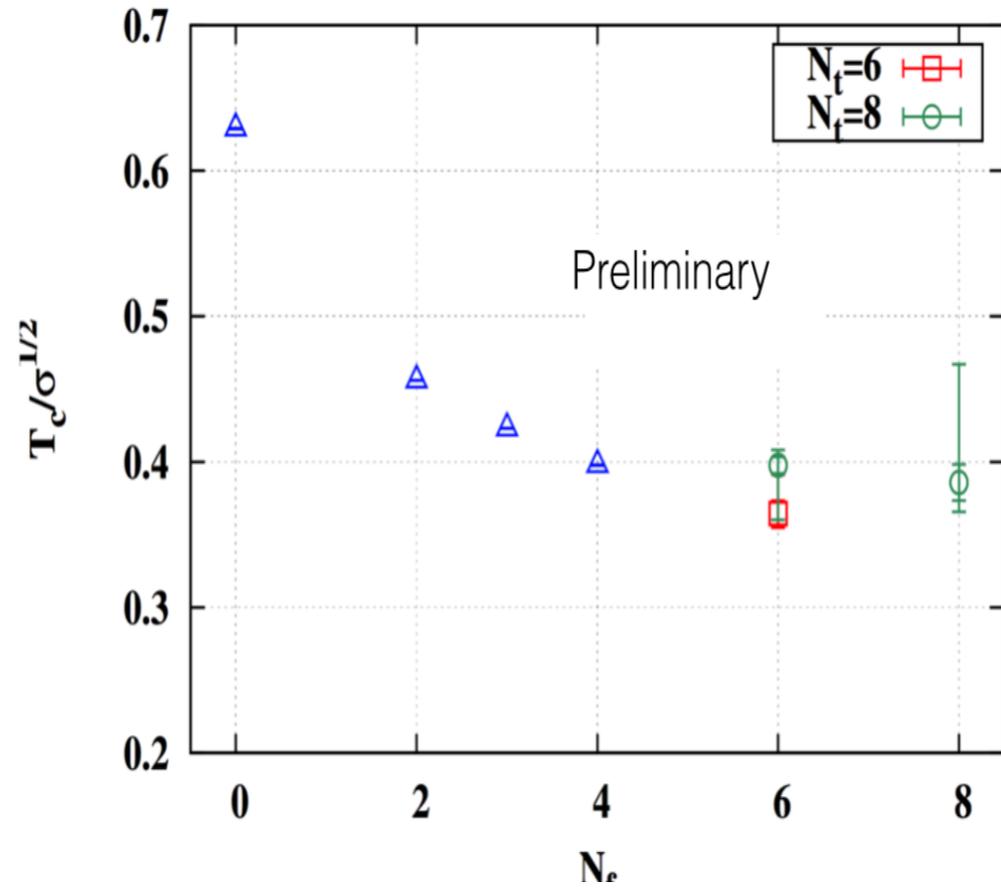


Again similar to the prediction of the WSS model:

$$\frac{T_c}{\sqrt{\sigma}} \propto (1 - \epsilon N_f / N_c)$$

communicated by F. Bigazzi

Hierarchy of scales in the near conformal phase



Hierarchy of scales

M

Λ_{UV}

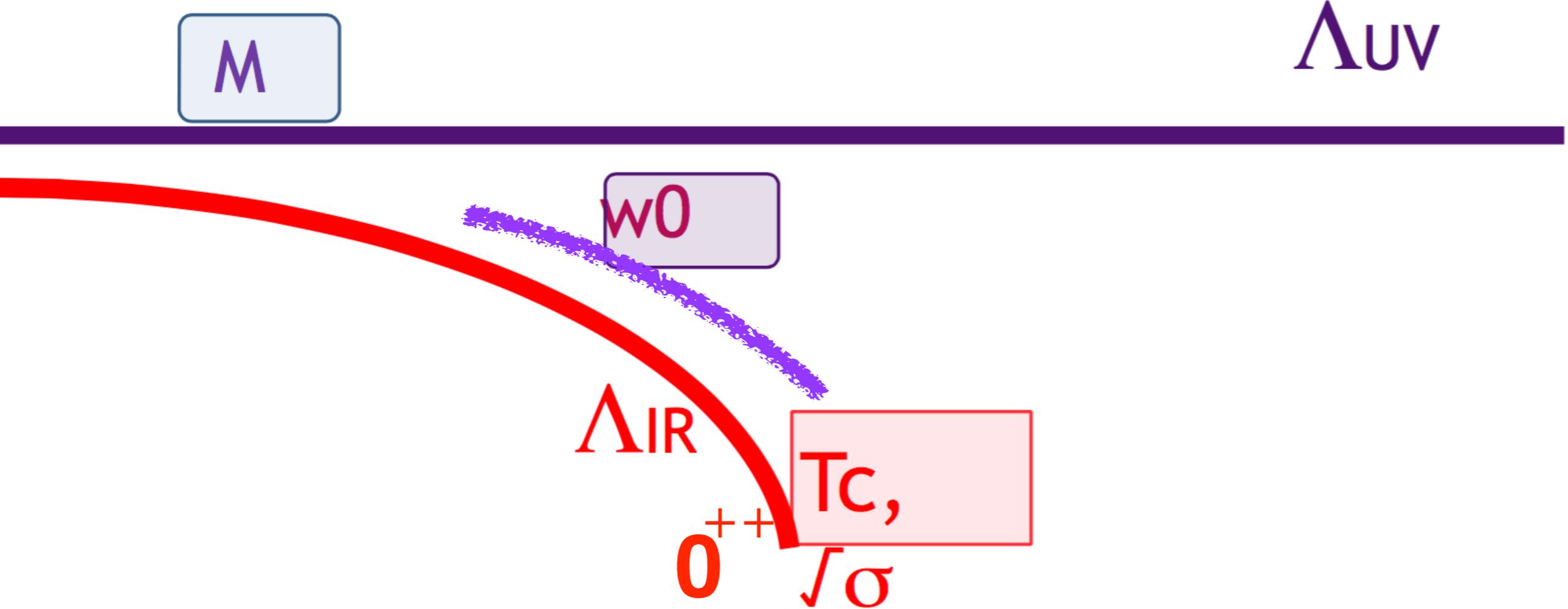
w_0

Λ_{IR}

$T_c,$

0^{++}

$\sqrt{\sigma}$

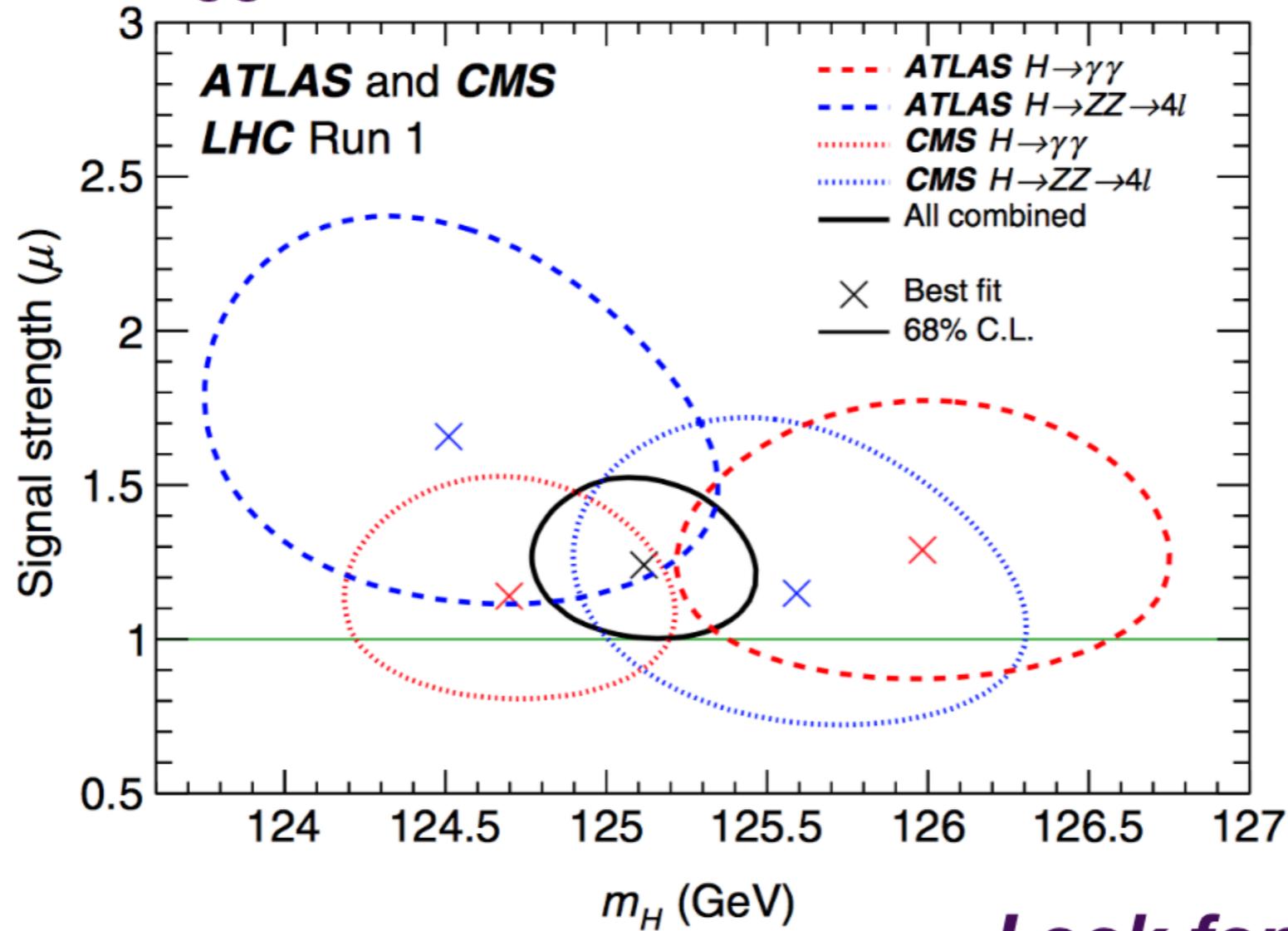


Short detour on phenomenology

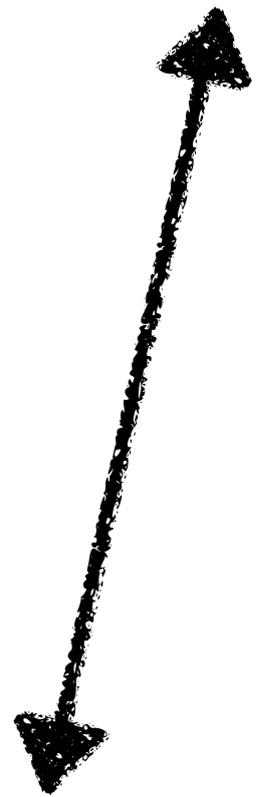


Beyond the Standard Model:

Can we find a theory which produces a narrow Higgs-like status?



Maybe
in the
preconformal
region?



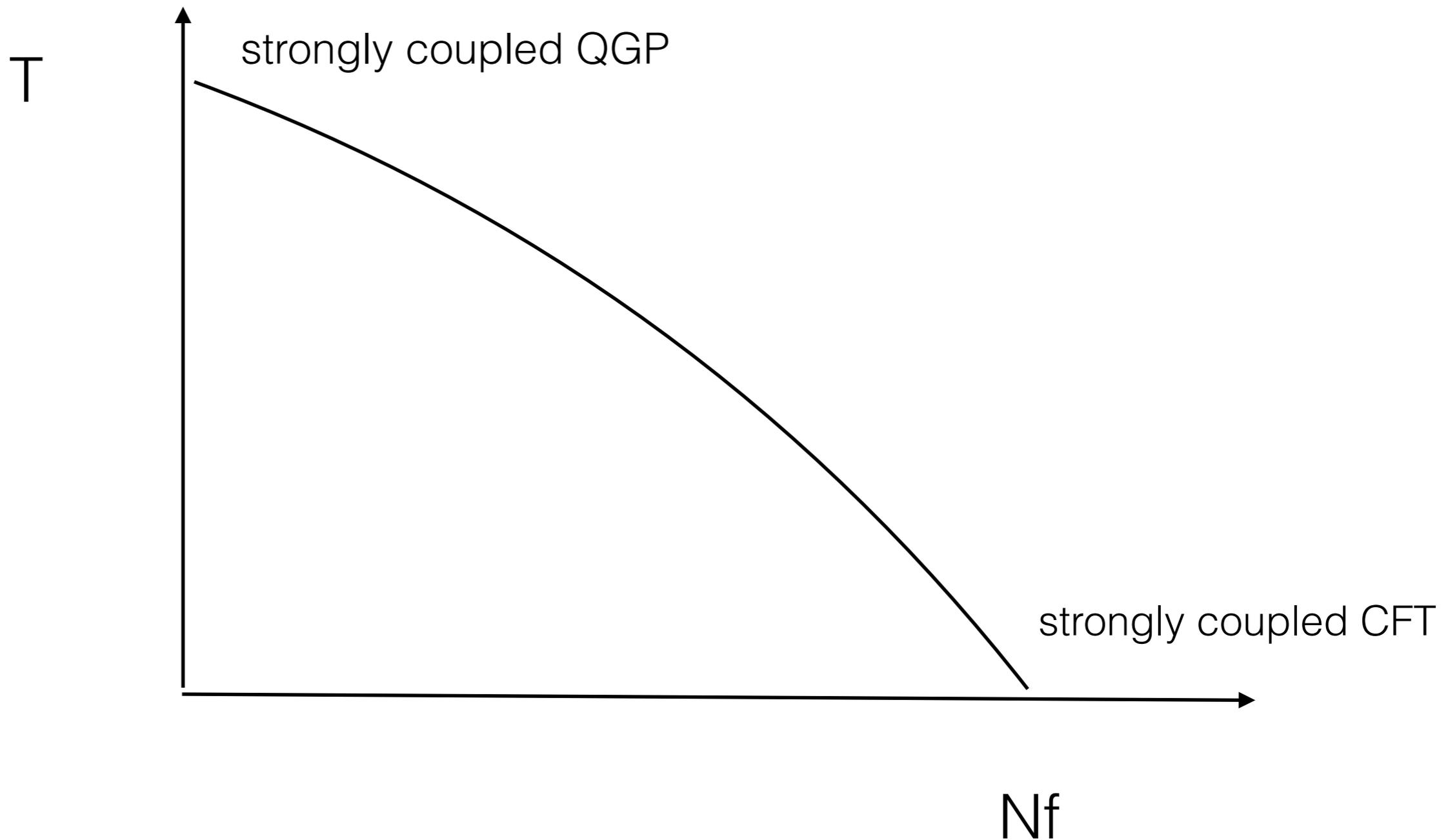
**Look for theories
with scale separation**

..as possible BSM candidates

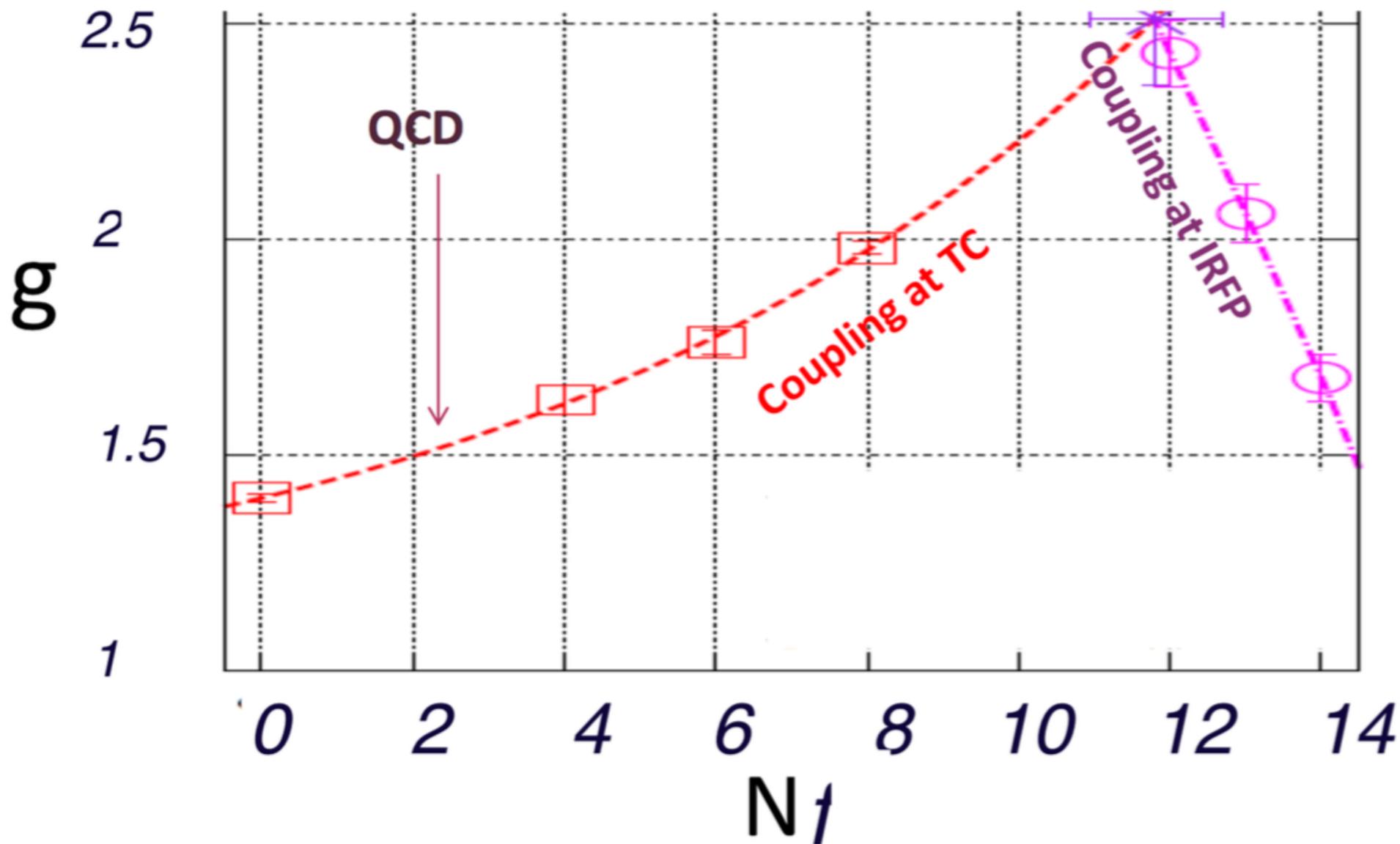


It should be a composite scalar particle, lighter than so-far unobserved composite vector states

Parting comments on the phase diagram



sQGP and strongly coupled conformal QCD
are continuously connected



The strength of the coupling increases with N_f

Conformality ubiquitous

Approach to the free field

Bulk viscosity set to zero

AdS/CFT methods

Speed of sound close to 0.3

Coupling slowly running :

*Hints of conformality also at
Strong coupling*

