Exotic models of the very early Universe

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Outline

- Introduction: why not hot Big Bang from the start?
- Motivation for exotica
- Obstacles:
  - Null Energy Condition (NEC)
    - Instabilities
  - Belinsky–Lifshits–Khalatnikov
    - Cure: ekpyrosis
- Violating NEC: generalized Galileons (aka Horndeski)

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Why are we confident that the hot cosmological epoch was not the first?

Key: cosmological perturbations

- density perturbations and associated gravitational potentials (3d scalar), observed;
- gravitational waves (3d tensor), not observed

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

Linear analysis appropriate. Go to Fourier space.

Wealth of data.
Primordial perturbations well understood at linear level.

Ad absurdum: causality

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^{\,2}$$

Expanding Universe:

$a(t) \propto t^{1/2}$ at radiation domination epoch, before $T \simeq 1$ eV, $t \simeq 50$ thousand years

$a(t) \propto t^{2/3}$ later, until recently.

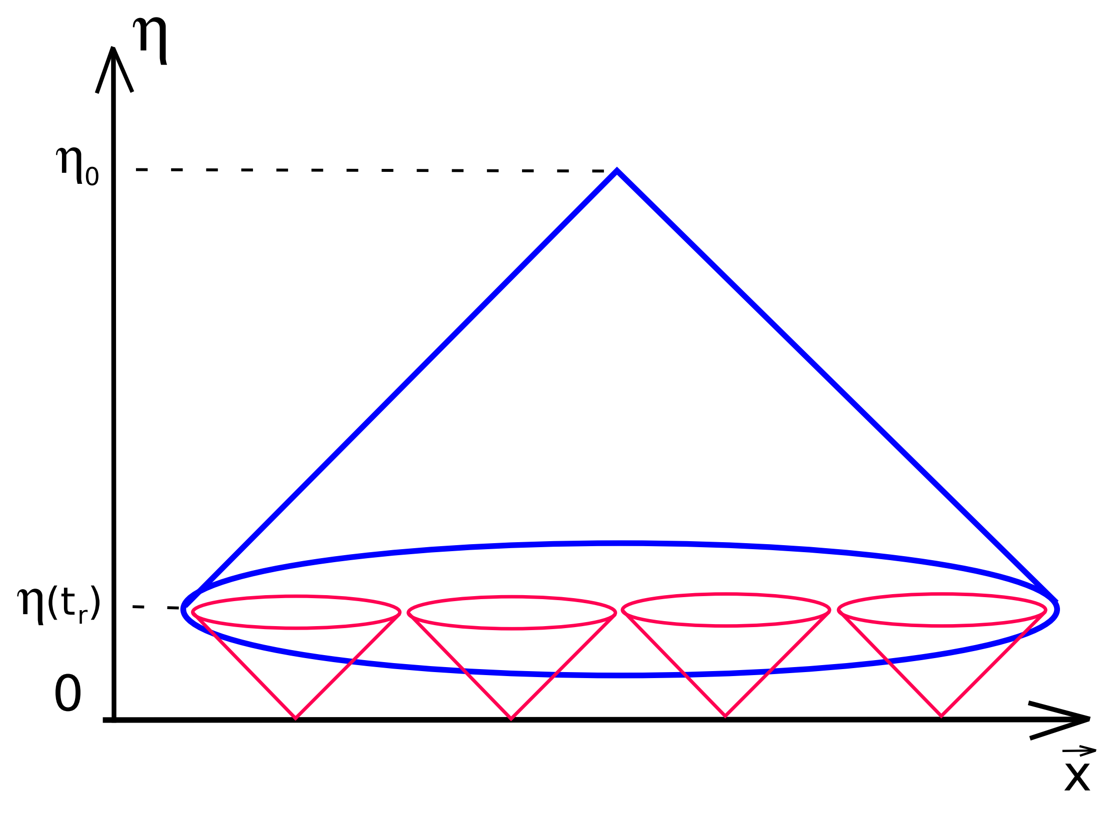
Assume that nothing preceeded hot epoch

Cosmological horizon: length that light travels from Big Bang moment,

$$l_H(t) = (2-3)ct$$

Causal structure of space-time in hot Big Bang theory (i.e., assuming that the Universe started right from the hot epoch)

$$\eta = \int \frac{dt}{a(t)} \, , \quad \text{conformal time}$$



Angular size of horizon at recombination $\approx 2^\circ$.

Causality $\Longrightarrow$ perturbations can be generated only when their wavelengths are smaller than horizon size.

Off-hand possibilities:

- Perturbations were generated at the hot cosmological epoch by some causal mechanism.

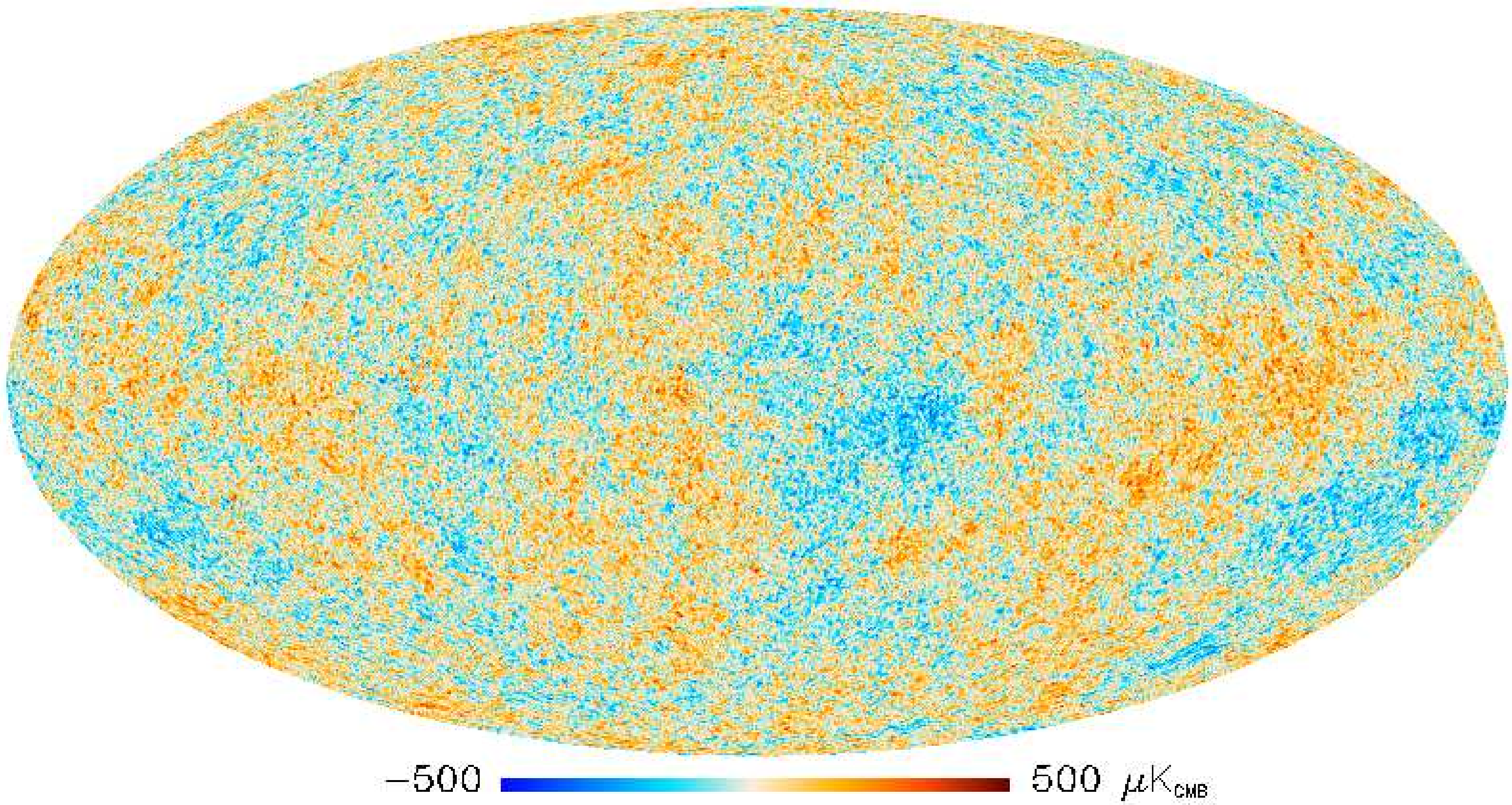
  E.g., seeded by topological defects (cosmic strings, etc.)

  N. Turok et.al.' 90s

  The only possibility, if expansion started from hot Big Bang.

- Hot epoch was preceded by some other epoch. Perturbations were generated then.

CMB: photographic picture of the Universe at recombination
$T = 3000$ K, $t = 380$ thousand years.



There are perturbations which were superhorizon at the time of recombination, angular scale $\gtrsim 2^\circ$. Causality: they could not be generated at hot epoch!

In more detail

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_H \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.

NB: Horizon entry occured after Big Bang Nucleosynthesis for perturbations of all relevant wavelengths $\Longleftrightarrow$ no guesswork.

Shorter wavelengths: perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase.

Reason: solutions to wave equation in superhorizon regime in expanding Universe

$$\frac{\delta\rho}{\rho} = \text{const} \qquad \text{and} \qquad \frac{\delta\rho}{\rho} = \frac{\text{const}}{t^{3/2}}$$

Assume that modes were superhorizon. Consistency of the picture: the Universe was not very inhomogeneous at early times, the initial condition is (up to amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \implies \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\implies$ phase of oscillations well defined.

Perturbations develop different phases by the time of photon last scattering (= recombination), depending on wave vector:

$$\frac{\delta\rho}{\rho}(t_r) \propto \cos\left(\int_0^{t_r} dt \; v_s \; \frac{k}{a(t)}\right)$$

(v_s = sound speed in baryon-photon plasma)

cf. Sakharov oscillations' 1965

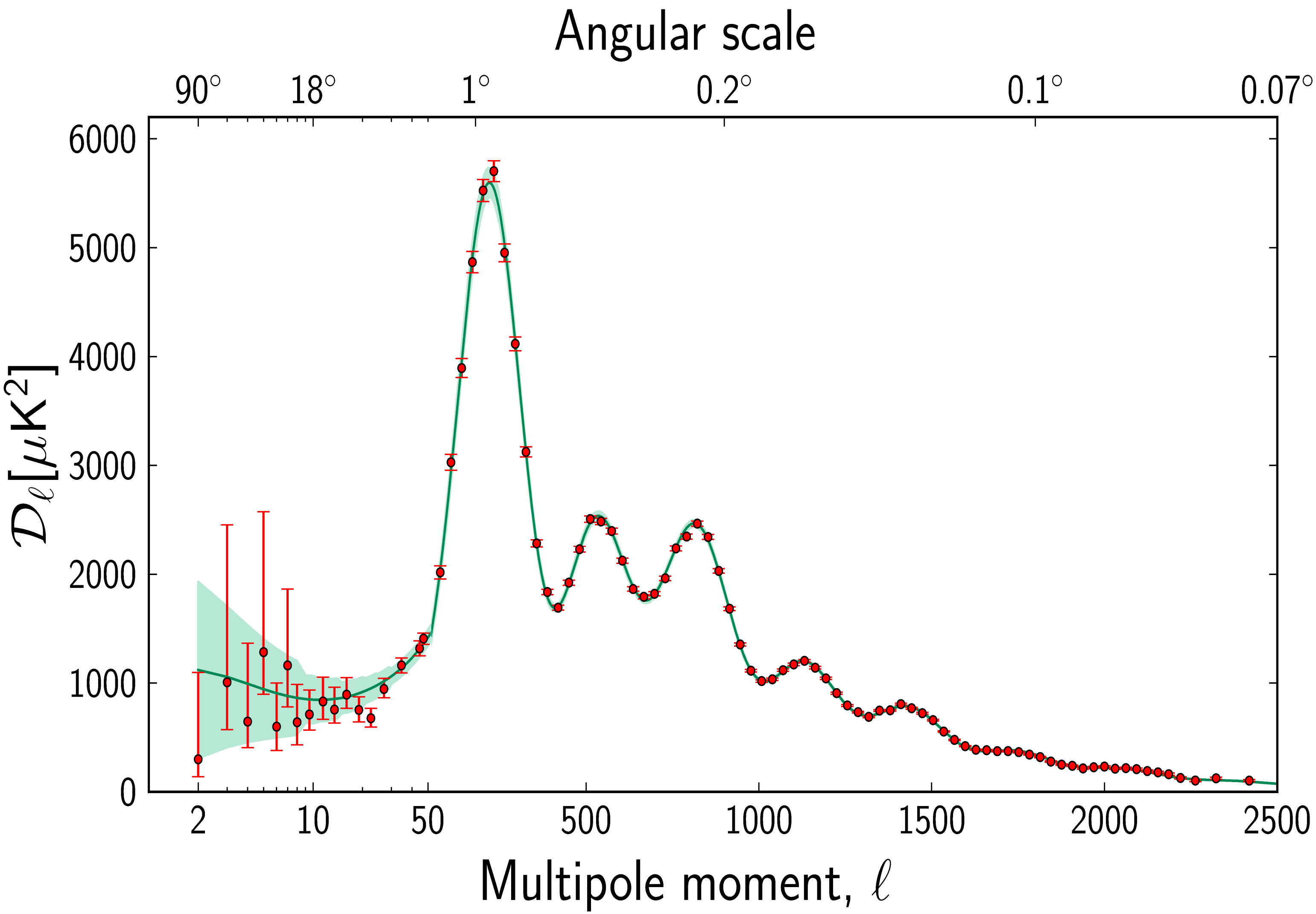
Oscillations in CMB temperature angular spectrum

Fourier decomposition of temperatue fluctuations over celestial sphere:

$$\delta T(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

$\langle a_{lm}^* a_{lm} \rangle = C_l$, temperature angular spectrum;

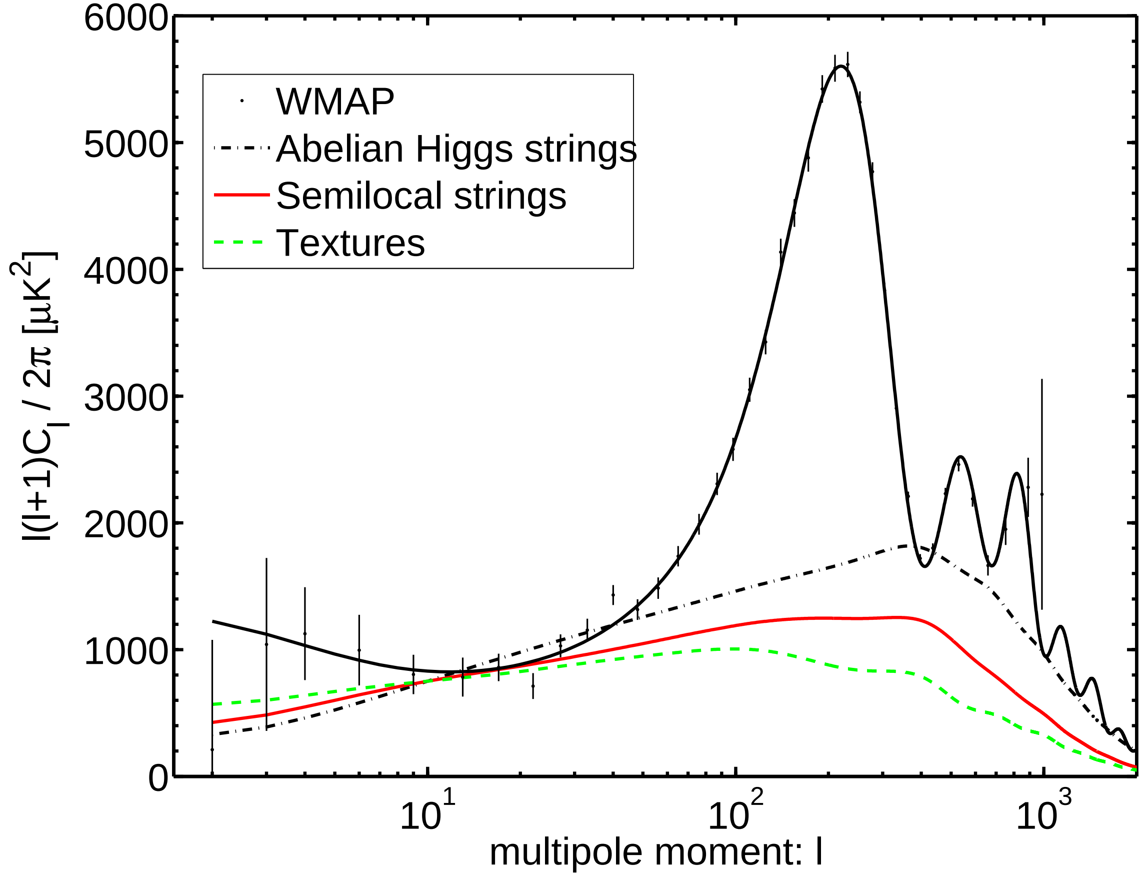
larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths



Planck

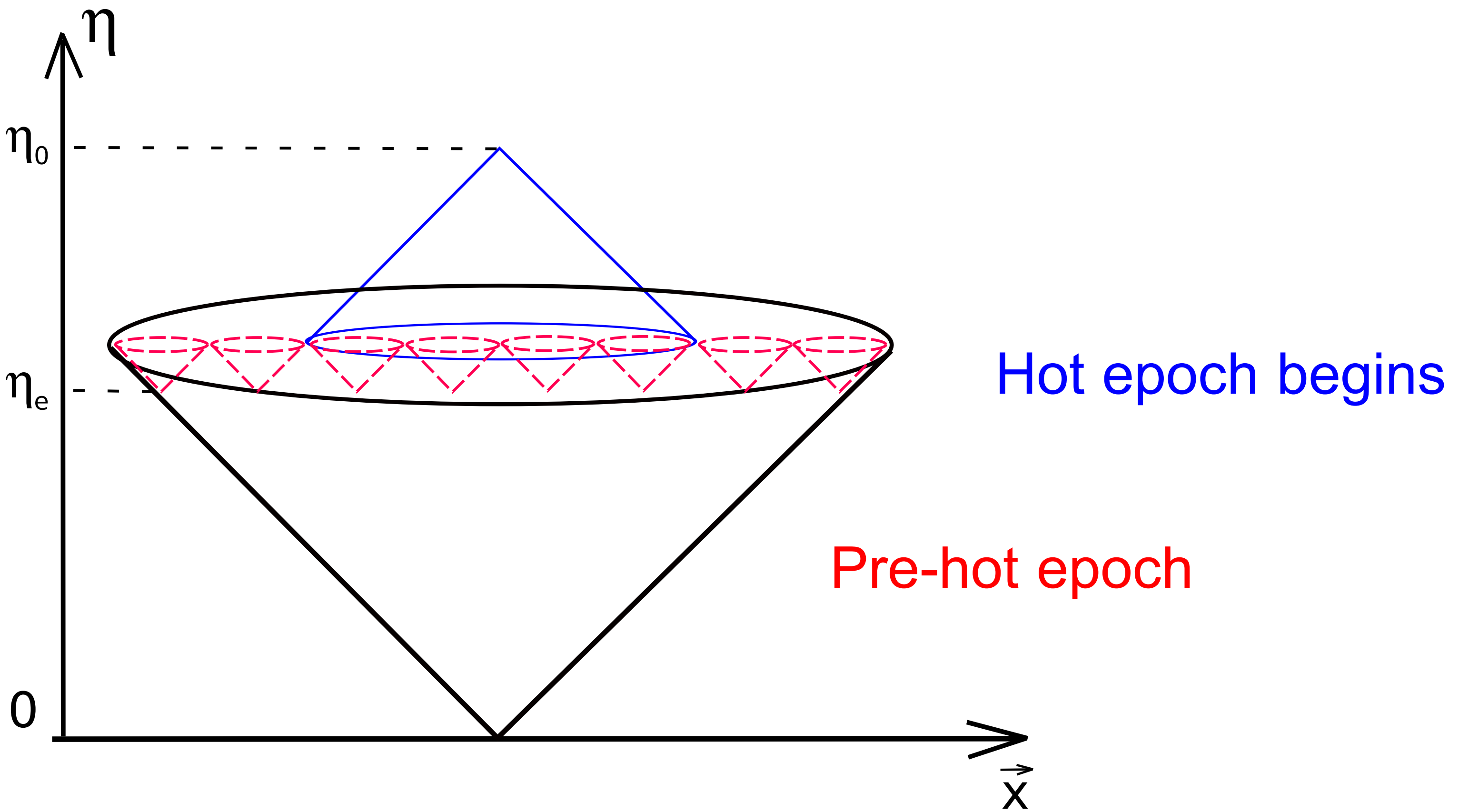
$$\mathscr{D}_l = \frac{l(l+1)}{2\pi} C_l$$

These properties would not be present if perturbations were generated at hot epoch in causal manner.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long (in conformal time) and unusual: perturbations were subhorizon early at that epoch, our visible part of the Universe was in a causally connected region.



Inflation does the job extremely well.

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Yet alternatives to inflation are of interest.

- It always makes sense to understand whether there are alternatives and what can they be

- Cosmological constant (CC) problem: relaxation.

  - "Natural" value of cosmological constant is at least Λ_{QCD}^4, which is 10^{44} times greater than actual value.

  - One option: anthropic considerations

  - If not, "CC" must be dynamical and relax to the very small value in the course of cosmological evolution.

    Whatever relaxation mechanism, towards the end of relaxation the Universe had to be much like at present – if energy density was much larger than present CC, which value to relax to?

Concrete proposals (admittedly very contrived):

V.R '1999; Steinhardt and Turok '2006; Khmelnitsky et. al. '2016

Alternatives to inflation:

- Bouncing Universe: contraction — bounce — expansion
- “Genesis”: start up from static, Minkowski state

Creminelli et.al.’06; ’10

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General obstruction: the Null Energy Condition, NEC

$$T_{\mu\nu} n^\mu n^\nu > 0$$

for any null vector n^μ, such that $n_\mu n^\mu = 0$.

- In the framework of classical General Relativity implies Penrose theorem:

Penrose' 1965

Once there is trapped surface, there is singularity in future.

Assumptions:

(i) The NEC holds

(ii) Cauchi hypersurface non-compact

Trapped surface:

a closed surface on which outward-pointing light rays actually converge (move inwards)

Spherically symmetric examples:

$$ds^2 = g_{00}dt^2 + 2g_{0R}dtdR + g_{RR}dR^2 - R^2 d\Omega^2$$

$4\pi R^2$: area of a sphere of constant t, R.

Trapped surface: R decreases along all light rays.

- Sphere inside horizon of Schwarzschild black hole
- Hubble sphere in contracting Universe $\Longrightarrow$

  Hubble sphere in expanding Universe = anti-trapped surface $\Longrightarrow$ singularity in the past.

  - No-go for bouncing Universe/Genesis scenarios?

- Homogeneous and isotropic spatially flat metric

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \ .$$

Hubble parameter $H = \dot{a}/a$.

- A combination of Einstein equations:

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

$\rho = T_{00}$ = energy density; $T_{ij} = \delta_{ij} p$ = effective pressure.

- The Null Energy Condition:

$T_{\mu\nu} n^\mu n^\nu \geq 0$, $n^\mu = (1, 1, 0, 0) \Longrightarrow \rho + p > 0 \Longrightarrow dH/dt < 0$,

Hubble parameter was greater early on.

At some moment in the past, there was a singularity, $H = \infty$.

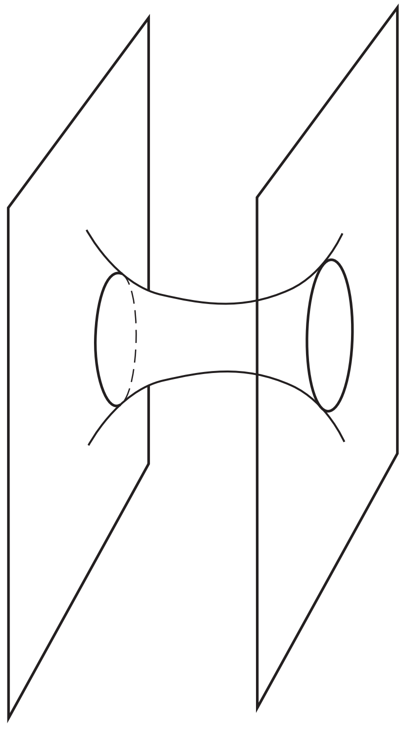
- Another side of the NEC: Covariant energy-momentum conservation:

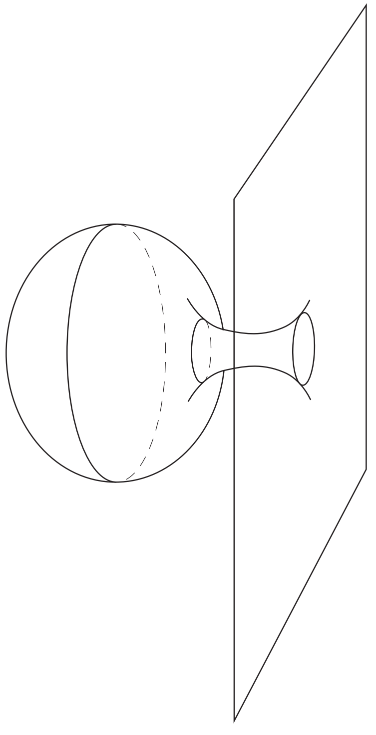
$$\frac{d\rho}{dt} = -3H(\rho + p)$$

NEC: energy density decreases during expansion, except for $p = -\rho$, cosmological constant.

Many other facets of the NEC,

like no-go for Lorentzian wormholes





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Can the Null Energy Condition be violated in classical field theory?

- Folklore until recently: NO!

PATHOLOGIES:

- Ghosts:

$$E = -\sqrt{p^2 + m^2}$$

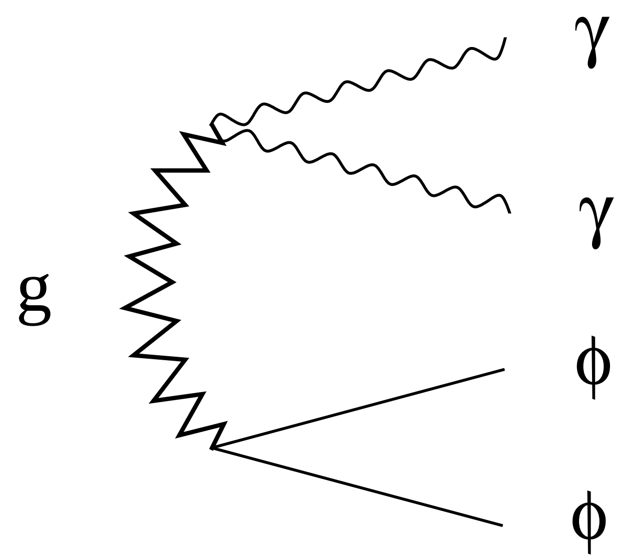
Example: theory with wrong sign of kinetic term,

$$\mathscr{L} = -(\partial\phi)^2 \quad \Longrightarrow \quad \rho = -\dot{\phi}^2 - (\nabla\phi)^2 \, , \qquad p = -\dot{\phi}^2 + (\nabla\phi)^2$$

$$\rho + p = -2\dot{\phi}^2 < 0$$

Catastrophic vacuum instability

Creation of ghosts plus ordinary particles from vacuum allowed:



Infinite phase space because of infinite volume of Lorentz group

$\Longrightarrow$ infinite rate

NB: Can be cured by Lorentz-violation

(but hard! – even though Lorentz-violation is inherent in cosmology)

Other pathologies

- Gradient instabilities:

$$E^2 = -(p^2 + m^2) \implies \varphi \propto \mathrm{e}^{|E|t}$$

- Superluminal propagation of excitations

Theory cannot descend from healthy Lorentz-invariant UV-complete theory

Adams et. al.' 2006

No-go theorem for theories with Lagrangians involving first derivatives of fields only (and minimal coupling to gravity)

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

$$L = F(X^{IJ}, \pi^I)$$

with $X^{IJ} = \partial_\mu \pi^I \partial^\mu \pi^J \Longrightarrow$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X^{IJ}} \partial_\mu \pi^I \partial_\nu \pi^J - g_{\mu\nu} F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} - F$$

$$T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} = 2\frac{\partial F}{\partial X^{IJ}} \dot{\pi}^I \dot{\pi}^J$$

NEC-violation: matrix $\partial F / \partial X_c^{IJ}$ non-positive definite.

But

Lagrangian for perturbations $\pi^I = \pi_c^I + \delta\pi^I$

$$L_{\delta\pi} = U_{IJ}\, \partial_t \delta\pi^I \cdot \partial_t \delta\pi^J - \frac{\partial F}{\partial X_c^{IJ}}\, \partial_i \delta\pi^I \cdot \partial_i \delta\pi^J + \dots$$

Gradient instabilities and/or ghosts

NB. Loophole: $\partial F / \partial X_c^{IJ}$ degenerate.

Higher derivative terms (understood in effective field theory sense) become important and help.

Ghost condensate

Arkani-Hamed et. al.' 2003

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Another problem for bouncing Universe:

Belinsky–Lifshits–Khalatnikov

Slightly anisotropic contraction

$$ds^2 = dt^2 - a^2(t) \cdot \sum_{a=1}^{3} \mathrm{e}^{2\beta_a(t)} dx^a dx^a \,, \qquad \sum_a \beta_a = 0 \,.$$

Einstein eqs give

$$\dot{\beta}_a = \frac{d_a}{a^3} \,, \quad d_a = \text{const} \,,$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{6a^6} \sum_a d_a^2 + \frac{8\pi}{3} G\rho \,.$$

“Normal” matter: $\rho \propto a^3$, a^4. Anisotropy dominates at small a,

$$a(t) = |t|^{1/3} \,, \quad \beta_a = d_a \ln|t| \,, \quad t < 0 \,.$$

Universe comes to bounce strongly anisotropic
(and inhomogeneous).

Cure: super-stiff matter ("ekpyrosis") $p = w\rho$, $w > 1$.

Covariant energy conservation $\Longrightarrow$ $\rho \propto a^{-3(1+w)}$, faster than a^{-6}. Matter dominates over anisotropy in "Friedmann eqn" $\Longrightarrow$

$$a(t) \propto |t|^{\frac{2}{3(1+w)}} \,, \quad a^{-3} \text{ less singular than } |t|^{-1} \,, \quad \beta_a \to \text{const} \,.$$

- Example: scalar field with negative exponential potential

Lehners et. al.' 07; Buchbinder, Khouri, Ovrut' 07; Creminelli, Senatore' 07

$$V(\phi) = -V_0 \, \mathrm{e}^{\phi/M} \,.$$

Attractor solution

$$a(t) = |t|^{16\pi \frac{M^2}{M_{Pl}^2}} \,, \quad \phi(t) = \text{const} - 2M \ln|t| \,, \quad w \propto \frac{M_{Pl}^2}{M^2} \gg 1$$

Positive energy density, consistent with expansion in future.

NB: Small $M \ll M_{Pl}$ $\Longrightarrow$ small $H \sim M^2/M_{Pl}^2 t^{-1}$, weak gravity regime: dynamics of ϕ does not feel expansion.

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Can the Null Energy Condition be violated in a simple and healthy way?

- Folklore until fairly recently: NO!

  Today: YES

Senatore' 2004;
V.R.' 2006;
Creminelli, Luty, Nicolis, Senatore' 2006

General properties of non-pathological NEC-violating field theories:

Non-standard kinetic terms

Non-trivial background, instability of Minkowski space-time

Example: scalar field, generalized Galileon $\pi(x^\mu)$,

$$L = F(X,\pi) + K(X,\pi)\cdot\Box\pi$$

$$\Box\pi \equiv \nabla_\mu\nabla^\mu\pi\ , \qquad X = (\partial_\mu\pi)^2$$

- Second order equations of motion (but L cannot be made first order by integration by parts)
- Generalization: Horndeski theory (1974) rediscovered several times

Fairlie, Govaerts, Morozov' 91;
Nicolis, Rattazzi, Trincherini' 09, ...

Minkowski:

$$L_n = K_n(X,\pi)\partial^{\mu_1}\partial_{[\mu_1}\pi\cdot\dots\cdot\partial^{\mu_n}\partial_{\mu_n]}\pi$$

Five Lagrangians in 4D, including $K_0 \equiv F$

Generalization to GR: L_0, L_1 trivial, $L_{n>1}$ non-trivial

Deffayet, Esposito-Farese, Vikman' 09