Phases of QCD, topology and axions - III

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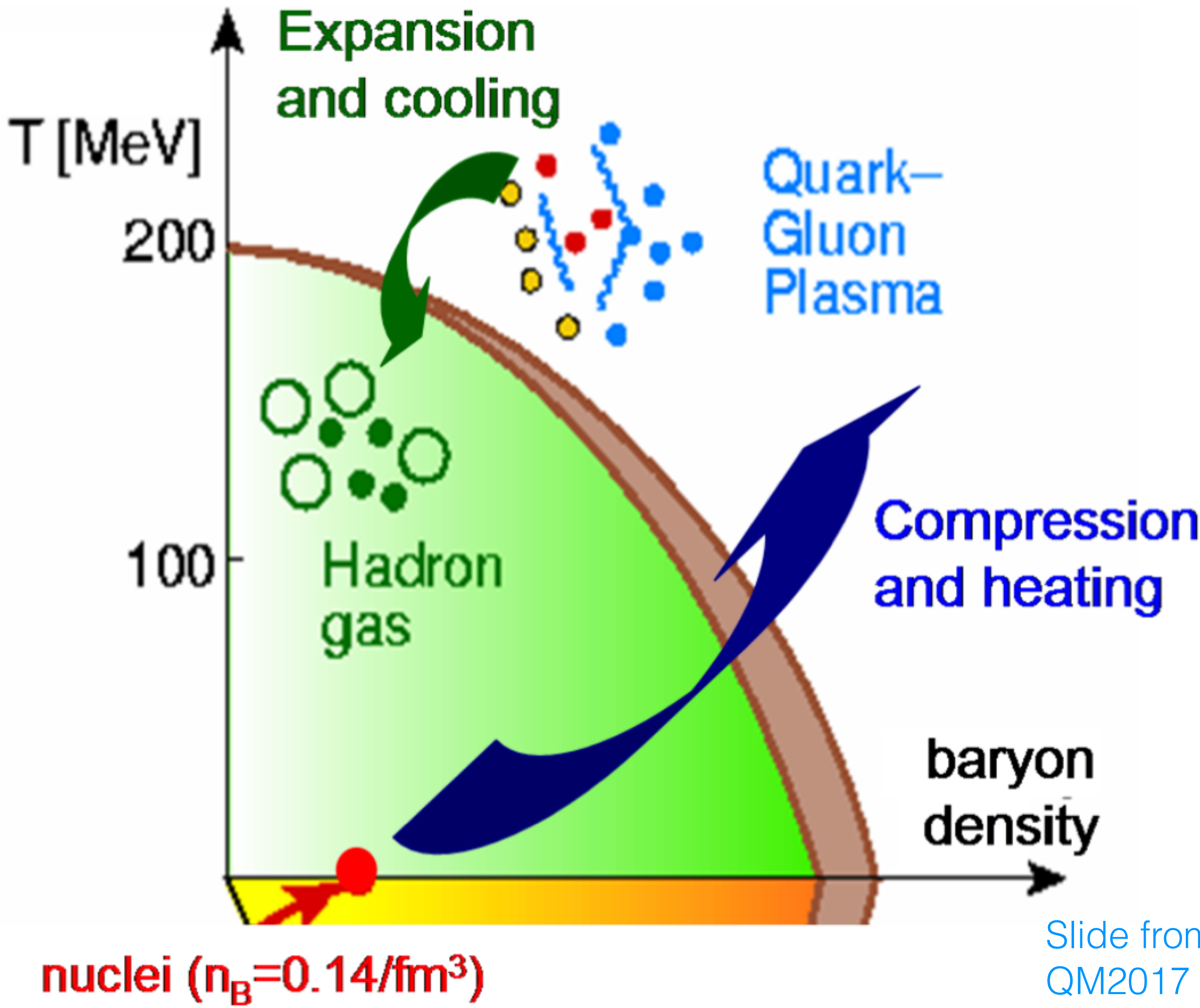
I Symmetries and phases
of QCD in the
Temperature, Nf space

II Results on the phase diagram

III Topology - broken phase

IV Topology - hot QCD & axions



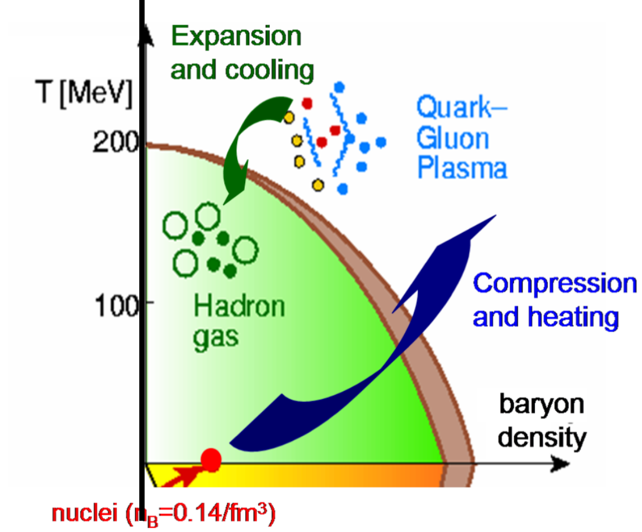
Slide from **Hanna Petersen**
QM2017 Plenary talk

Temperatures:

150 MeV < T < 500 MeV

..and beyond

Quark Gluon Plasma:
Topology



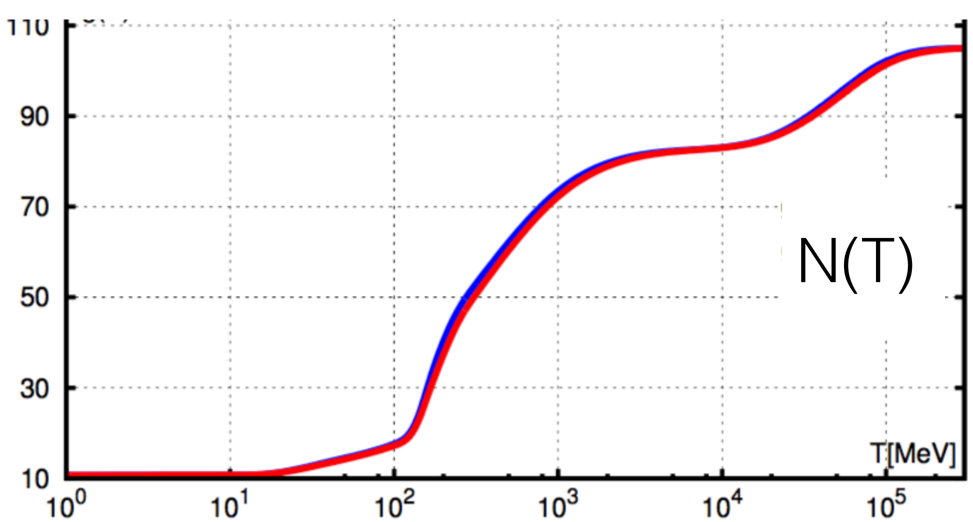
Time from Big Bang

Temperatures

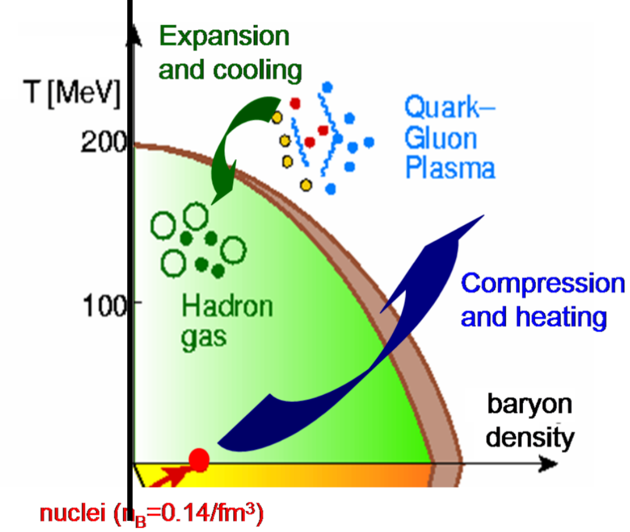
150 MeV < T < 500 MeV

..and beyond

Temperature and Time from BigBang
are linked by the Equation of State



Quark Gluon Plasma:
Topology



Time from Big Bang

Temperature

Hubble parameter

$$H(T) \simeq T^2/M_P$$

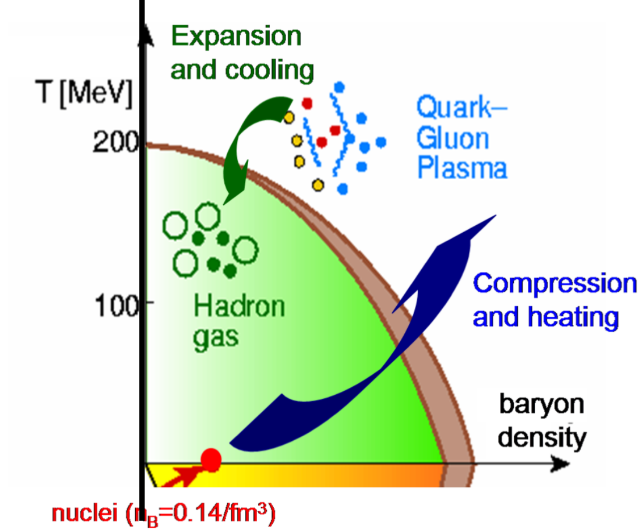
Axions's freezout

$$3H(T) = m_a(T)$$

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

Quark Gluon Plasma: Topology

Axions' mass and density today



Outline for today:

-Topology: θ *term, topological charge*

- Resolution of the UA(1) puzzle, η' mass

- Why is topology challenging? Methods

- Topology at T=0 - results

- Hot QCD topology : introduction

QCD topology, long standing focus of strong interaction:

-learning about the structure of the (s)QGP
-fundamental symmetries, strongCP problem —> axions
-hampered by technical difficulties

Recent developments:

-methodological progress: gradient flow, chiral fermions
-first results for dynamical fermions at high temperature:

Trunin *et al.* **J.Phys.Conf.Ser. 668 (2016) no.1, 012123**
Bonati *et al.* **JHEP 1603 (2016) 155**
Borsany *et al.* **Nature 539 (2016) no.7627, 69-71**
Petreczky *et al.* **Phys.Lett. B762 (2016) 498-505**
Burger *et al.* **Nucl. Phys. A, in press**
Taniguchi *et al.* **Phys.Rev. D95 (2017) no.5, 054502**

TmFT

+work in progress

θ term and $U_A(1)$ problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2} \mathrm{tr} F\tilde{F}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)]$$

$$\frac{\partial^2 F(\theta,T)}{\partial\theta^2}\bigg|_{\theta=0} \equiv \chi(T) = (< Q^2 > - < Q >^2)/V$$

The θ dependence solves the $U_A(1)$ problem:

Approximate symmetry: $q \longrightarrow e^{i\alpha\gamma_5} q$

Would be broken by the (spontaneously generated) $\bar{q}q$:

the candidate Goldstone is the η'

Heavy!! (900 MeV)

BUT:

the divergence of the current $j_5^\mu = \bar{q}\gamma_5\gamma_\mu q,$

$$\partial_\mu j_5^\mu = m\bar{q}\gamma_5 q + \frac{1}{32\pi^2} F\tilde{F}.$$

Contains another term

The $U_A(1)$ symmetry is explicit broken

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c^2)
Pion[6]	π^+	π^-	$u\bar{d}$	139.570 18 ±0.000 35
Pion[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a]	134.9766 ±0.0006
Eta meson[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a]	547.862 ±0.018
Eta prime meson[9]	$\eta'(958)$	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a]	957.78 ±0.06
Kaon[12]	K^+	K^-	$u\bar{s}$	493.677 ±0.016
Kaon[13]	K^0	$\bar{K}^0$	$d\bar{s}$	497.614 ±0.024

....

The θ dependence solves the $U_A(1)$ problem: ......provided that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F}$$ is different from zero.

It can be proven that $\frac{1}{32\pi^2}\int d^4x F\tilde{F}$ = Q (topological charge)

and

$$Q = n_+ - n_-$$

……provided that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F}$$

is different from zero.

It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$

Gluonic definition

and

$$Q = n_+ - n_-$$

Fermionic definition

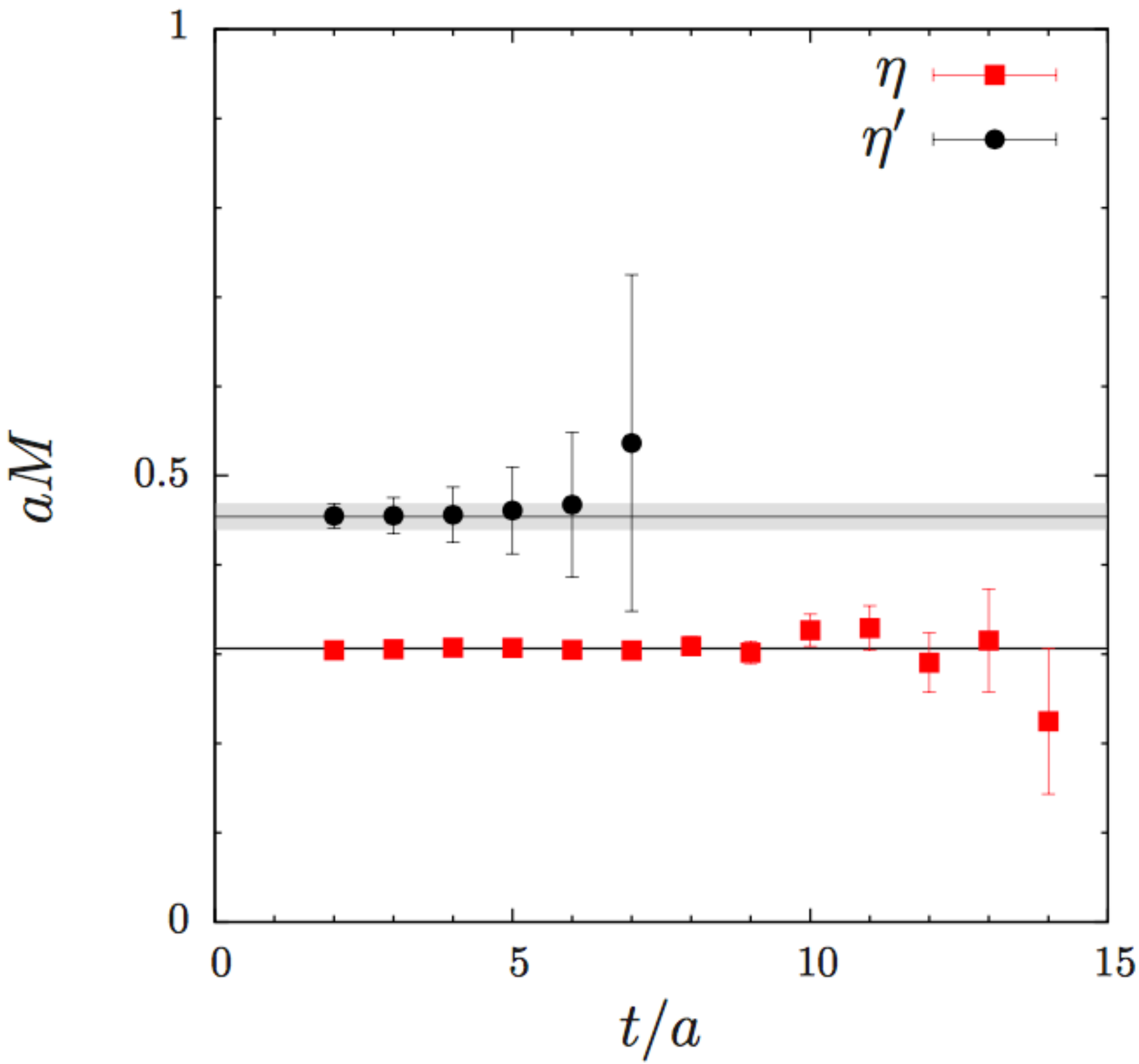
The η' mass may now be computed from the decay of the correlation

$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x)\tilde{F}(x)F(y)\tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2}\chi_t^{\mathrm{qu}}$$

Contemporary studies of η, η'



Otnnad, Urbach, Michael (ETMC) 2013

$$M_\eta = 551(8)_{\rm stat}(6)_{\rm sys}\,.$$

$$M_{\eta'} = 1006(54)_{\rm stat}(38)_{\rm sys}(+61)_{\rm ex}$$

OK!

A note on computation:

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x\, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

Sign problem

Approach similar in spirit to Taylor expansion for chempot

$$Z_{QCD}(\theta,T) = \int [dA][d\psi][d\bar{\psi}]\exp\left(-T\sum_t d^3x\, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta,T)$$

Simulations at $\theta = 0$

$$\left.\frac{\partial^2 F(\theta,T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T), = (< Q^2 > - < Q >^2)/V$$

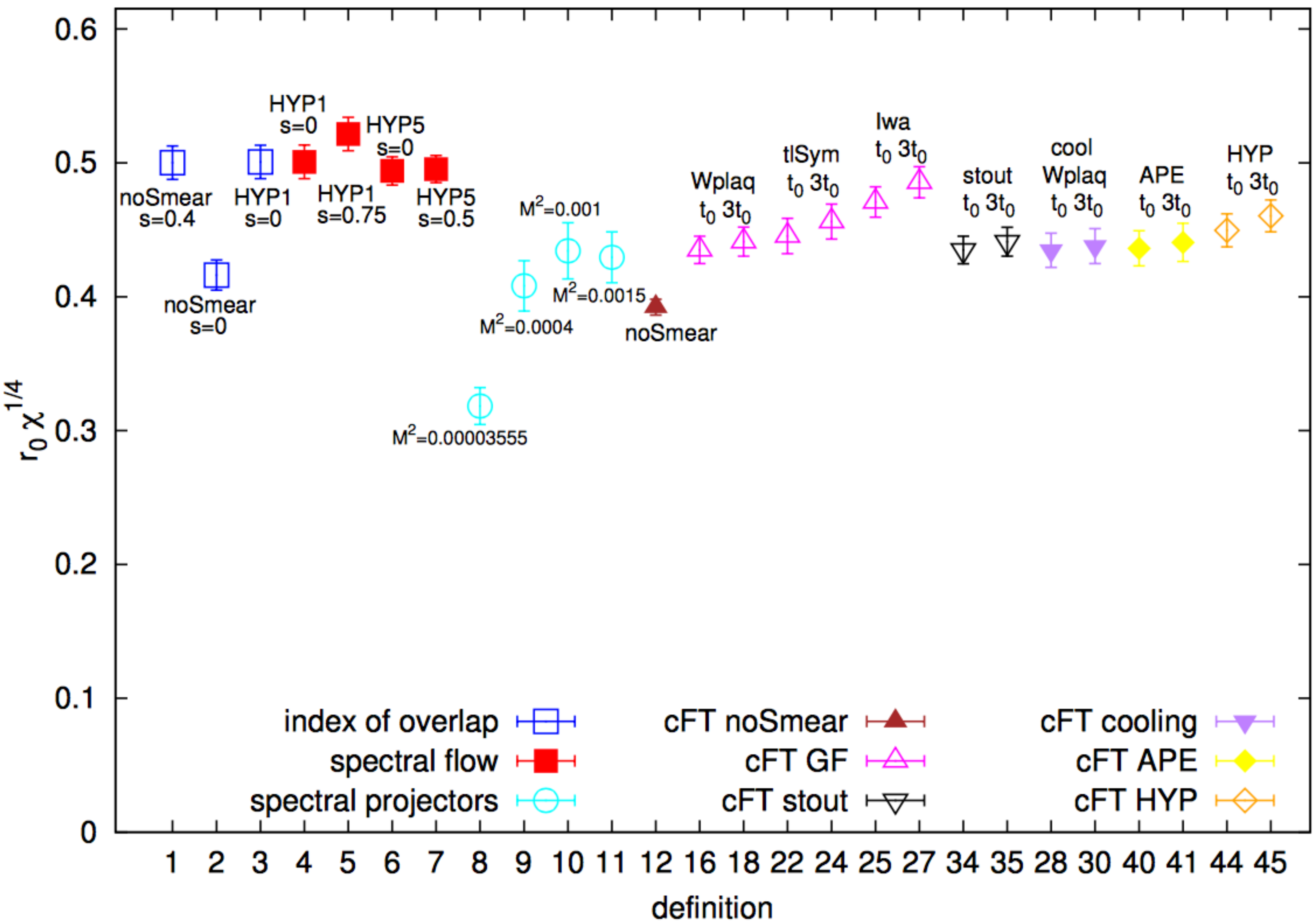
Comparison of topological charge definitions in Lattice QCD

Constantia Alexandrou[a,b], Andreas Athenodorou[b,a], Krzysztof Cichy[c,d], Arthur Dromard[e], Elena Garcia-Ramos[f,g], Karl Jansen[f], Urs Wenger[h], Falk Zimmermann[i]

2017

Comparison of topological charge definitions

nr	full name	smearing type	short name	type
1	index of overlap Dirac operator $s = 0.4$	–	index nonSmear $s = 0.4$	F
2	index of overlap Dirac operator $s = 0.0$	–	index nonSmear $s = 0$	F
3	index of overlap Dirac operator $s = 0.0$	HYP1	index HYP1 $s = 0$	F
4	Wilson-Dirac op. spectral flow $s = 0.0$	HYP1	SF HYP1 $s = 0.0$	F
5	Wilson-Dirac op. spectral flow $s = 0.75$	HYP1	SF HYP1 $s = 0.75$	F
6	Wilson-Dirac op. spectral flow $s = 0.0$	HYP5	SF HYP5 $s = 0.0$	F
7	Wilson-Dirac op. spectral flow $s = 0.5$	HYP5	SF HYP5 $s = 0.5$	F
8	spectral projectors $M^2 = 0.00003555$	–	spec. proj. $M^2 = 0.0000355$	F
9	spectral projectors $M^2 = 0.0004$	–	spec. proj. $M^2 = 0.0004$	F
10	spectral projectors $M^2 = 0.0010$	–	spec. proj. $M^2 = 0.0010$	F
11	spectral projectors $M^2 = 0.0015$	–	spec. proj. $M^2 = 0.0015$	F
12	field theoretic (clover)	–	cFT nonSmear	G
13	field theoretic (plaquette)	GF (Wplaq,t_0)	pFT GF Wplaq t_0	G
14	field theoretic (plaquette)	GF (Wplaq,$2t_0$)	pFT GF Wplaq $2t_0$	G
15	field theoretic (plaquette)	GF (Wplaq,$3t_0$)	pFT GF Wplaq $3t_0$	G
16	field theoretic (clover)	GF (Wplaq,t_0)	cFT GF Wplaq t_0	G
17	field theoretic (clover)	GF (Wplaq,$2t_0$)	cFT GF Wplaq $2t_0$	G
18	field theoretic (clover)	GF (Wplaq,$3t_0$)	cFT GF Wplaq $3t_0$	G
19	field theoretic (improved)	GF (Wplaq,t_0)	iFT GF Wplaq t_0	G
20	field theoretic (improved)	GF (Wplaq,$2t_0$)	iFT GF Wplaq $2t_0$	G
21	field theoretic (improved)	GF (Wplaq,$3t_0$)	iFT GF Wplaq $3t_0$	G
22	field theoretic (clover)	GF (tlSym,t_0)	cFT GF tlSym t_0	G
23	field theoretic (clover)	GF (tlSym,$2t_0$)	cFT GF tlSym $2t_0$	G
24	field theoretic (clover)	GF (tlSym,$3t_0$)	cFT GF tlSym $3t_0$	G
25	field theoretic (clover)	GF (Iwa,t_0)	cFT GF Iwa t_0	G
26	field theoretic (clover)	GF (Iwa,$2t_0$)	cFT GF Iwa $2t_0$	G
27	field theoretic (clover)	GF (Iwa,$3t_0$)	cFT GF Iwa $3t_0$	G
28	field theoretic (clover)	cool (Wplaq,t_0)	cFT cool (GF Wplaq t_0)	G
29	field theoretic (clover)	cool (Wplaq,$3t_0$)	cFT cool (GF Wplaq $3t_0$)	G
30	field theoretic (clover)	cool (tlSym,t_0)	cFT cool (GF tlSym t_0)	G
31	field theoretic (clover)	cool (tlSym,$3t_0$)	cFT cool (GF tlSym $3t_0$)	G
32	field theoretic (clover)	cool (Iwa,t_0)	cFT cool (GF Iwa t_0)	G
33	field theoretic (clover)	cool (Iwa,$3t_0$)	cFT cool (GF Iwa $3t_0$)	G
34	field theoretic (clover)	stout (0.01,t_0)	cFT stout 0.01 (GF Wplaq t_0)	G
35	field theoretic (clover)	stout (0.01,$3t_0$)	cFT stout 0.01 (GF Wplaq $3t_0$)	G
36	field theoretic (clover)	stout (0.1,t_0)	cFT stout 0.1 (GF Wplaq t_0)	G
37	field theoretic (clover)	stout (0.1,$3t_0$)	cFT stout 0.1 (GF Wplaq $3t_0$)	G
38	field theoretic (clover)	APE (0.4,t_0)	cFT APE 0.4 (GF Wplaq t_0)	G
39	field theoretic (clover)	APE (0.4,$3t_0$)	cFT APE 0.4 (GF Wplaq $3t_0$)	G
40	field theoretic (clover)	APE (0.5,t_0)	cFT APE 0.5 (GF Wplaq t_0)	G
41	field theoretic (clover)	APE (0.5,$3t_0$)	cFT APE 0.5 (GF Wplaq $3t_0$)	G
42	field theoretic (clover)	APE (0.6,t_0)	cFT APE 0.6 (GF Wplaq t_0)	G
43	field theoretic (clover)	APE (0.6,$3t_0$)	cFT APE 0.6 (GF Wplaq $3t_0$)	G
44	field theoretic (clover)	HYP (t_0)	cFT HYP (GF Wplaq t_0)	G
45	field theoretic (clover)	HYP ($3t_0$)	cFT HYP (GF Wplaq $3t_0$)	G



0.6
0.5
0.4
0.3
0.2
0.1
0
$r_0 \chi^{1/4}$
1 2 3 4 5 6 7 8 9 10 11 12 16 18 22 24 25 27 34 35 28 30 40 41 44 45
definition
noSmear s=0.4
noSmear s=0
HYP1 s=0
HYP1 s=0
HYP5 s=0
HYP1 s=0.75
HYP5 s=0.5
M²=0.001
M²=0.0015
M²=0.0004
M²=0.00003555
noSmear
Wplaq t_0 $3t_0$
tlSym t_0 $3t_0$
Iwa t_0 $3t_0$
stout t_0 $3t_0$
cool Wplaq t_0 $3t_0$
APE t_0 $3t_0$
HYP t_0 $3t_0$
index of overlap
spectral flow
spectral projectors
cFT noSmear
cFT GF
cFT stout
cFT cooling
cFT APE
cFT HYP

T=0 results satisfactory

(of course there is room for improvement)

From now on:

High Temperature

The two faces of Hot QCD topology

Window to Axions

Property of Quark Gluon Plasma

Tomorrow

Our setup at a glance

Hot QCD and
Nf 2+1+1 twisted mass
Wilson fermions

up,down
in the isospin limit

Physical strange

Physical charm

Why Nf = 2 + 1 + 1? Why Wilson twisted?

QCD Symmetries, lattice and the real world

NB: Doubling

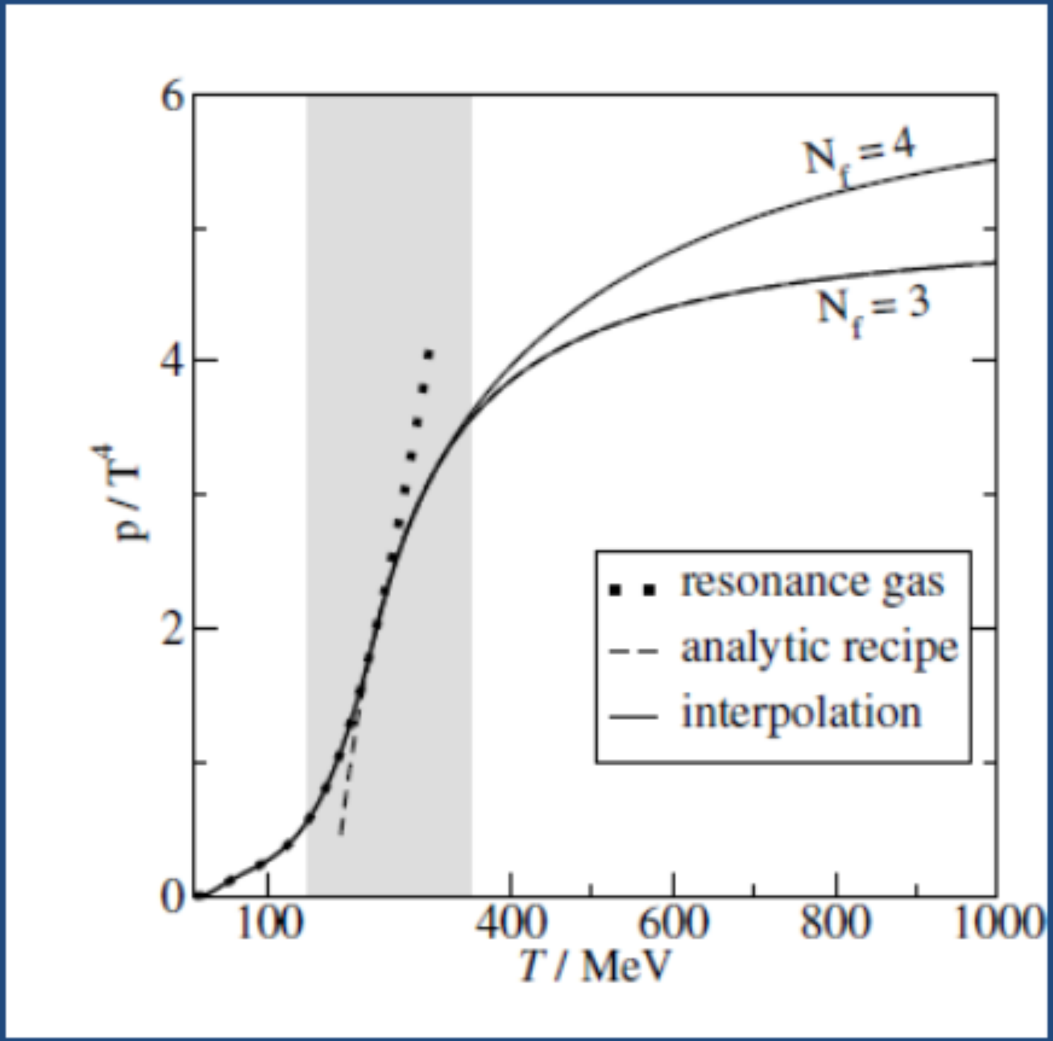
Good compromise

Why Nf = 2 +1 +1 ?

Tc ≈200MeV

340 –380 MeV RHIC AuAu 200 GeV	420-480 MeV LHC 2.76 TeV	500- 600MeV LHC hot spots 2.76 TeV	→ 1 GeV LHC 7 TeV

Quark Gluon Plasma @ Colliders



Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

Laine Schroeder 2006

Setup

Fixed varying scale

For each lattice spacing we explore a range of temperatures 150MeV — 500 MeV by varying Nt

We repeat this for three different lattice spacings following ETMC T=0 simulations.

Advantages: we rely on the setup of ETMC T=0 simulations. Scale is set once for all.

Disadvantages: mismatch of temperatures - need interpolation before taking the continuum limit

Four pion masses

Number of flavours	$m_{\pi^\pm}$
$N_f = 2+1+1$	210
	260
	370
	470
$N_f = 2$	360
	430

$T=0$ (ETMC) nomenclature	β	a [fm] [6]	N_σ^3	N_τ	T [MeV]	# confs.
A60.24	1.90	0.0936(38)	24^3	5	422(17)	585
				6	351(14)	1370
				7	301(12)	341
				8	263(11)	970
				9	234(10)	577
				10	211(9)	525
				11	192(8)	227
				12	176(7)	1052
			32^3	13	162(7)	294
				14	151(6)	1988
B55.32	1.95	0.0823(37)	32^3	5	479(22)	595
				6	400(18)	345
				7	342(15)	327
				8	300(13)	233
				9	266(12)	453
				10	240(11)	295
				11	218(10)	667
				12	200(9)	1102
				13	184(8)	308
				14	171(8)	1304
				15	160(7)	456
				16	150(7)	823
D45.32	2.10	0.0646(26)	32^3	6	509(20)	403
				7	436(18)	412
				8	382(15)	416
				10	305(12)	420
				12	255(10)	380
				14	218(9)	793
				16	191(8)	626
			40^3	18	170(7)	599
			48^3	20	153(6)	582