Spectral quantities in thermal QCD

II: mesons, transport, baryons

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Thermal QCD

all Green functions related by analyticity and KMS conditions

- QCD: strongly interacting system
- do not rely on perturbation theory
- lattice simulations
- yield numerically determined euclidean correlators

relation between Euclidean correlators and spectral functions

$$G^E(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega)$$

analyse for mesons and baryons

Thermal spectral functions

what to expect?

- already encountered single-particle spectral function

$$\rho_{\mathbf{k}}(\omega) = 2\pi\epsilon(\omega)\delta\left(\omega^2 - \omega_{\mathbf{k}}^2\right)$$

- single-particle peak at $\omega = \pm\omega_{\mathbf{k}}$
- relevant for QCD at $T = 0$: spectrum of hadrons

increase the temperature:

- in-medium effects
- thermal broadening
- deconfinement
- symmetry restoration
- transport
- ...

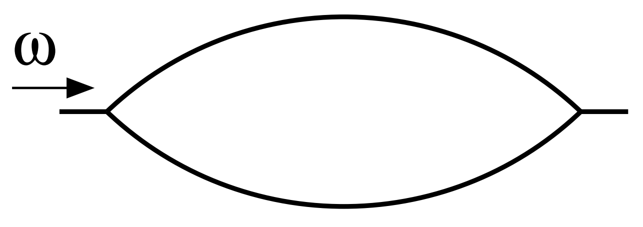
discuss this for mesons and baryons

Mesons

- quark $+$ anti-quark: simplest operator $O_H = \bar{\psi}\Gamma_H\psi$
- Γ_H depends on the channel:

scalar	$\Gamma_H = \mathbb{1}$
pseudoscalar	$\Gamma_H = \gamma_5$
vector	$\Gamma_H = \gamma_\mu$
axial-vector	$\Gamma_H = \gamma_\mu\gamma_5$

- simple diagram



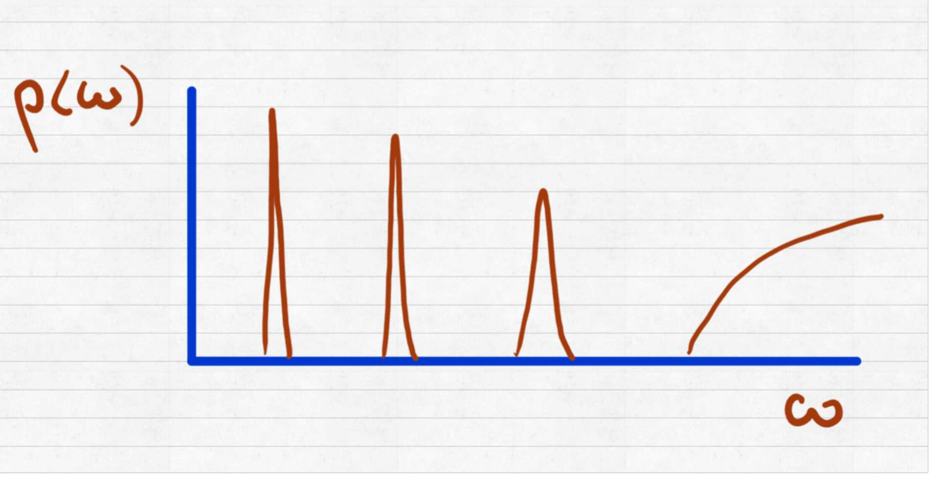
- for flavour singlets: also disconnected diagrams (typically not included)

Mesonic spectral functions

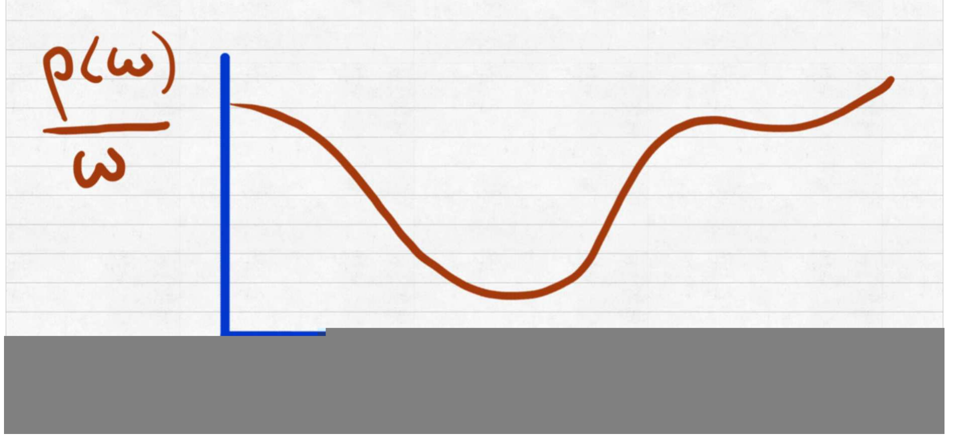
what to expect?

consider two extremes:

- at $T = 0$: QCD spectrum single-particle peaks at $\omega = M$ + excited states



- at $T \gg T_c$: deconfined plasma (quasi-)free quarks and gluons



Example: Vector channel

electromagnetic current $j_\mu(t, \mathbf{x}) = \bar{\psi}(t, \mathbf{x})\gamma_\mu\psi(t, \mathbf{x})$

spectral function $\rho_{\mu\nu}(t - t', \mathbf{x} - \mathbf{x}') = \langle[j_\mu(t, \mathbf{x}), j_\nu(t', \mathbf{x}')]\rangle$

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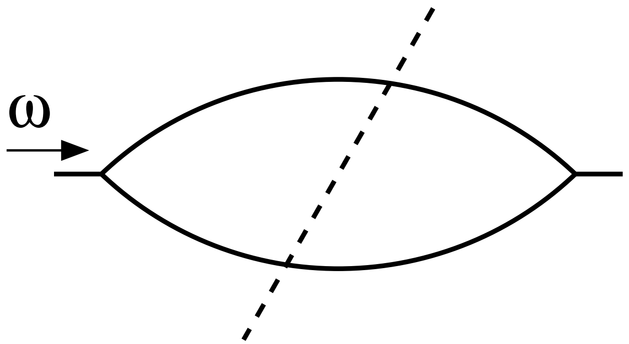
- photon production, rate $\sim n_\text{B}(k)/k\,\rho_{\mu\mu}(k,\mathbf{k})$
- dilepton production, rate $\sim n_\text{B}(\omega)/M^2\,\rho_{\mu\mu}(\omega,\mathbf{k})$
  with $M^2 = \omega^2 - \mathbf{k}^2$ and $m_\ell \sim 0$

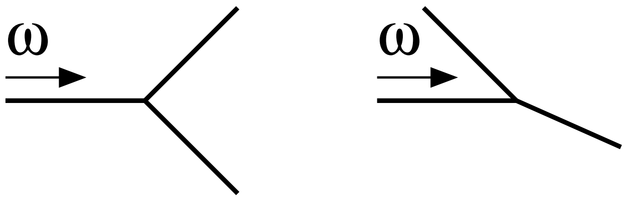
Mesonic spectral functions

what to expect at $T \gg T_c$? GA & Martínez Resco, hep-lat/0507004

- use perturbation theory: lowest-order diagram
- same computation as imaginary part of self-energy
- spectral function: cut diagram



- put internal lines onshell
- two processes:



- decay: $\omega^2 > \mathbf{p}^2 + 4m^2$
- scattering: $\omega^2 < \mathbf{p}^2$
  below the lightcone only, Landau damping

Mesonic spectral functions

meson spectral function at leading order in g^2

$$\rho_H(\omega,\mathbf{p}) = 2\pi N_c \int_{\mathbf{k}} \delta(\mathbf{r}-\mathbf{p}-\mathbf{k}) \Bigg\{$$

$$\left(a_H^{(1)} + a_H^{(2)}\frac{\mathbf{k}\cdot\mathbf{r}}{\omega_{\mathbf{k}}\omega_{\mathbf{r}}} + a_H^{(3)}\frac{m^2}{\omega_{\mathbf{k}}\omega_{\mathbf{r}}}\right)\left[n_F(\omega_{\mathbf{k}}) - n_F(\omega_{\mathbf{r}})\right]\delta(\omega+\omega_{\mathbf{k}}-\omega_{\mathbf{r}})$$

$$+\left(a_H^{(1)} - a_H^{(2)}\frac{\mathbf{k}\cdot\mathbf{r}}{\omega_{\mathbf{k}}\omega_{\mathbf{r}}} - a_H^{(3)}\frac{m^2}{\omega_{\mathbf{k}}\omega_{\mathbf{r}}}\right)\left[1 - n_F(\omega_{\mathbf{k}}) - n_F(\omega_{\mathbf{r}})\right]\delta(\omega-\omega_{\mathbf{k}}-\omega_{\mathbf{r}})$$

$$-(\omega \to -\omega)\Bigg\}$$

- coefficients $a_H^{(i)}$ depend on the channel
- scattering:

$$n_F(\omega_{\mathbf{k}}) - n_F(\omega_{\mathbf{r}}) = n_F(\omega_{\mathbf{k}})[1-n_F(\omega_{\mathbf{r}})] - [1-n_F(\omega_{\mathbf{k}})]n_F(\omega_{\mathbf{r}})$$

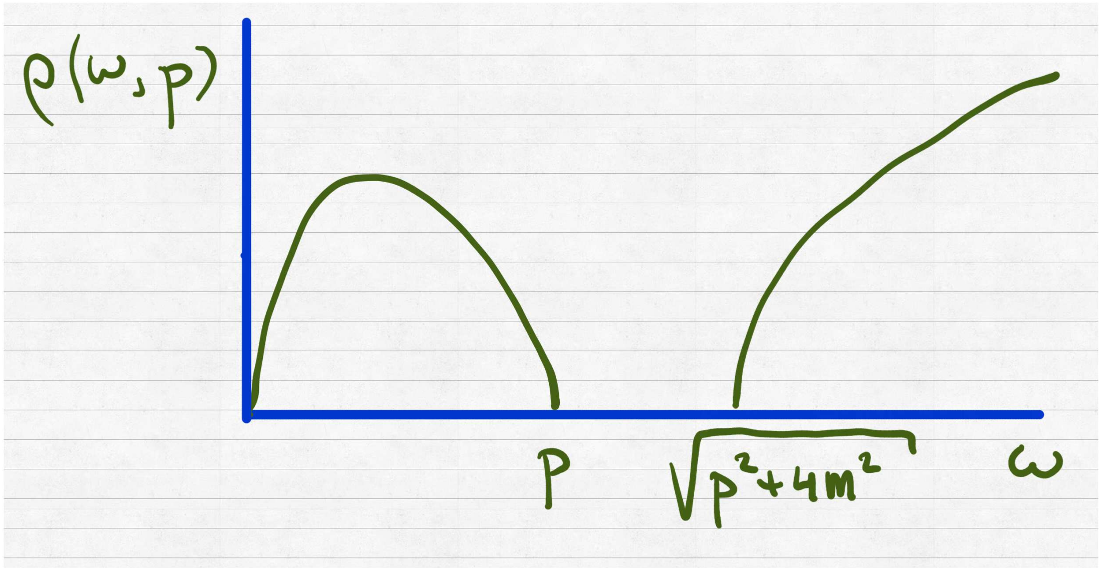
- decay: $1 - n_F(\omega_{\mathbf{k}}) - n_F(\omega_{\mathbf{r}})$

$$= [1-n_F(\omega_{\mathbf{k}})][1-n_F(\omega_{\mathbf{r}})] - n_F(\omega_{\mathbf{k}})n_F(\omega_{\mathbf{r}})$$

Mesonic spectral functions

what to expect at $T \gg T_c$?

- decay: $\omega^2 > \mathbf{p}^2 + 4m^2$
- scattering: $\omega^2 < \mathbf{p}^2$ below the lightcone



- higher-order interactions will fill in the gap

consider one particular higher-order effect

Hydrodynamic limit

- higher-order diagrams: new physics enters
- transport and hydrodynamics: vector channel
- diffusion of conserved charge
- long (transport) time scales $\tau_{\rm tr} = 1/\Gamma \sim 1/g^4T$
- form of spectral function dictated by diffusion equation and current conservation

$$\partial_t n(\mathbf{x},t) = D\nabla^2 n(\mathbf{x},t) \qquad\qquad \partial_t n(\mathbf{x},t) + \nabla\cdot\mathbf{j}(\mathbf{x},t) = 0$$

- diagrams: extensive resummation, at leading order
- transport peak at zero momentum

$$\rho_{ii}(\omega,\mathbf{0}) \sim \frac{\Gamma\omega}{\omega^2+\Gamma^2}$$

- slope at $\omega = 0$: conductivity $\sigma = D\chi \sim 1/\Gamma$

Conductivity/diffusion

- electrical conductivity σ
- charge susceptibility χ
- both σ and χ proportional to EM factor

$$C_{\text{em}} = e^2 \sum_f q_f^2 \qquad\qquad q_f = \frac{2}{3}, -\frac{1}{3}$$

- diffusion coefficient $D = \sigma/\chi$
- C_{em} cancels
- in SU(N_c) theories, factors of N_c cancel
- finite large N_c limit
- weak coupling: $D \sim 1/g^4 T$
- strong coupling: $D = 1/2\pi T$ (holography)

Conductivity/diffusion

- linear response: Kubo relation

$$\sigma = \lim_{\omega \to 0} \frac{1}{6\omega} \rho_{ii}(\omega, \mathbf{0})$$

- spectral function

$$\rho_{\mu\nu}(t, \mathbf{x}) = \langle [j_\mu(t, \mathbf{x}), j_\nu(0, \mathbf{0})] \rangle$$

- current-current spectral function, j_μ is EM current

some FASTSUM results (see below for details)

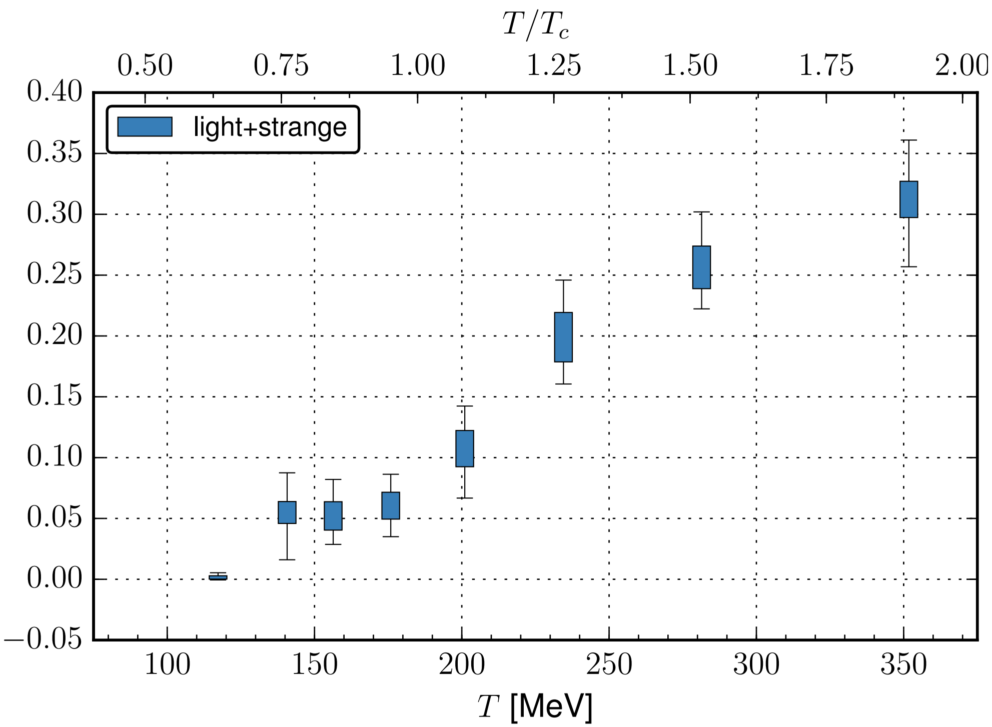
PRL 111 (2013) 172001 [arXiv:1307.6763 [hep-lat]]

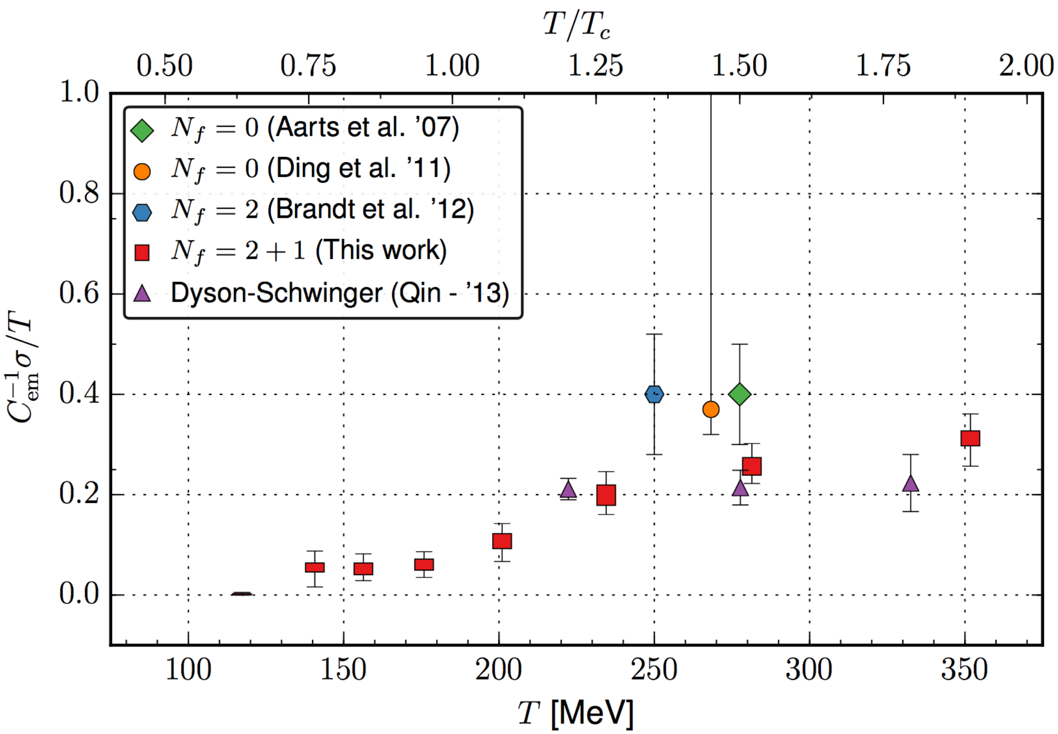
JHEP 02 (2015) 186 [arXiv:1412.6411 [hep-lat]]

see also lectures by Olaf

Conductivity

- conductivity $C_{\mathrm{em}}^{-1}\sigma/T$ $\qquad C_{\mathrm{em}} = e^2 \sum_f q_f^2$



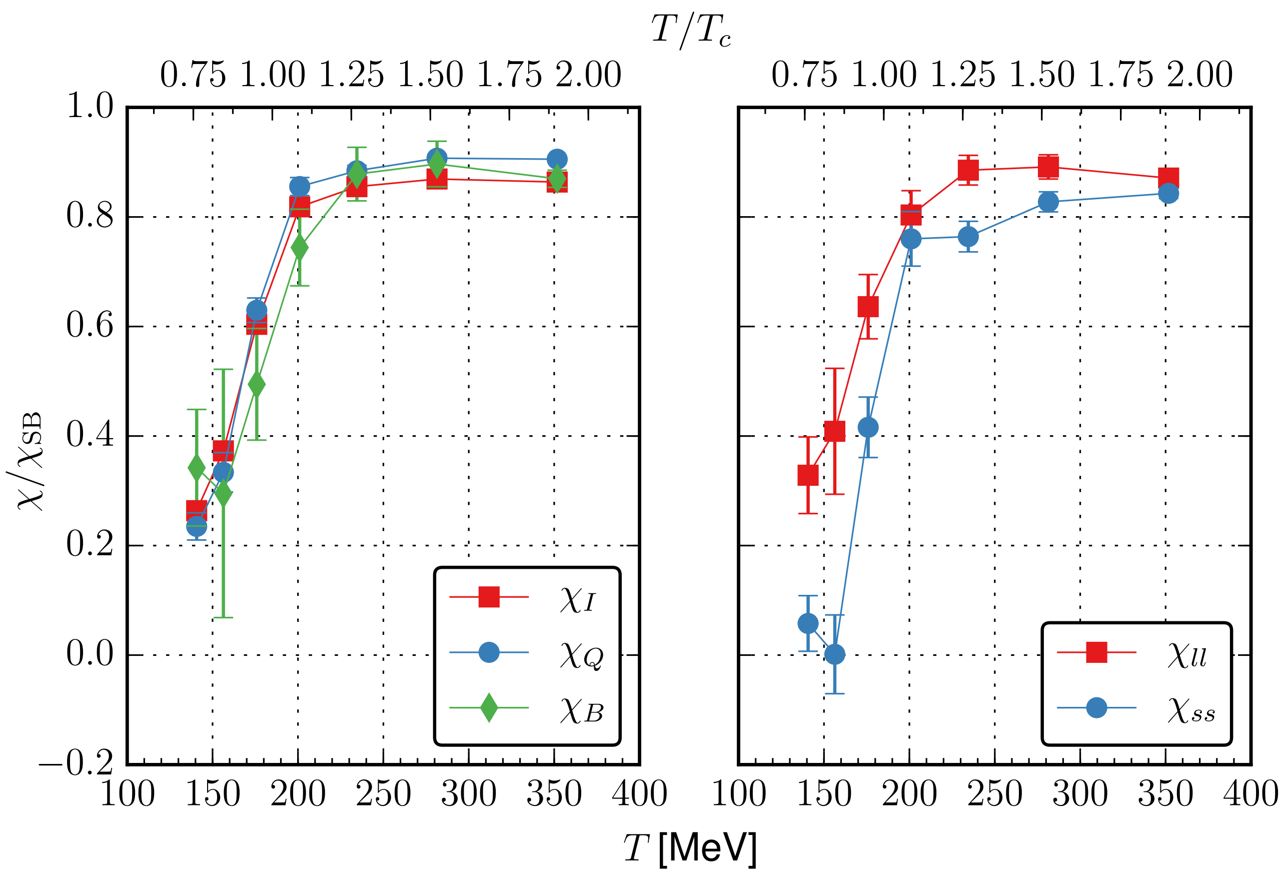


- temperature dependent
- agreement with previous results above T_c

Ding et al 2011: Kaczmarek, Brandt et al 2012: Francis

Susceptibilies

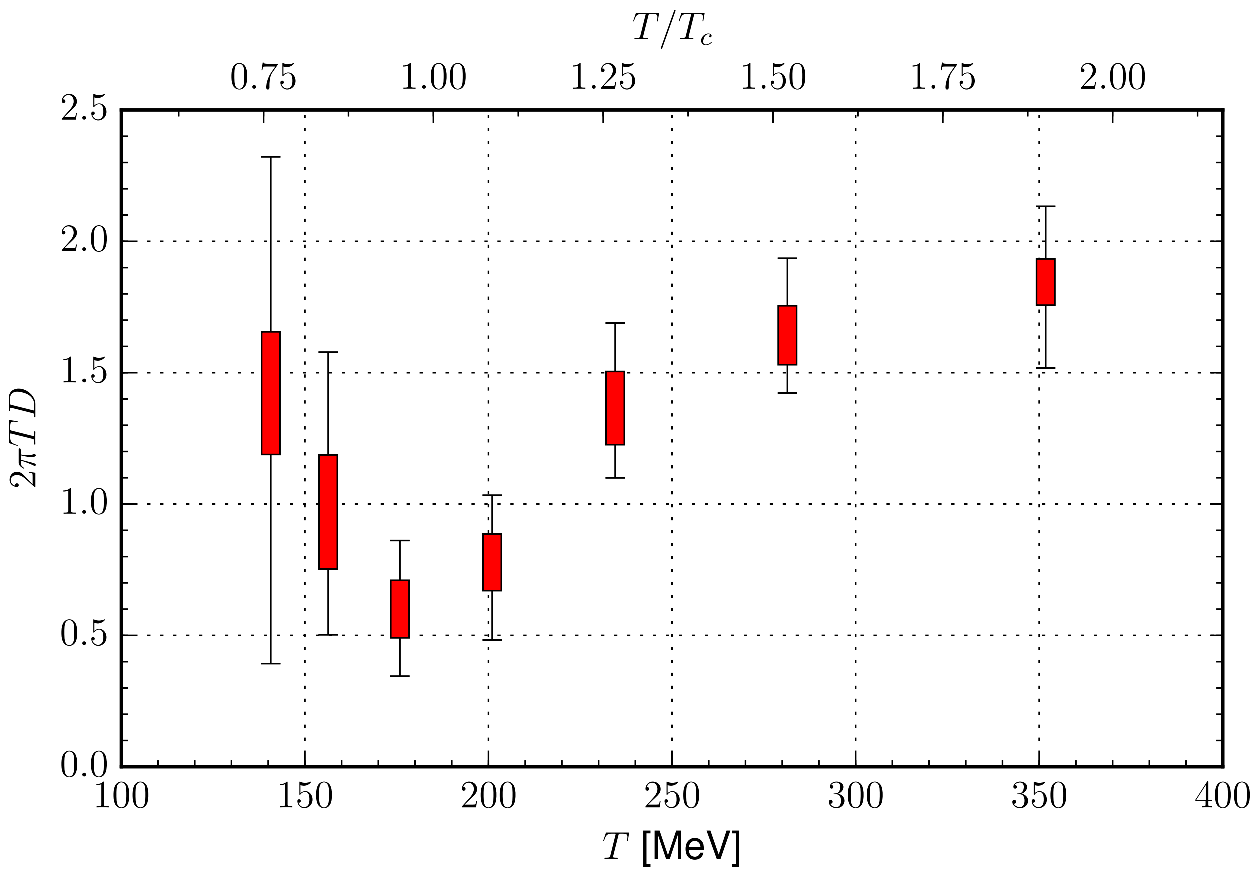
- fluctuations of isospin, electrical charge, baryon number, flavour



- agreement with previous (mostly staggered) results
- some flavour dependence

Diffusion coefficient

- combination of results: $D = \sigma/\chi_Q$



- consistent with strongly coupled plasma, $2\pi TD \sim 1$
- minimum around transition, c.f. η/s

Mesons in a medium

mesons in a medium very well studied

- hadronic phase: thermal broadening, mass shift
- QGP: deconfinement/dissolution/melting
- quarkonia survival as thermometer
- transport: conductivity/dileptons from vector current
- chiral symmetry restoration

relatively easy on the lattice

- high-precision correlators

what about baryons?

Baryons in a medium

lattice studies of baryons at finite temperature very limited

- screening masses De Tar and Kogut 1987
- ... with a small chemical potential QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators Datta, Gupta, Mathur et al 2013

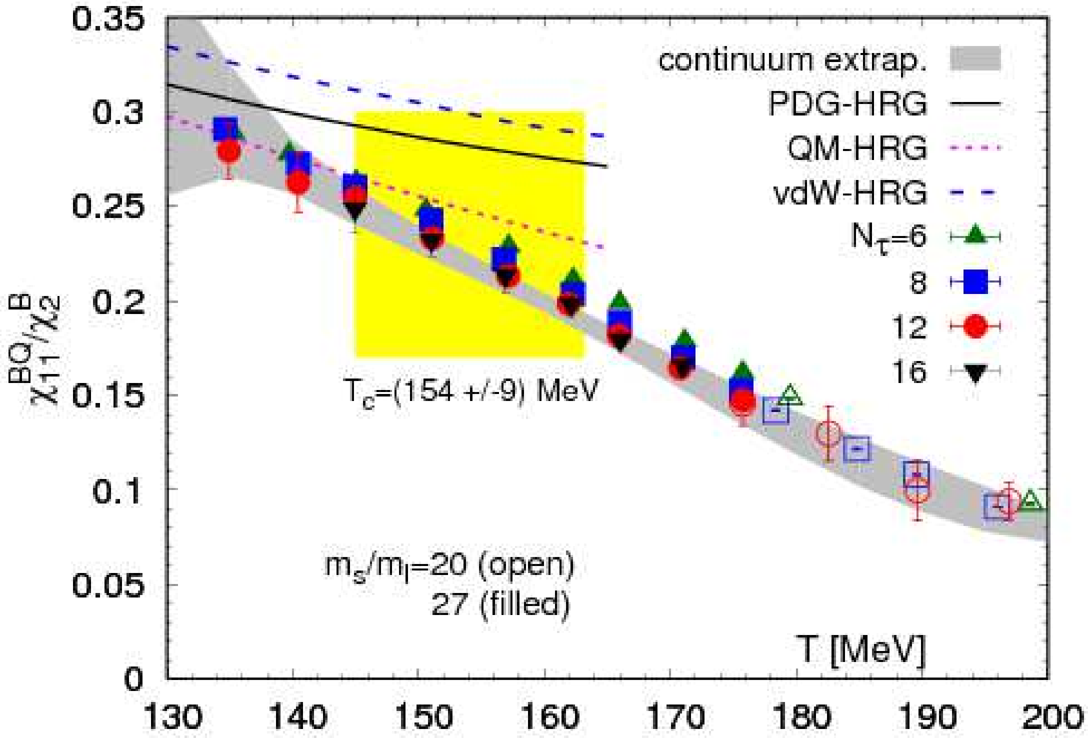
not much more ...

- effective models, mostly at $T \sim 0$ and nuclear density
  $\Rightarrow$ parity doubling models De Tar and Kunihiro 1989

but understanding highly relevant for e.g. hadron resonance gas (HRG) descriptions in confined phase

Baryons and HRG

ratio of fluctuations: $\langle BQ \rangle / \langle BB \rangle$
fluctuations of charged baryons / fluctuations of all baryons



Karsch (HotQCD)

arXiv:1706.01620

standard HRG
is somewhat off

- what is the source of this discrepancy?
- more states? residual interactions? in-medium effects?

Outline

baryons across the deconfinement transition:

- some basic thermal field theory
- lattice QCD – FASTSUM collaboration
- baryon correlators
- in-medium effects below T_c
- parity doubling above T_c
- spectral functions

FASTSUM: PRD 92 (2015) 014503 [arXiv:1502.03603 [hep-lat]]

+ JHEP 06 (2017) 034 [arXiv:1703.09246 [hep-lat]]

+ in preparation

Baryons

correlators $\qquad G^{\alpha\alpha'}(x) = \langle O^{\alpha}(x)\,\overline{O}^{\alpha'}(0)\rangle$

examples: N, Δ, Ω baryons

$$O^{\alpha}_{N}(x) = \epsilon_{abc}\, u^{\alpha}_{a}(x) \left(d^{T}_{b}(x) C\gamma_5 u_c(x) \right)$$

$$O^{\alpha}_{\Delta,i}(x) = \epsilon_{abc} \left[2u^{\alpha}_{a}(x) \left(d^{T}_{b}(x) C\gamma_i u_c(x) \right) + d^{\alpha}_{a}(x) \left(u^{T}_{b}(x) C\gamma_i u_c(x) \right) \right]$$

$$O^{\alpha}_{\Omega,i}(x) = \epsilon_{abc}\, s^{\alpha}_{a}(x) \left(s^{T}_{b}(x) C\gamma_i s_c(x) \right)$$

with C charge conjugation matrix:

$$C^{\dagger}C = \mathbb{1} \qquad \gamma^{T}_{\mu} = -C\gamma_{\mu}C^{-1} \qquad C^{T} = -C^{-1}$$

action on fermionic operator:

$$\mathcal{C}O\mathcal{C}^{-1} = O^{(c)} = C^{-1}\bar{O}^{T} \qquad \mathcal{C}\bar{O}\mathcal{C}^{-1} = \bar{O}^{(c)} = -O^{T}C$$

Baryons

- essential difference with mesons: role of parity

$$\mathcal{P}O(\tau,\mathbf{x})\mathcal{P}^{-1} = \gamma_4 O(\tau,-\mathbf{x})$$

- positive/negative parity operators

$$O_\pm(x) = P_\pm O(x) \qquad\qquad P_\pm = \frac{1}{2}(1\pm\gamma_4)$$

- no parity doubling in Nature: nucleon ground state

  positive parity: $m_+ = m_N \;= 0.939$ GeV

  negative parity: $m_- = m_{N^*} = 1.535$ GeV

- thread: what happens as temperature increases?

Reminder: spectral properties – bosons

- bosonic operators

$$G(\tau, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega)\rho(\omega, \mathbf{p})$$

- kernel $(\tilde{\tau} = \tau - 1/2T)$

$$K_{\rm boson}(\tau, \omega) = \frac{\cosh(\omega\tilde{\tau})}{\sinh(\omega/2T)} = [1 + n_B(\omega)]\, e^{-\omega\tau} + n_B(\omega) e^{\omega\tau}$$

- kernel symmetric around $\tau = 1/2T$, odd in ω
- singular as $\omega \to 0$

$$\lim_{\omega\to 0} K_{\rm boson}(\tau, \omega) = \frac{2T}{\omega}$$

- relevant for transport GA & Martínez Resco, hep-ph/0203177

Spectral properties: fermions

$$G^{\alpha\alpha'}(\tau,\mathbf{p}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau,\omega)\rho^{\alpha\alpha'}(\omega,\mathbf{p})$$

with

$$G^{\alpha\alpha'}(x-x') = \langle O^{\alpha}(x)\overline{O}^{\alpha'}(x')\rangle$$

$$\rho^{\alpha\alpha'}(x-x') = \langle\{O^{\alpha}(x),\overline{O}^{\alpha'}(x')\}\rangle$$

- fermionic Matsubara frequencies

$$K(\tau,\omega) = T\sum_n \frac{e^{-i\omega_n\tau}}{\omega - i\omega_n} = \frac{e^{-\omega\tau}}{1+e^{-\omega/T}} = e^{-\omega\tau}\left[1-n_F(\omega)\right]$$

- kernel not symmetric, instead

$$K(1/T-\tau,\omega) = K(\tau,-\omega)$$

Kernels

- bosons $(\tilde{\tau} = \tau - 1/2T)$

$$K_{\rm boson}(\tau,\omega) = \frac{\cosh(\omega\tilde{\tau})}{\sinh(\omega/2T)} = [1 + n_B(\omega)]\, e^{-\omega\tau} + n_B(\omega) e^{\omega\tau}$$

- fermions: even and odd terms

$$K(\tau,\omega) = \frac{1}{2}\left[K_{\rm e}(\tau,\omega) + K_{\rm o}(\tau,\omega)\right],$$

$$K_{\rm e}(\tau,\omega) = \frac{\cosh(\omega\tilde{\tau})}{\cosh(\omega/2T)} = [1 - n_F(\omega)]\, e^{-\omega\tau} + n_F(\omega) e^{\omega\tau}$$

$$K_{\rm o}(\tau,\omega) = -\frac{\sinh(\omega\tilde{\tau})}{\cosh(\omega/2T)} = [1 - n_F(\omega)]\, e^{-\omega\tau} - n_F(\omega) e^{\omega\tau}$$

- no singular behaviour $2T/\omega$ for fermions, no transport subtlety

Reminder: spectral properties – bosons

- spectral decomposition

$$\rho(x) = \langle [O(x), O^\dagger(0)] \rangle = \frac{1}{Z} \mathrm{Tr}\, e^{-\beta H} [O(x), O^\dagger(0)]$$
$$= \frac{1}{Z} \sum_n e^{-\beta E_n} \langle n | [O(x), O^\dagger(0)] | n \rangle$$

- write out commutator and insert complete set of states $\sum_m |m\rangle\langle m| = \mathbb{1}$
- use $O(x) = e^{-ik\cdot x} O(0) e^{ik\cdot x}$, with $k^0 = H$
- go to momentum space

$$\rho(p) = \frac{1}{Z} \sum_{n,m} \left(e^{-k_n^0/T} - e^{-k_m^0/T} \right) |\langle n|O(0)|m\rangle|^2 \, (2\pi)^4 \delta^{(4)}(p + k_n - k_m)$$

- if $O^\dagger = \pm O \quad \Rightarrow \quad \omega\rho(\omega, \mathbf{p}) \geq 0$ positivity

Spectral decomposition: Positivity

$$\rho^{\alpha\beta}(x) = \sum \gamma_\mu^{\alpha\beta}\rho_\mu(x) + 1\!\!1^{\alpha\beta}\rho_m(x)$$

- take trace with γ_4, $P_\pm = (1\!\!1 \pm \gamma_4)/2$:

$$\rho_4(p) = \frac{1}{Z}\sum_{n,m,\alpha}\left(e^{-k_n^0/T} + e^{-k_m^0/T}\right)\frac{1}{4}\left|\langle n|O^\alpha(0)|m\rangle\right|^2 (2\pi)^4\delta^{(4)}(p+k_n-k_m)$$

$$\rho_\pm(p) = \frac{\pm 1}{Z}\sum_{n,m,\alpha}\left(e^{-k_n^0/T} + e^{-k_m^0/T}\right)\frac{1}{4}\left|\langle n|O_\pm^\alpha(0)|m\rangle\right|^2 (2\pi)^4\delta^{(4)}(p+k_n-k_m)$$

- $\rho_4(p), \pm\rho_\pm(p) \geq 0$ for all ω

- take trace with $1\!\!1$

$$\rho_m(p) = [\rho_+(p) + \rho_-(p)]/4$$

not sign definite

Spectral decomposition: Positivity

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Next lecture

some more formal properties of baryon correlators

and then on to recent lattice results