Universität Bielefeld

Lattice QCD results on soft and hard probes of strongly interacting matter

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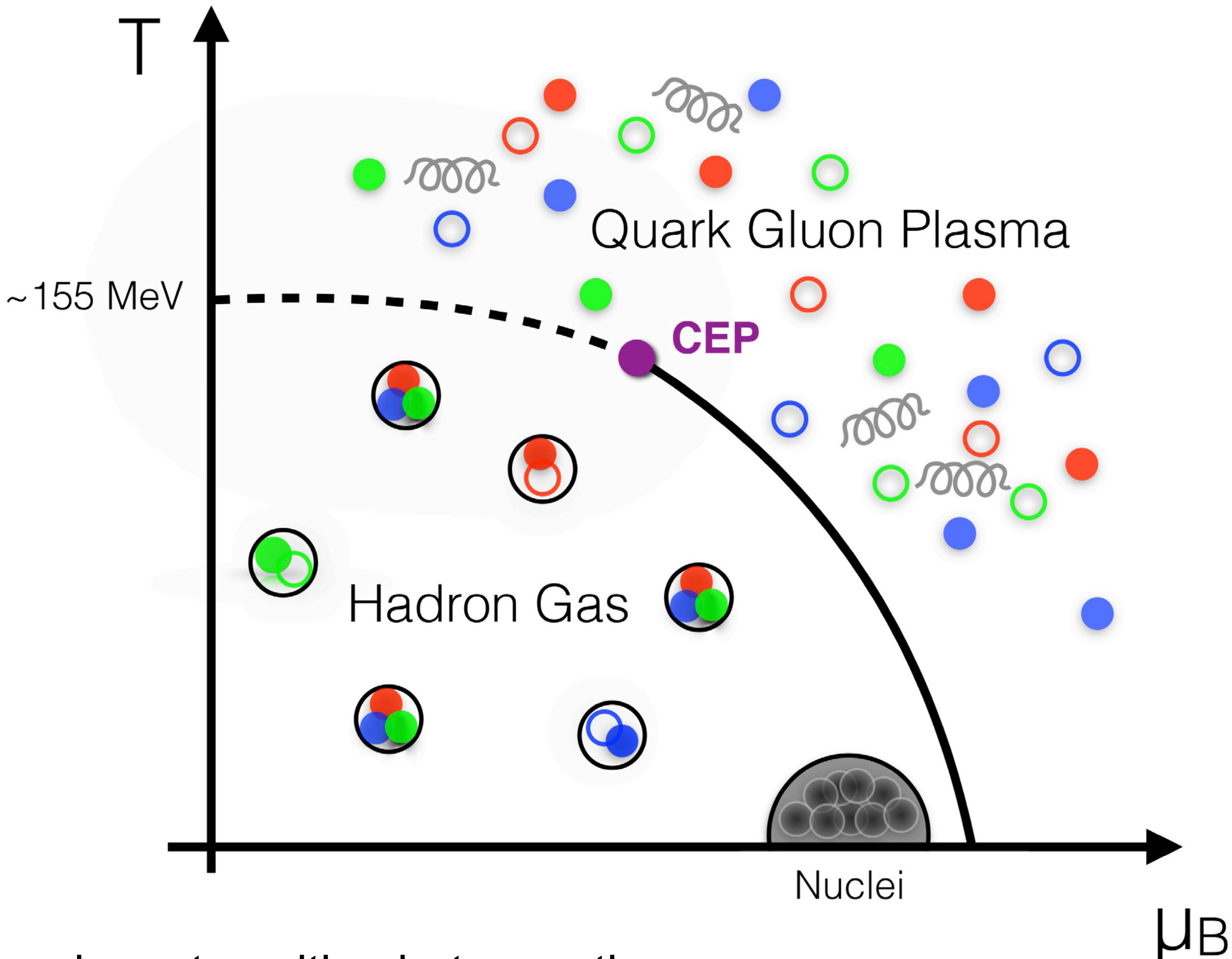
I) Equation of State at $\mu_B > 0$ and cumulants of conserved charges
- → thermal conditions at freeze-out
- → radius of convergence and critical point
- → skewness and kurtosis of net-baryon number

II) Spectral and Transport properties in the QGP
- → thermal dilepton and photon rates
- → electrical conductivity and heavy-quark diffusion

Helmholtz International Summer School
"Hadron Structure and Hadronic Matter, and Lattice QCD“
Dubna, 20.08.-02.09.2017

QCD phase diagram in the temperature (T) and baryon chemical potential (μ_B) plane



At small μ_B not a phase transition but a continuous crossover

→ **how does the physics change from Hadron Gas to QGP**

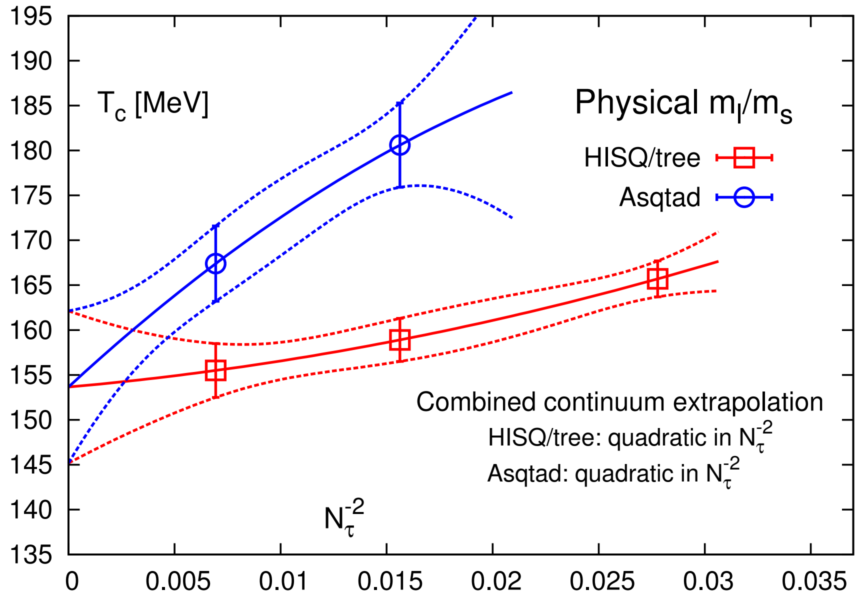
At $\mu_B > 0$ second order critical end point may exist, followed by line of 1st-order transitions

→ **location of the CEP? Critical behavior close to the CEP?**

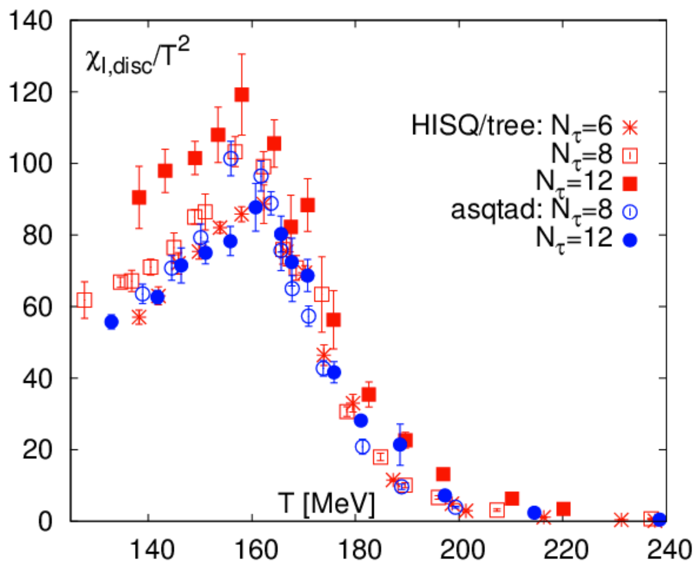
Well defined pseudo-critical temperature

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

consistent results from continuum extrapolated results of different lattice discretizations



[A. Bazavov et al. (hotQCD), PRD85 (2012) 054503]



from location of the peak in the fluctuations of the chiral condensate

$$\chi_l = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_l^2}$$

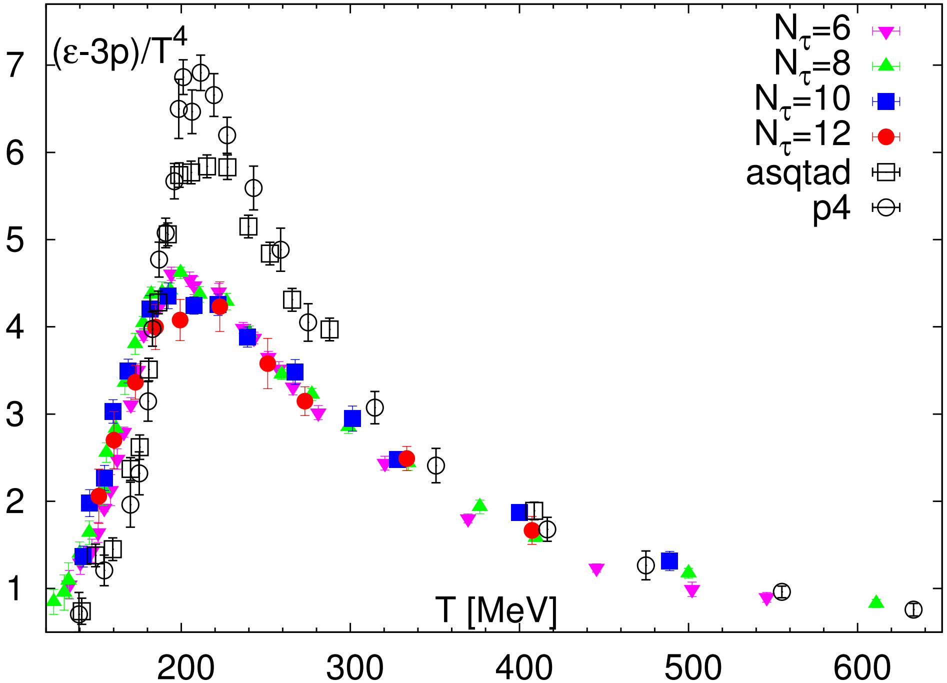
thermodynamic quantities obtained from derivatives of the partition function

$$Z(\beta, N_\sigma, N_\tau) = \int \prod_{x,\mu} dU_{x,\mu} e^{-S(U)} \qquad S(U) = \beta S_G(U) - S_F(U)$$

using trace of the energy momentum tensor:

$$\Theta^{\mu\mu} = \epsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a}$$

HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503]



$$\frac{\epsilon - 3p}{T^4} \equiv \frac{\Theta^{\mu\mu}_G(T)}{T^4} + \frac{\Theta^{\mu\mu}_F(T)}{T^4},$$

$$\frac{\Theta^{\mu\mu}_G(T)}{T^4} = R_\beta\left[\langle s_G\rangle_0 - \langle s_G\rangle_\tau\right] N_\tau^4 ,$$

$$\frac{\Theta^{\mu\mu}_F(T)}{T^4} = -R_\beta R_m [2m_l\left(\langle\bar\psi\psi\rangle_{l,0} - \langle\bar\psi\psi\rangle_{l,\tau}\right) + m_s\left(\langle\bar\psi\psi\rangle_{s,0} - \langle\bar\psi\psi\rangle_{s,\tau}\right)] N_\tau^4 .$$

need the beta-function:
(on a line of constant physics)

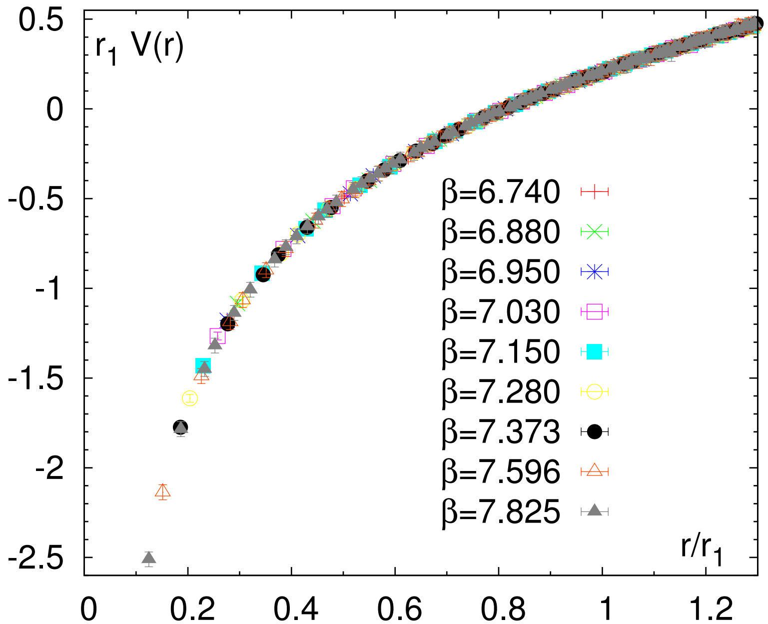
$$R_\beta(\beta) = a\frac{\mathrm{d}\beta}{\mathrm{d}a} \qquad R_m(\beta) = \frac{1}{m_s(\beta)}\frac{\mathrm{d}m_s(\beta)}{\mathrm{d}\beta}$$

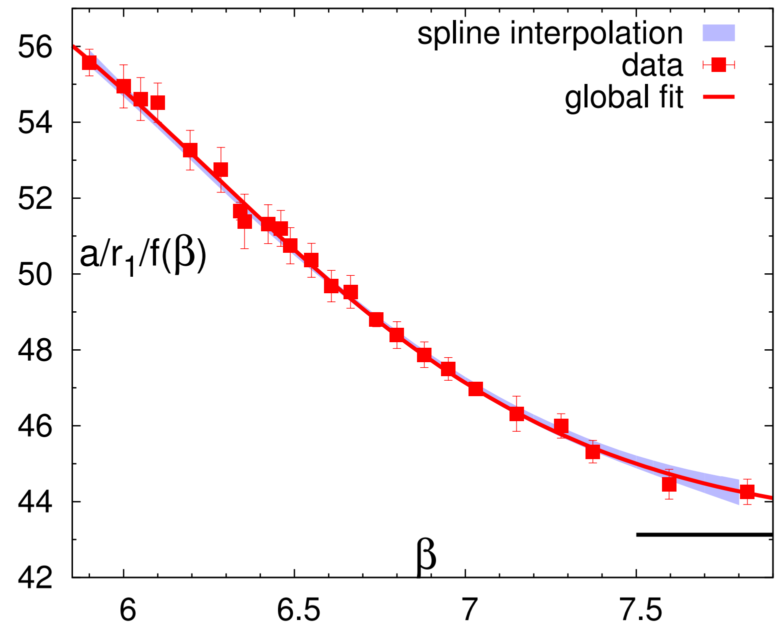
pressure calculated using integral method:

$$\frac{p(T)}{T^4} = \frac{p_0}{T_0^4} + \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}}{T'^5}$$

Sommer scale from heavy quark potential: $r^2 \frac{dV}{dr}\Big|_{r_1} = 1.0$

HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503]





Interpolation of r_1:

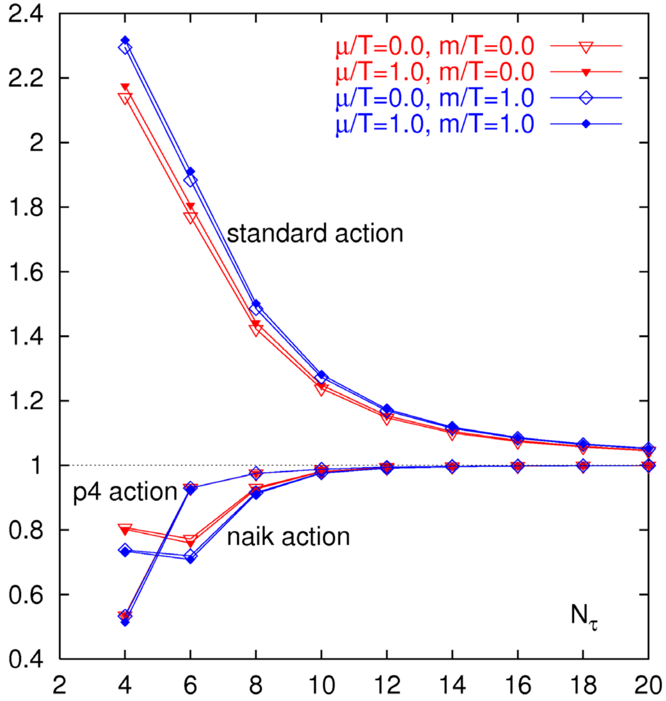
$$\frac{a}{r_1} = \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}$$

$$f(\beta) = \left(\frac{10 b_0}{\beta}\right)^{-b_1/(2b_0^2)} \exp(-\beta/(20 b_0)) \qquad b_0 = 9/(16\pi^2) \quad b_1 = 1/(4\pi^4)$$

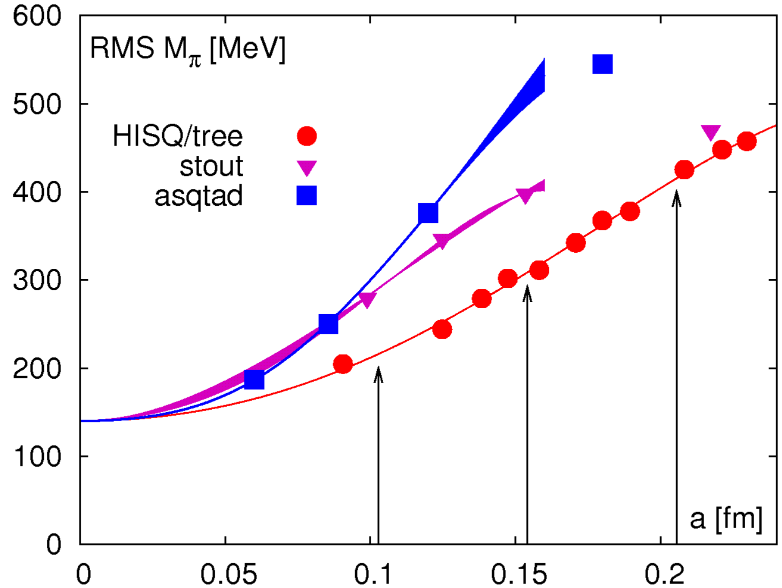
→ beta-function: $R_\beta(\beta) = \frac{r_1}{a}\left(\frac{\mathrm{d}(r_1/a)}{\mathrm{d}\beta}\right)^{-1}$

→ **improve thermodynamics and flavor symmetry:**

Pressure of the ideal quark gas

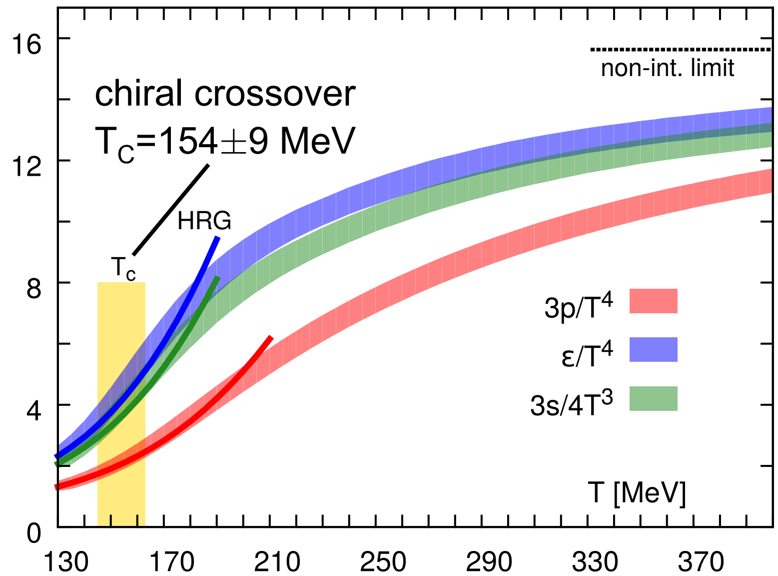


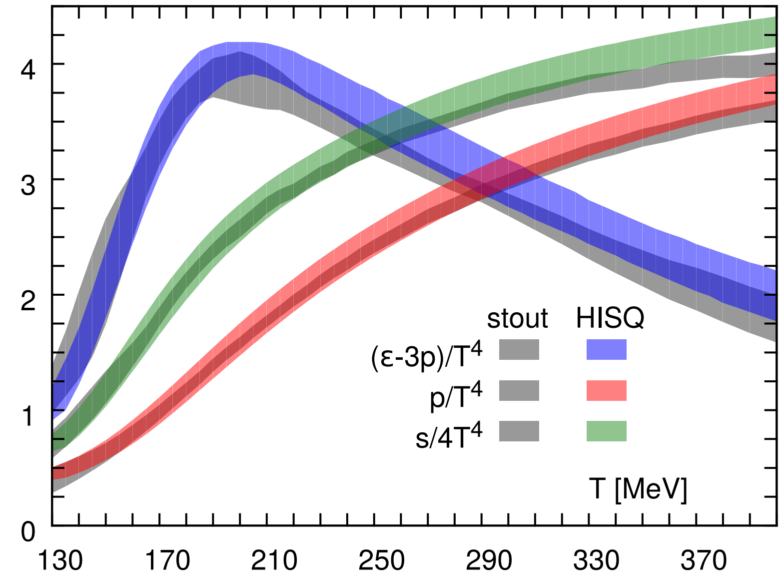
Root mean square mass



p4, asqtad, HISQ, stout

Continuum extrapolated results of **pressure** & **energy density** & **entropy density**





HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503] **stout:** [S. Borsanyi et al. (BMW), PLB730, 99 (2014)]

consistent results from hotQCD (HISQ) and Budapest-Wuppertal (stout)

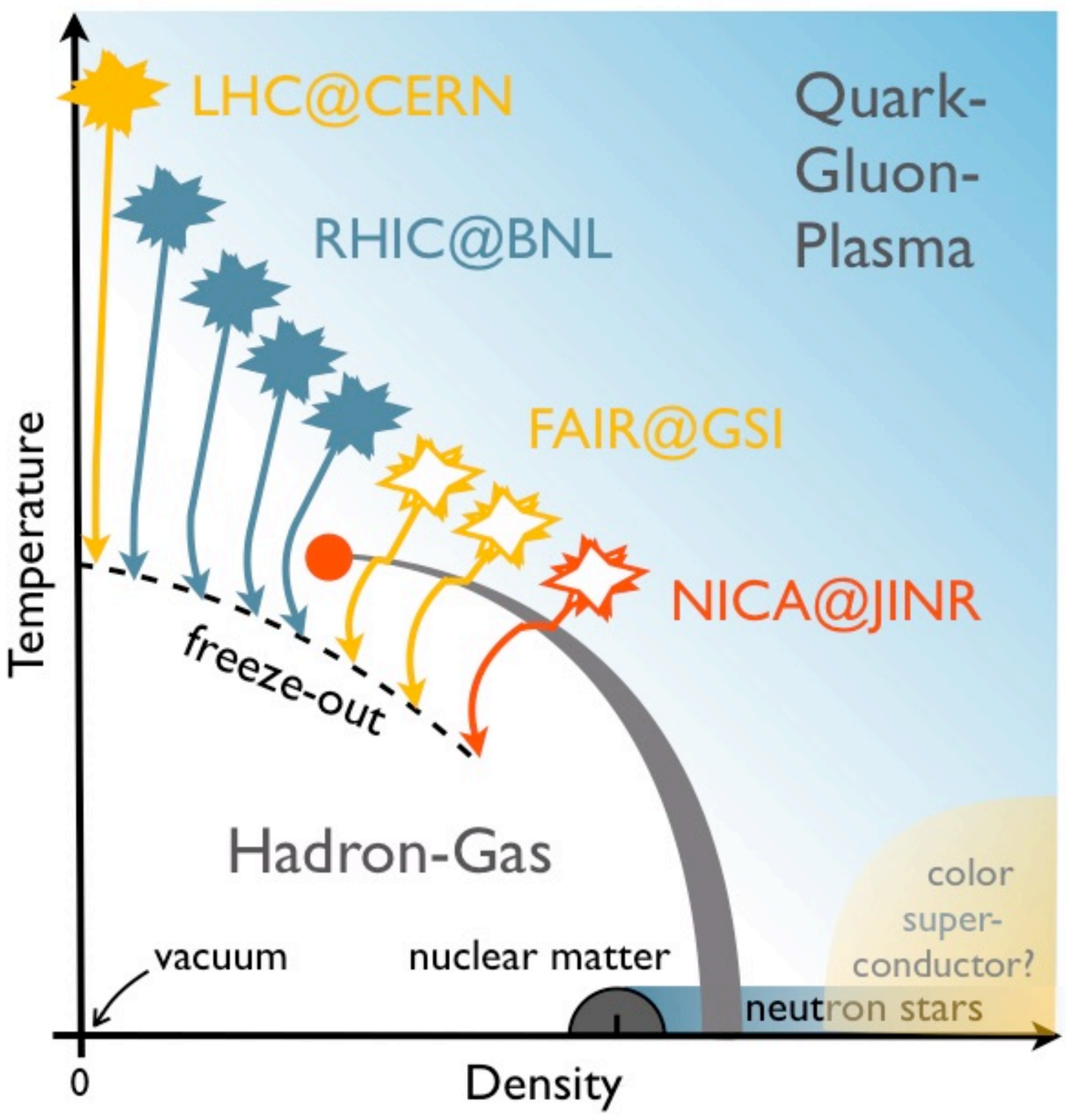
Hadron resonance gas (HRG) model using all known hadronic resonances from PDG

describes the EoS quite well up to cross-over region

QCD results systematically above HRG

QCD is quite different from HRG thermodynamics at T > 160MeV

Experimental studies of the QCD phase diagram



Most quantities are measured at freeze-out

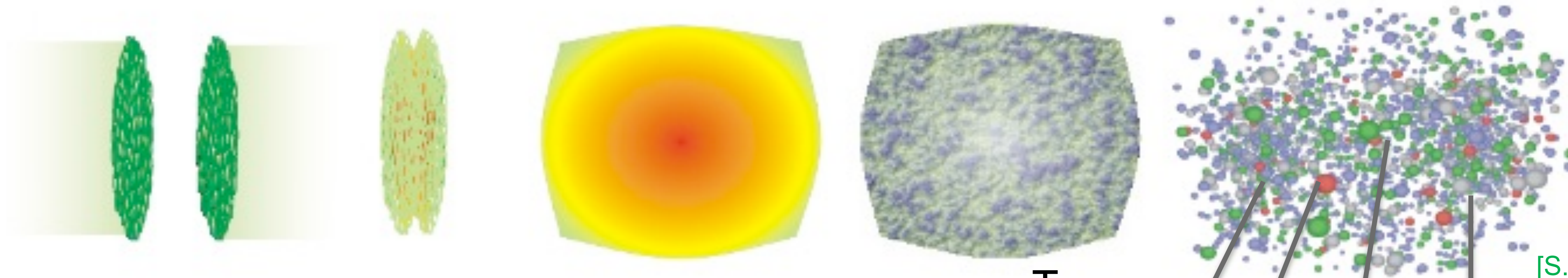
→ Hadronic fluctuations
→ Hadronic abundances

Hadron resonance gas (HRG) a good description of the hadronic phase?

Compare HRG to QCD

Use QCD to describe physics at freeze-out

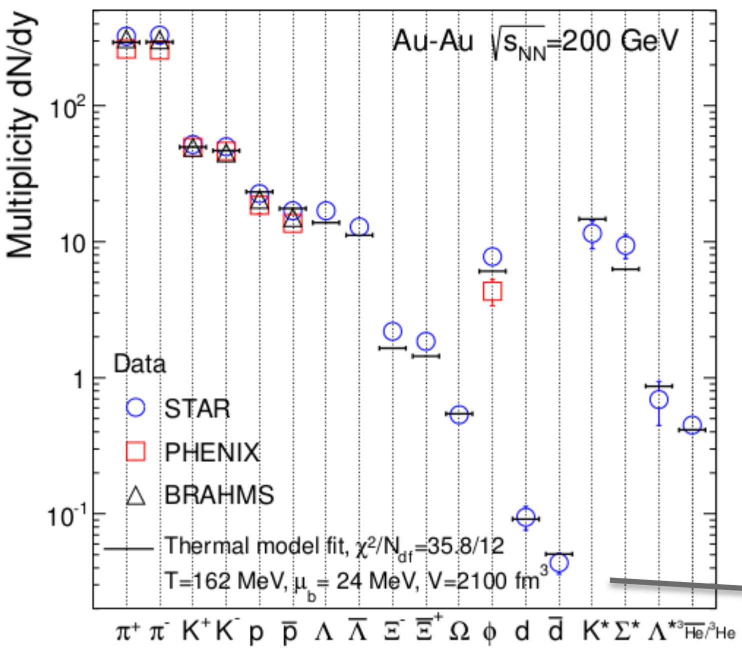
Heavy Ion Collision **QGP** **Expansion+Cooling** **Hadronization**



[S.Bass]

T_c, μ_c

Hadron Resonance Gas (HRG)
to describe
hadron yields at the freeze-out



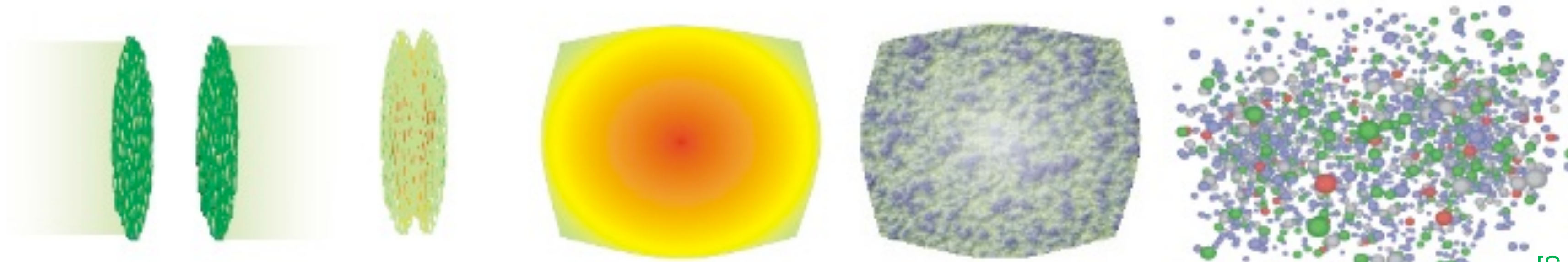
[Andronic et.al., Nucl.Phys.A904(2013)535c]

hadrons measured
in the detectors

"observed" mainly at the
freeze-out stage of the HIC

T^f, μ^f

Heavy Ion Collision QGP Expansion+Cooling Hadronization



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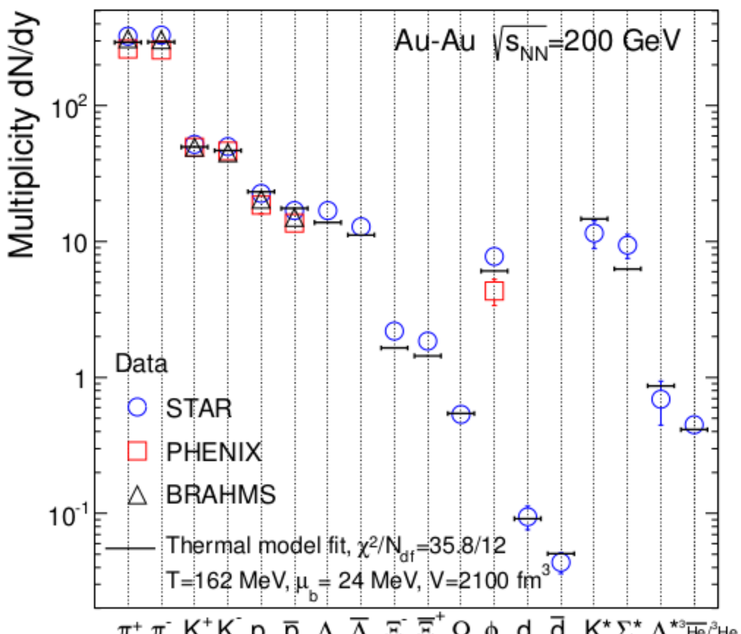

[Andronic et.al., Nucl.Phys.A904(2013)535c]

HRG: thermal gas of uncorrelated hadrons

partial pressure of each hadron:

$$\hat{P}_h \sim f(\hat{m}_h) \cosh\left[B_h\hat{\mu}_h + Q_h\hat{\mu}_Q + S_h\hat{\mu}_S + C_h\hat{\mu}_C\right]$$

total pressure given by the sum over all (known) hadrons

$$\hat{P}_{total} = \sum_{all\ \ hadrons} \hat{P}_h$$

are we sensitive to this?

→ use thermodynamics instead of HRG
to describe freeze-out

HRG: thermal gas of uncorrelated hadrons

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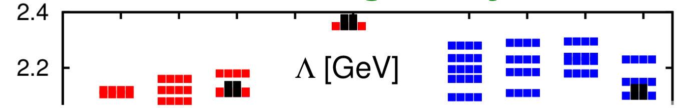
$$\hat{P}_h \sim f(\hat{m}_h) \cosh\left[B_h \hat{\mu}_h + Q_h \hat{\mu}_Q + S_h \hat{\mu}_S\right]$$

total pressure given by the sum over all (known) hadrons

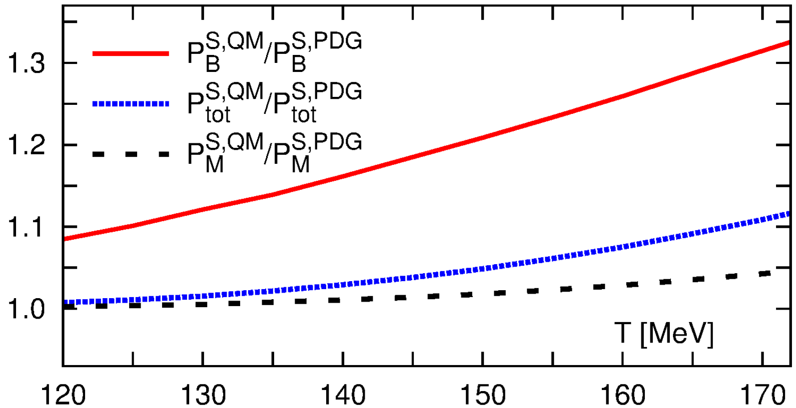
$$\hat{P}_{total} = \sum_{all\ \ hadrons} \hat{P}_h$$

are we sensitive to this?

Quark Model predicts more strange baryons:



[Capstick-Isgur, PRD34 (1986) 2809]



[Bielefeld-BNL-CCNU, PRL113(2014)072001]

$$P_{tot}^{S,X} = P_M^{S,X} + P_B^{S,X}$$

X = { QM resonances, PDG resonances

$$P_{M/B}^{S,X}(T,\vec{\mu}) = \frac{T^4}{2\pi^2} \sum_{i\in X} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \times \cosh\left(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S\right)$$

large enhancement of the partial baryonic pressure from additional strange baryons

PDG-HRG uses states listed in the particle data tables

QM-HRG uses states calculated in the quark model

[see also P.Alba, R.Bellwied et al., PRD 96 (2017) 034517]

use QCD thermodynamics instead of HRG to describe physics at freeze-out

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

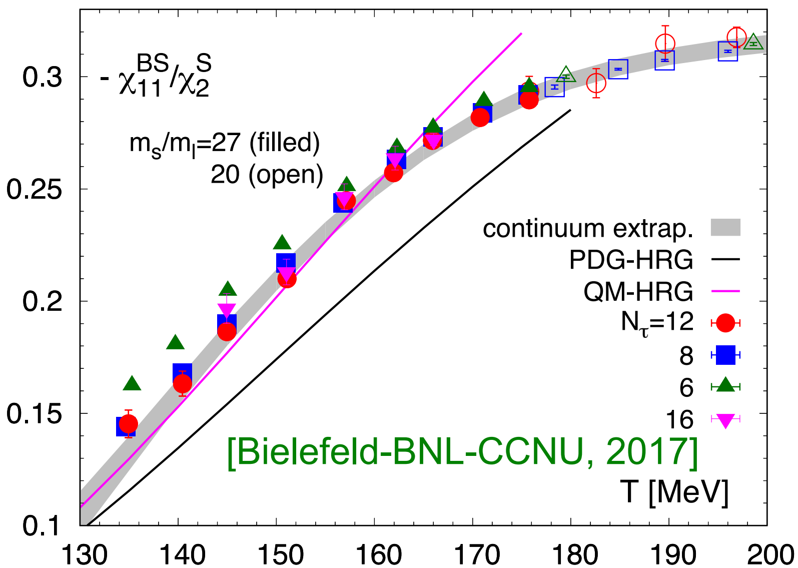
defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{(i+j+k)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\vec{\mu}=0}$

correlations of strangeness with baryon number fluctuations:

$$\chi_{11}^{BS} = \left.\frac{\partial^2[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B \partial\hat{\mu}_S}\right|_{\vec{\mu}=0}$$

second cumulant of net strangeness fluctuations:

$$\chi_2^S = \left.\frac{\partial^2[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_S^2}\right|_{\vec{\mu}=0}$$



suitable ratios like

$$\frac{\chi_{11}^{BS}}{\chi_2^S}$$

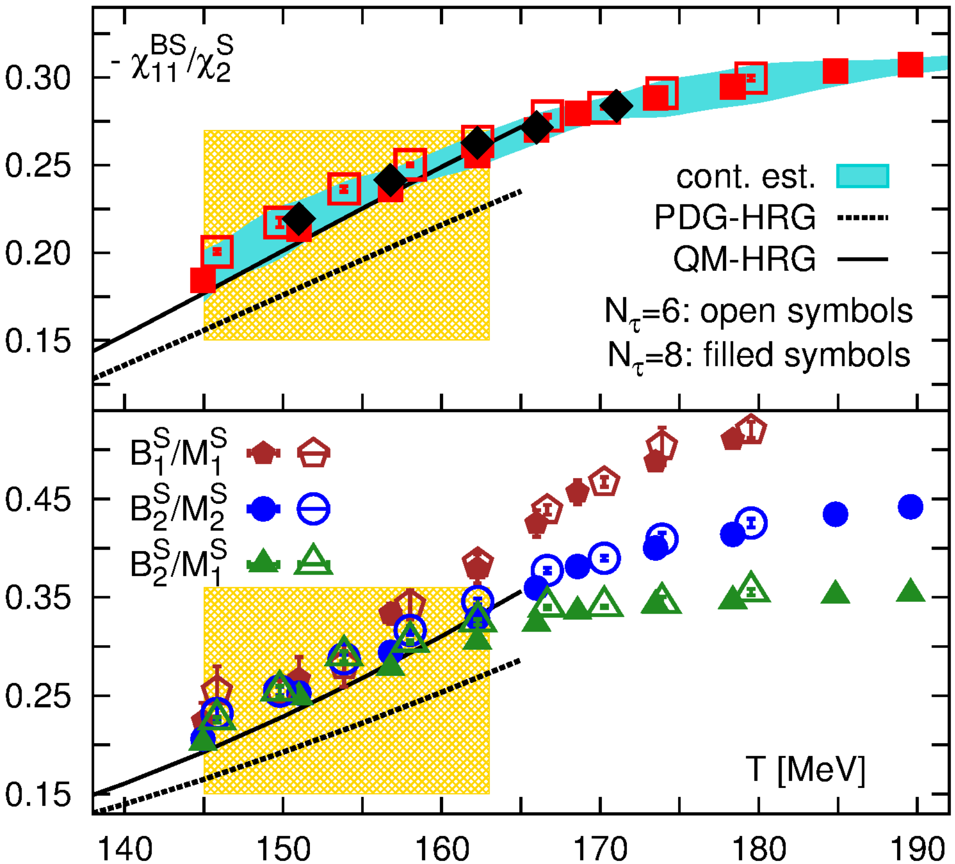
strange baryon density (in a hadron gas)

dominated by strange mesons

are sensitive probes of the strangeness carrying degrees of freedom

$$\frac{\mu_S}{\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S} + \mathcal{O}(\mu^2)$$

$$\chi_{klm}^{BQS} = \left. \frac{\partial^{(k+l+m)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^k \partial\hat{\mu}_Q^l \partial\hat{\mu}_S^m} \right|_{\vec{\mu}=0}$$



individual pressure-observables

for open strange mesons (P_M^S in HRG):

$$M_1^S = \chi_2^S - \chi_{22}^{BS}$$

$$M_2^S = \frac{1}{12}\left(\chi_4^S + 11\chi_2^S\right) + \frac{1}{2}\left(\chi_{11}^{BS} + \chi_{13}^{BS}\right)$$

for strange baryons (P_B^S in HRG):

$$B_1^S = -\frac{1}{6}\left(11\chi_{11}^{BS} + 6\chi_{22}^{BS} + \chi_{13}^{BS}\right)$$

$$B_2^S = \frac{1}{12}\left(\chi_4^S - \chi_2^S\right) - \frac{1}{3}\left(4\chi_{11}^{BS} - \chi_{13}^{BS}\right)$$

all give identical results in a gas of uncorrelated hadrons

yield widely different results when the degrees of freedom are quarks

→ QM-HRG model calculations are in good agreement with LQCD up to the chiral crossover region

→ evidence for the existence of additional strange baryons

and their thermodynamic importance below the QCD crossover

Implications for strangeness freeze-out

initial nuclei in a heavy ion collision are net strangeness free + iso-spin asymmetry

$$\langle n_Q \rangle = r \langle n_B \rangle$$

→ the HRG at the chemical freeze-out must also be strangeness neutral

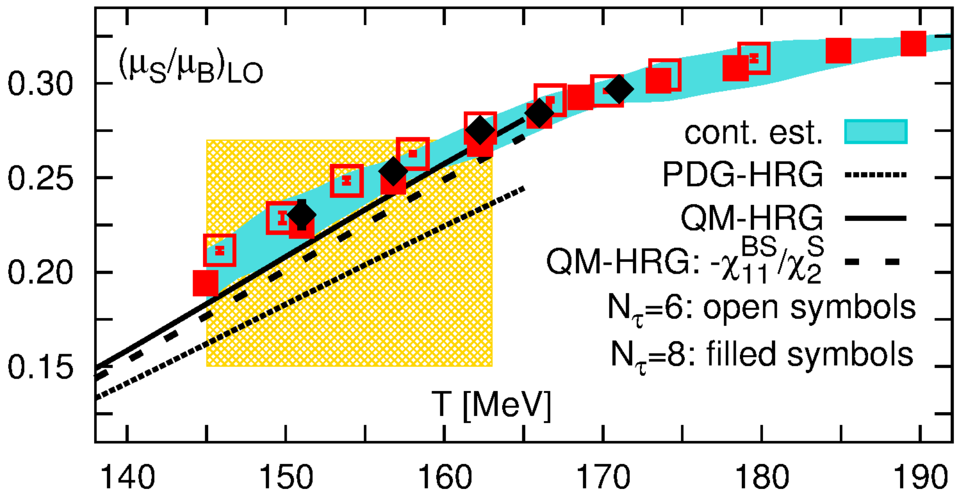
→ thermal parameters T, μ_B and μ_S are related

$$\frac{\mu_S}{\mu_B} = s_1(T) + s_3(T) \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

small for $\mu_B \lesssim 200 MeV$

$$\left(\frac{\mu_S}{\mu_B}\right)_{\rm LO} \equiv s_1(T) = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}$$

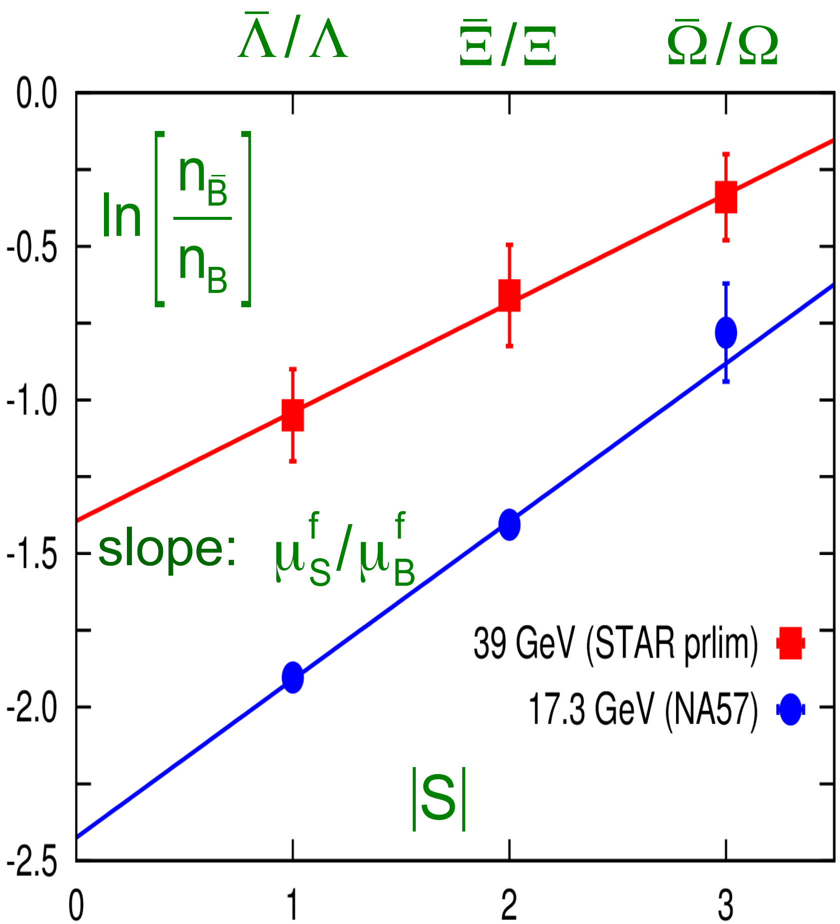
small correction from nonzero electric charge chemical potential



Lattice QCD results well reproduced by QM-HRG in the crossover region

for a given μ_S/μ_B QM-HRG would give a smaller temperature compared to PDG-HRG relative yields of strange anti-baryons ($\overline{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

$$R_H \equiv \frac{\overline{H}_S}{H_S} = \mathrm{e}^{-2(\mu_B^f/T^f)\left(1-(\mu_S^f/\mu_B^f)|S|\right)}$$



only assumes that hadron yields are thermal

compare results for μ_B/T and μ_S/μ_B to Lattice QCD

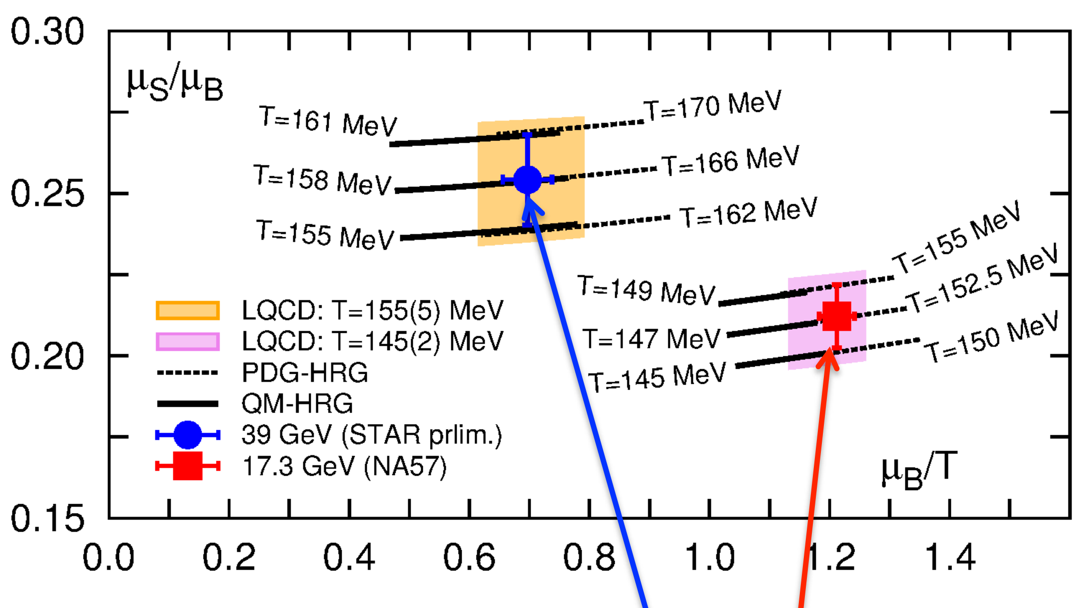
to obtain **freeze-out *T***

Implications for strangeness freeze-out

relative yields of strange anti-baryons ($\overline{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

$$R_H \equiv \frac{\overline{H}_S}{H_S} = \mathrm{e}^{-2(\mu_B^f/T^f)\left(1-(\mu_S^f/\mu_B^f)|S|\right)}$$

and compared to Lattice QCD or HRG to determine freeze-out temperature T^f:



Experiment STAR/NA57

relative yields of strange anti-baryons ($\bar{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

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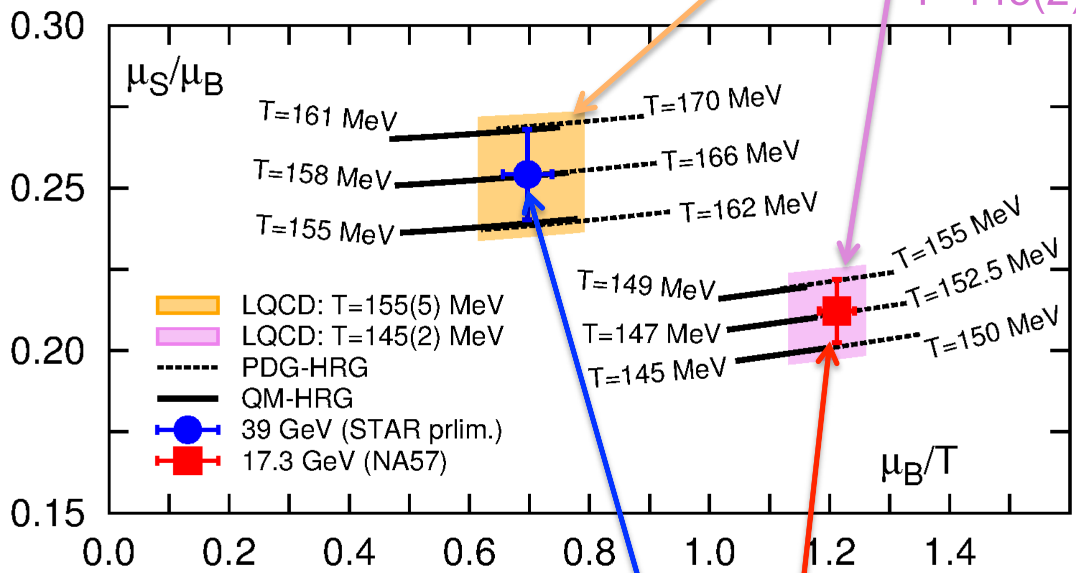
Lattice QCD

T=155(5) MeV

T=145(2) MeV



freeze-out temperature obtained from a comparison of experimental data and Lattice QCD results

Experiment STAR/NA57

Implications for strangeness freeze-out

relative yields of strange anti-baryons ($\overline{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

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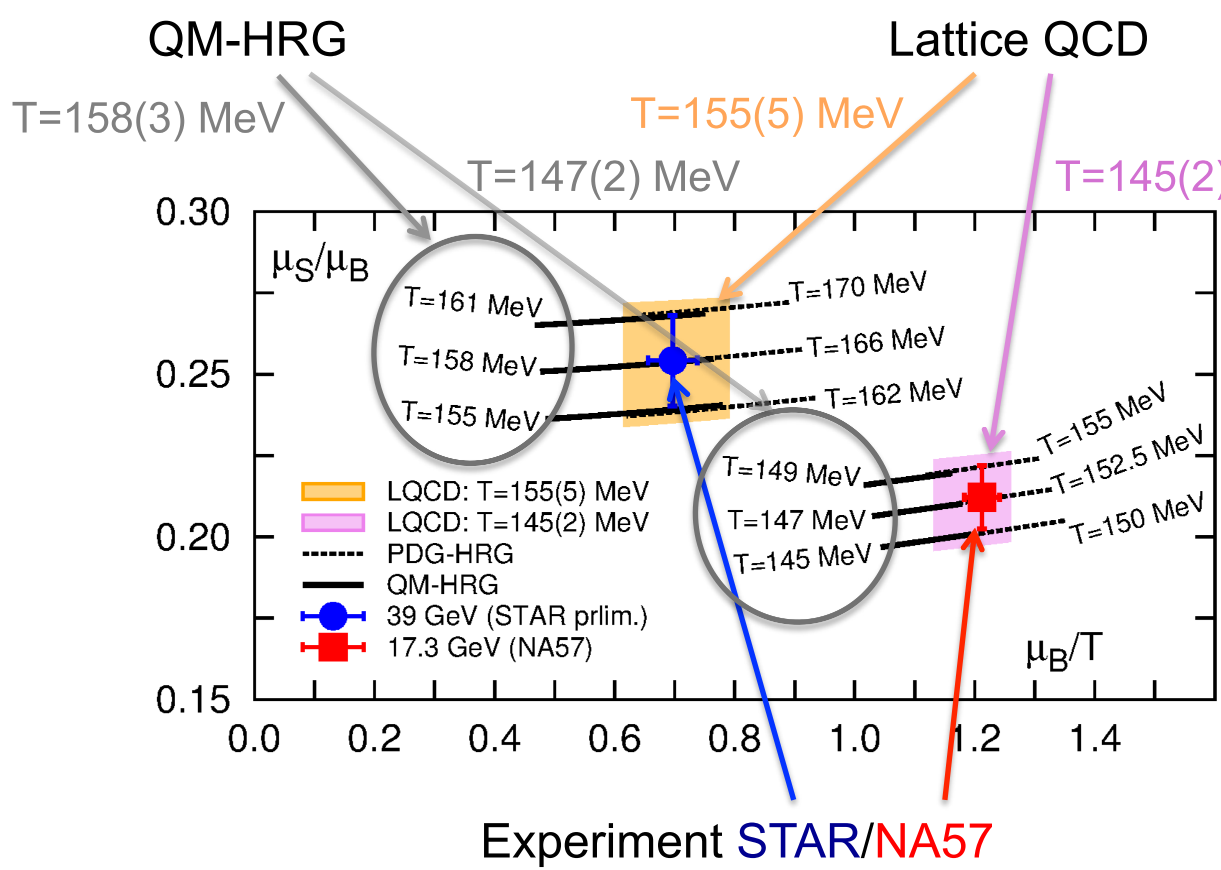
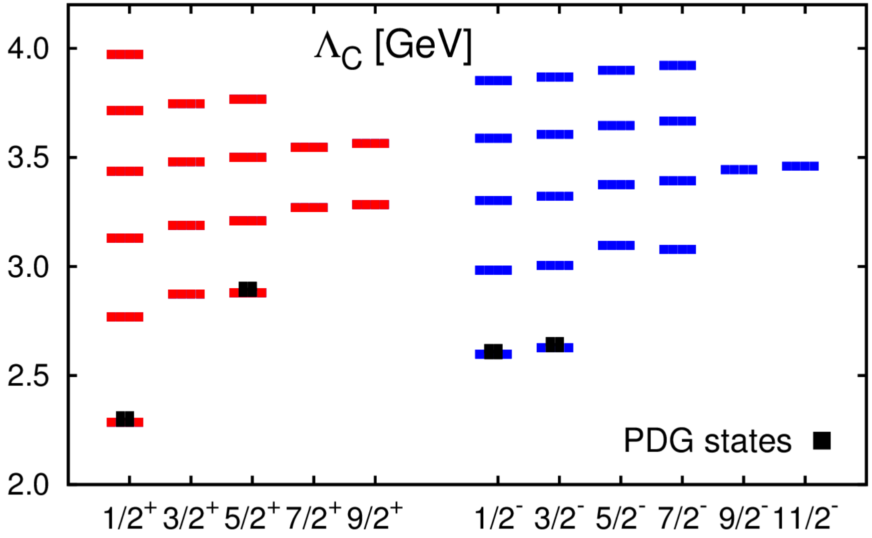


freeze-out temperature obtained from QM-HRG agrees well with Lattice QCD results

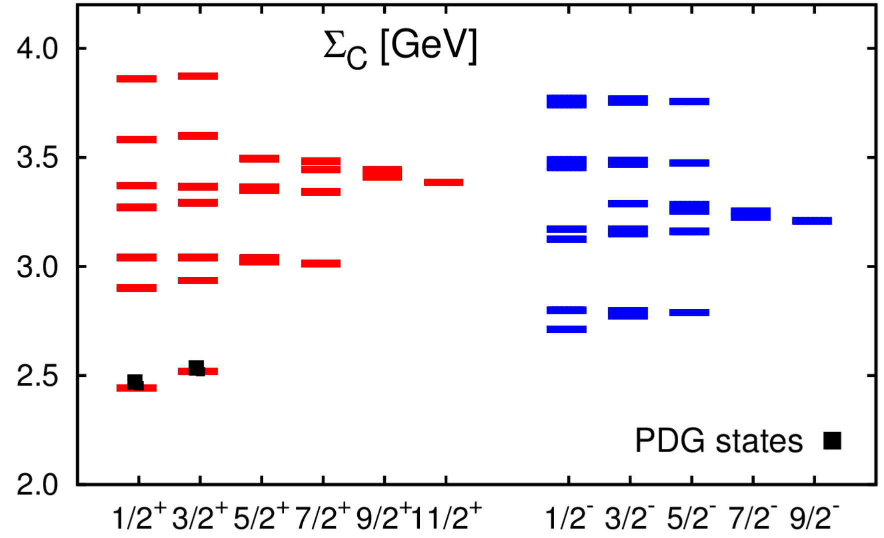
important to include so-far undiscovered strange baryons in HRG

charm baryons

Quark Model



Quark Model



[Capstick-Isgur, Phys.Rev.D34 (1986) 2809]

in the following

PDG will denote results using states listed in the particle data tables

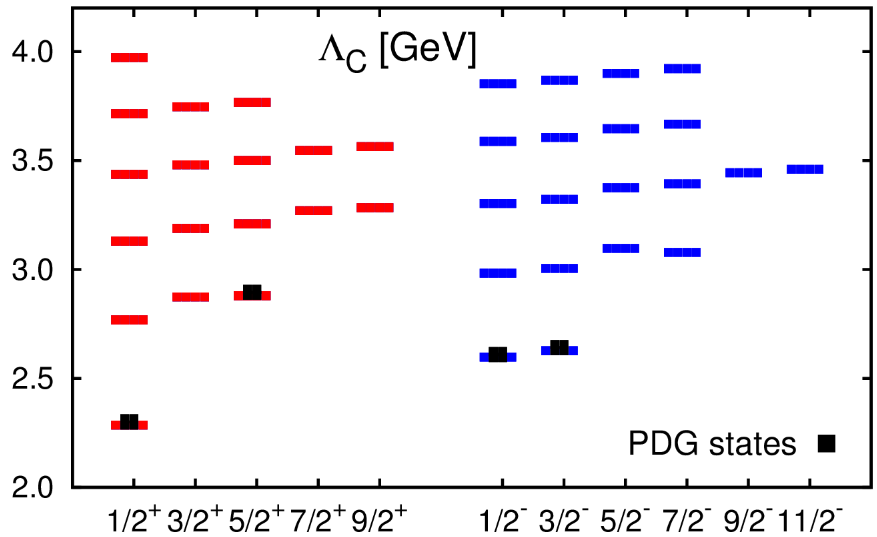
QM will denote results using states calculated in the quark model

QM-3 all resonances up to 3.0 GeV

QM-3.5 all resonances up to 3.5 GeV

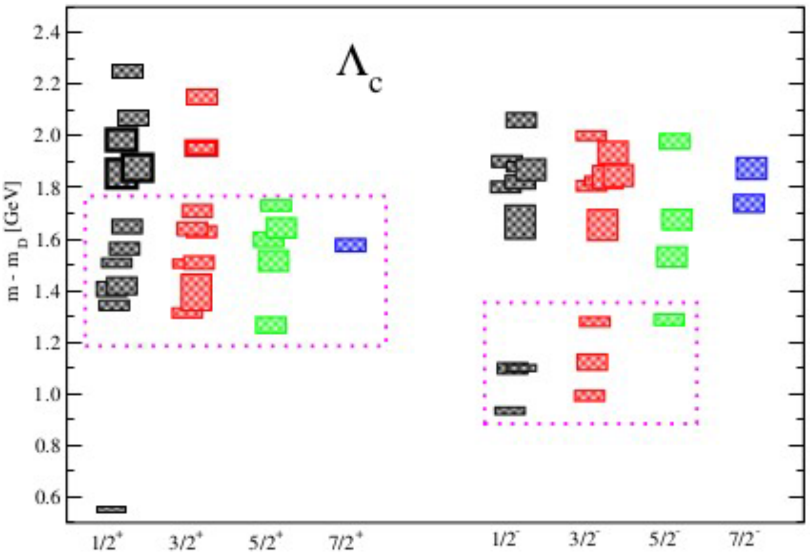
charm baryons

Quark Model



[Capstick-Isgur, Phys.Rev.D34 (1986) 2809]

Lattice QCD



[Padmanath et al., arXiv 1311.4806]

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QM-3 all resonances up to 3.0 GeV

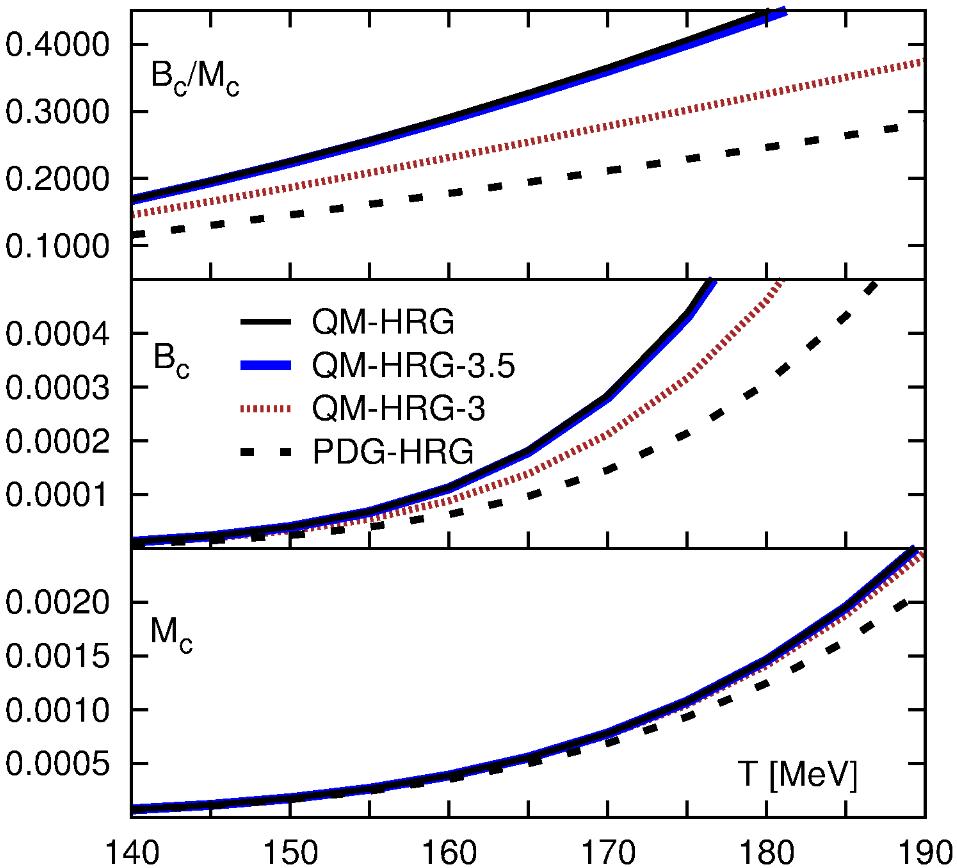
QM-3.5 all resonances up to 3.5 GeV

partial pressure P of all open charm hadrons

can be separated into mesonic P_M and baryonic P_B components

$$P_{\mathrm{tot}}^{C,X} = P_M^{C,X} + P_B^{C,X}$$

$$P_{M/B}^{C,X}(T,\vec{\mu}) = \frac{T^4}{2\pi^2} \sum_{i\in X} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \times \cosh\left(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C\right)$$



X = {
QM resonances
QM-3.5 resonances up to 3.5 GeV
QM-3 resonances up to 3.0 GeV
PDG resonances

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klmn}^{BQSC}(T) \left(\frac{\mu_B}{T}\right)^k \left(\frac{\mu_Q}{T}\right)^l \left(\frac{\mu_S}{T}\right)^m \left(\frac{\mu_C}{T}\right)^n$$

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left. \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n} \right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

$$P^C = P_M^C \cosh(\hat{\mu}_C) + \sum_{k=1,2,3} P_B^{C=k} \cosh(B\hat{\mu}_B + k\hat{\mu}_C)$$

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

$$\chi_{mn}^{BC} = B^m P_B^{C=1} + B^m 2^n P_B^{C=2} + B^m 3^n P_B^{C=3} \simeq B^m P_B^{C=1}$$

relative contribution of C=2 and C=3 baryons negligible

ratios independent of the detailed spectrum and sensitive to special sectors:

charmed baryon sector

$$\frac{\chi_{mn}^{BC}}{\chi_{m+1,n-1}^{BC}} = B^{-1}$$

=1 when DoF are hadronic

=3 when DoF are quarks

$$\frac{\chi_{mn}^{BC}}{\chi_{m,n+2}^{BC}} = 1$$ always

Correlations of conserved charges – open charm sector

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

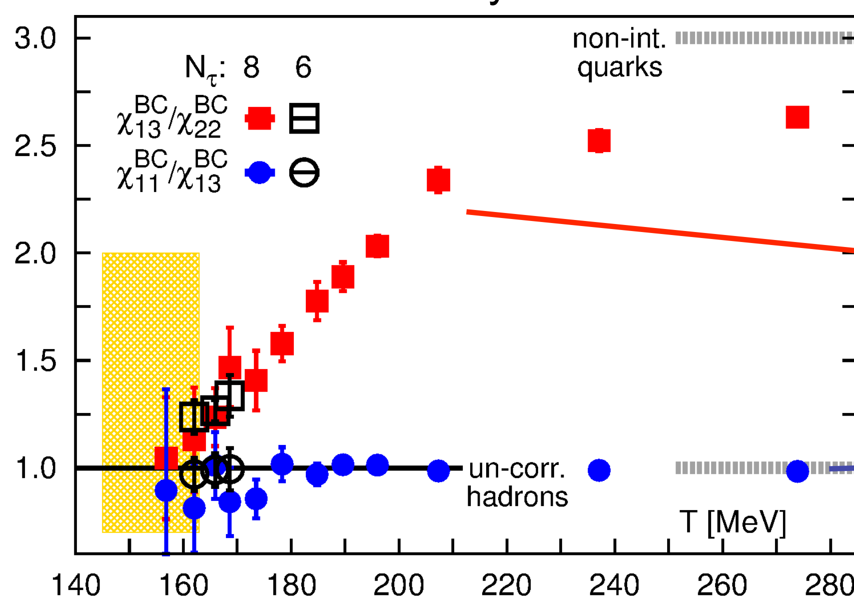
$32^3 \times 8$ and $24^3 \times 6$ lattices with $m_l = m_s/20$ and physical m_s and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charmed baryon sector



$$\frac{\chi_{mn}^{BC}}{\chi_{m+1,n-1}^{BC}} = B^{-1}$$

$$\frac{\chi_{mn}^{BC}}{\chi_{m,n+2}^{BC}} = 1$$

→ **indications that charmed baryons start to dissolve already close to the chiral crossover**

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klmn}^{BQSC}(T) \left(\frac{\mu_B}{T}\right)^k \left(\frac{\mu_Q}{T}\right)^l \left(\frac{\mu_S}{T}\right)^m \left(\frac{\mu_C}{T}\right)^n$$

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

$$P^C = P_M^C \cosh(\hat{\mu}_C) + \sum_{k=1,2,3} P_B^{C=k} \cosh(B\hat{\mu}_B + k\hat{\mu}_C)$$

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

$$\chi_{mn}^{B=1,C} = P_B^{C=1} + 2^n P_B^{C=2} + 3^n P_B^{C=3} \simeq P_B^{C=1}$$

$$\chi_k^C = P_M^C + 2^n P_B^{C=2} + 3^n P_B^{C=3} \simeq P_M^C + P_B^{C=1}$$

ratios independent of the detailed spectrum and sensitive to special sectors:

partial pressure of open-charm mesons:

open charm meson sector

$$P_M^C = \chi_2^C - \chi_{22}^{BC} = \chi_4^C - \chi_{13}^{BC}$$

$$\frac{\chi_4^C}{\chi_2^C} = 1$$

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

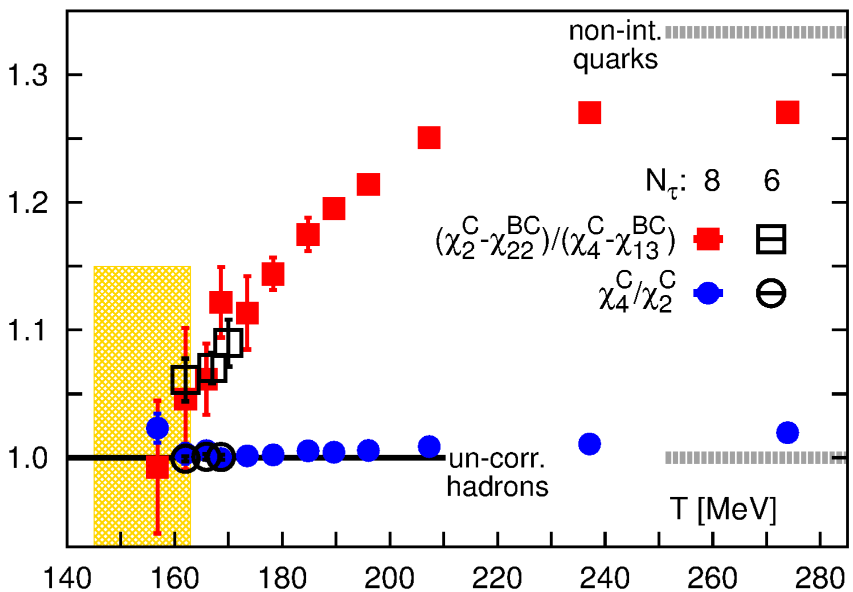
$32^3 \times 8$ and $24^3 \times 6$ lattices with $m_l = m_s/20$ and physical m_s and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left. \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n} \right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

open charm meson sector



partial pressure of open-charm mesons:

$$P_M^C = \chi_2^C - \chi_{22}^{BC} = \chi_4^C - \chi_{13}^{BC}$$

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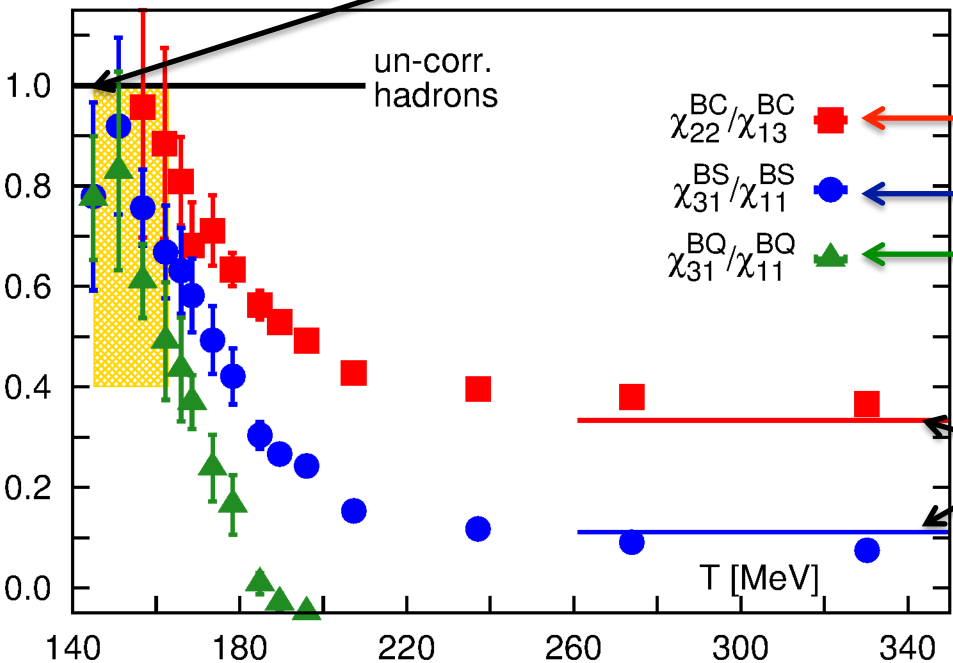
→ indications that open charm mesons start to dissolve already close to the chiral crossover

ratios of BC, BS, and BQ correlations
unity in a gas of uncorrelated hadrons



charm sector

strange sector

light quark sector

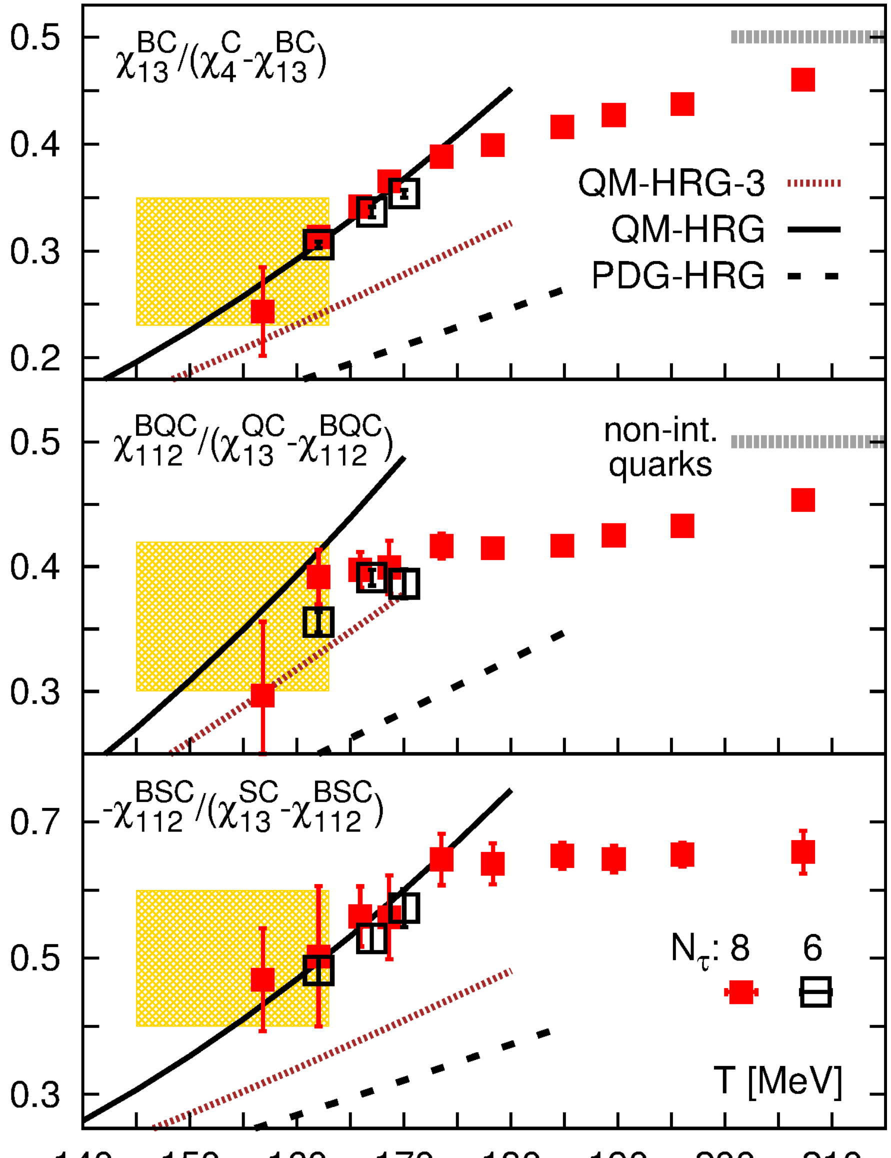
uncorrelated quark gas

A. Bazavov et al., Phys.Lett.B 737 (2014) 210]

→ **charmed hadrons start to deconfine around the chiral crossover region**

→ **strange hadrons start to deconfine around the chiral crossover region**

charmed pressure ratios are sensitive to the charm hadron spectrum



A. Bazavov et al., Phys.Lett.B 737 (2014) 210]

charmed baryon to meson ratio

$$R_{13}^{BC} = \frac{\chi_{13}^{BC}}{M_C} = \frac{B_C}{M_C}$$

$$M_C \simeq \chi_4^C - \chi_{13}^{BC}$$

charged charmed baryon to meson ratio

$$R_{13}^{QC} = \frac{\chi_{112}^{BQC}}{M_{QC}}$$

$$M_{QC} \simeq \chi_{13}^{QC} - \chi_{112}^{BQC}$$

strange charmed baryon to meson ratio

$$R_{13}^{SC} = -\frac{\chi_{112}^{BSC}}{M_{SC}}$$

$$M_{SC} \simeq \chi_{13}^{SC} - \chi_{112}^{BSC}$$

→ important to include so-far undiscovered open charm hadrons in HRG

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

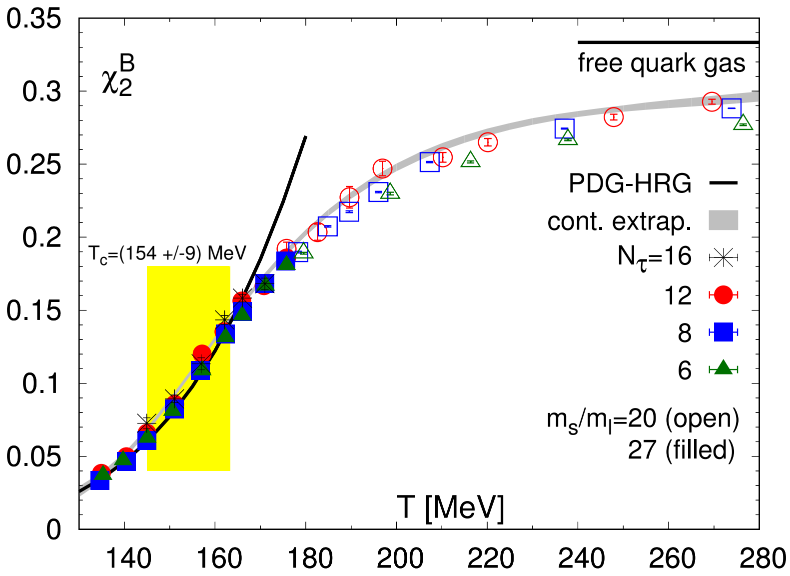
defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{(i+j+k)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\vec{\mu}=0}$

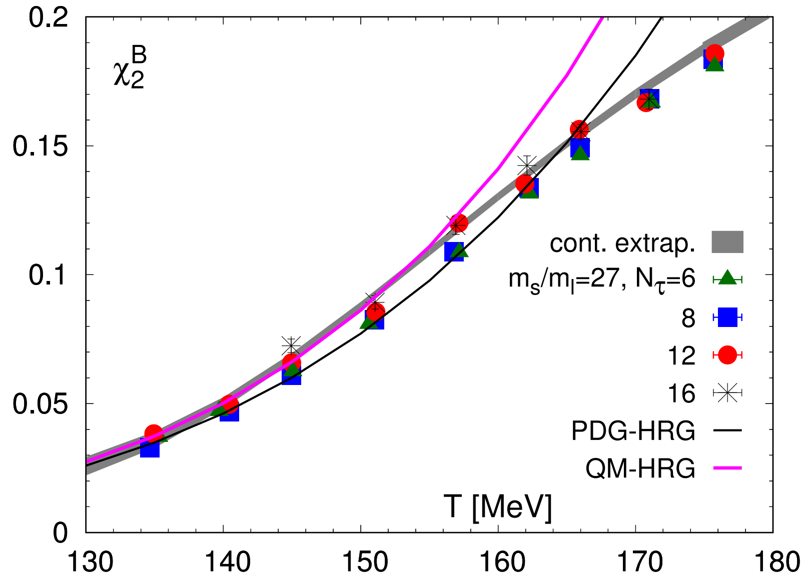
for $\mu_Q = \mu_S = 0$ this simplifies to

variance of net-baryon number distribution

kurtosis*variance $\kappa_B \sigma_B^2$

$$\frac{\Delta P(T)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right) + \mathcal{O}\left(\mu_B^6\right)$$





[Bielefeld-BNL-CCNU, PRD 95 (2017) 054504]

good agreement with HRG in crossover region

using the extended QM-HRG model

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

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for $\mu_Q = \mu_S = 0$ this simplifies to

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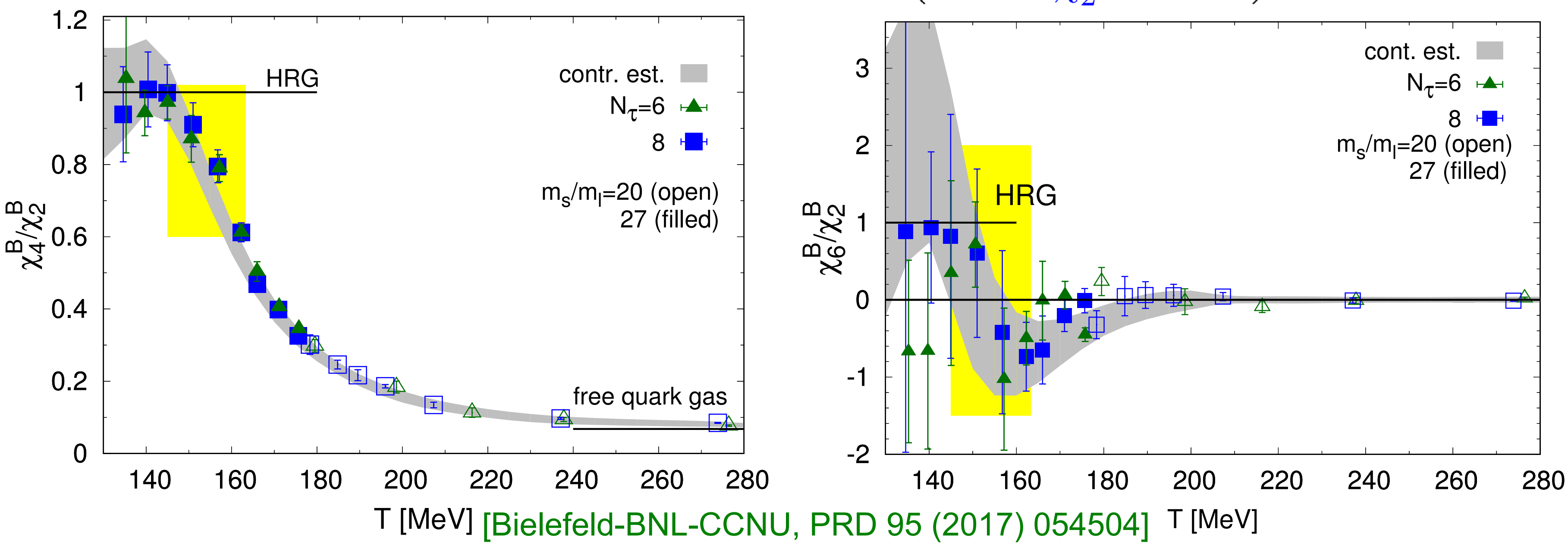


[Bielefeld-BNL-CCNU, PRD 95 (2017) 054504]

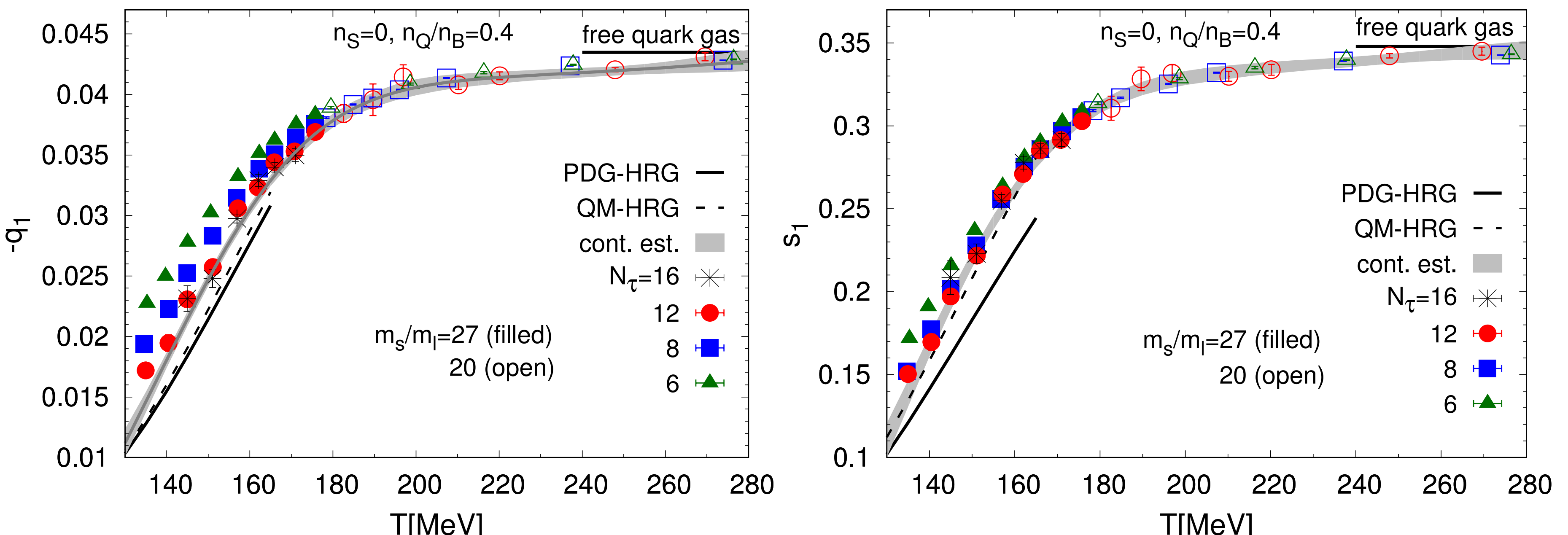
good agreement with HRG up to crossover region deviations from HRG in crossover region

Constraints in heavy-ion collisions: $n_S = 0 \quad , \quad \frac{n_Q}{n_B} = r = 0.4$

strangeness neutrality and fixed **electric charge to baryon-number ratio**

→ contraints for the chemical potentials μ_Q, μ_S and μ_B

$$\begin{aligned}\hat{\mu}_Q(T,\mu_B) &= q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3 + q_5(T)\hat{\mu}_B^5 + \dots \\ \hat{\mu}_S(T,\mu_B) &= s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3 + s_5(T)\hat{\mu}_B^5 + \dots\end{aligned}$$

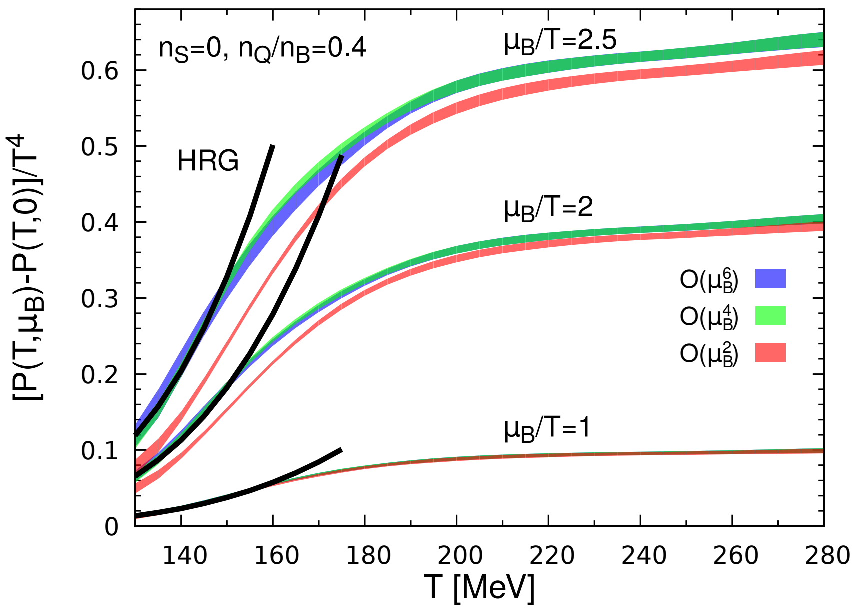


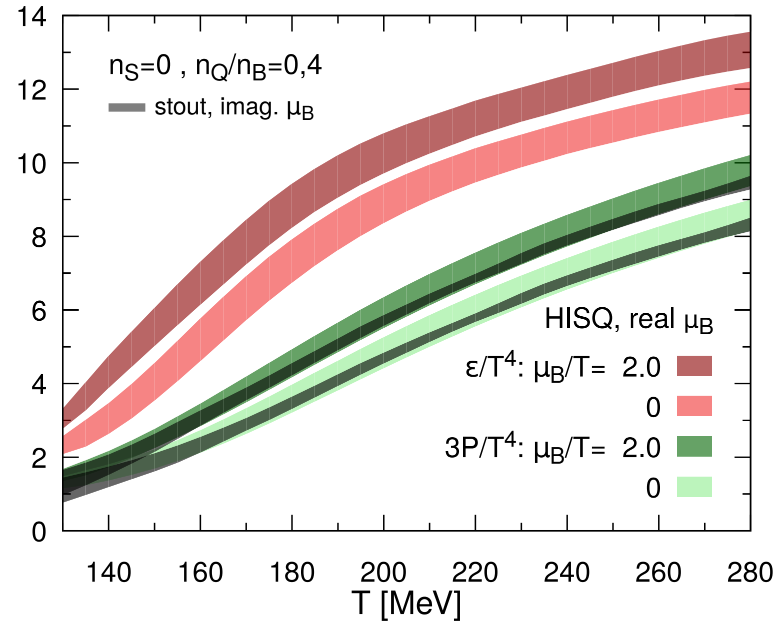
[Bielefeld-BNL-CCNU, PRD 95 (2017) 054504]

Constraints in heavy-ion collisions: $n_S = 0 \quad , \quad \frac{n_Q}{n_B} = r = 0.4$

strangeness neutrality and fixed **electric charge to baryon-number ratio**

Continuum estimated results of **pressure** & **energy density** up to 6th-order:





Parametrization of the EoS for T∈ [130MeV,280MeV]
& μ_B/T ≥ 0 in [Bielefeld-BNL-CCNU, PRD 95(2017)054504]

HISQ: [Bielefeld-BNL-CCNU, PRD95(2017)054504]
stout: [Wuppertal-Budapest, arXiv:1607.02493]

Consistent results from two different actions, HISQ and stout, in the continuum

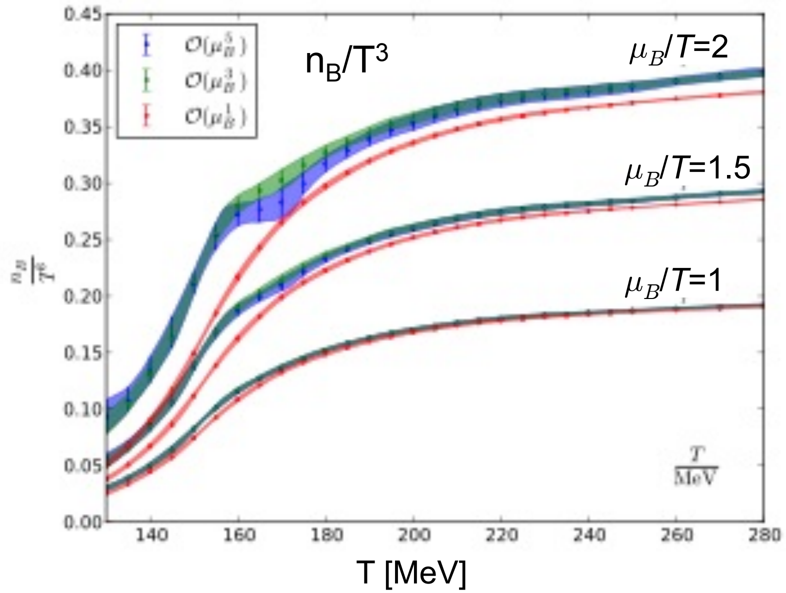
→ Equation of state well controlled for $\mu_B/T \lesssim 2$ ←→ $\sqrt{s_{NN}} \geq 14.5\ GeV$

Equation of state of (2+1)-flavor QCD - $\mu_B/T > 0$

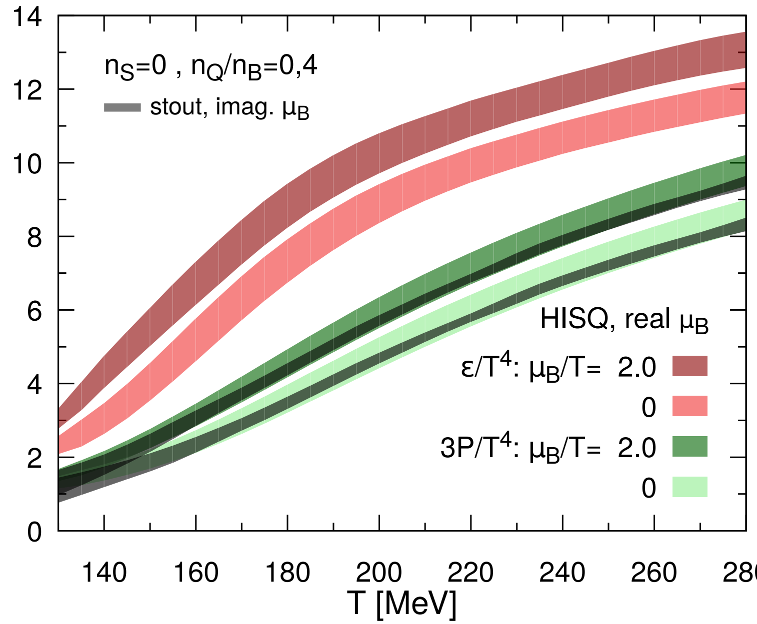
Constraints in heavy-ion collisions: $n_S = 0 \quad , \quad \frac{n_Q}{n_B} = r = 0.4$

strangeness neutrality and fixed **electric charge to baryon-number ratio**

Continuum estimated results of **pressure** & **energy density** up to 6th-order:



stout: [Wuppertal-Budapest]



HISQ: [Bielefeld-BNL-CCNU, arXiv:1701.04325]

stout: [Wuppertal-Budapest, arXiv:1607.02493]

Consistent results from two different actions, HISQ and stout, in the continuum

→ Equation of state well controlled for $\mu_B/T \lesssim 2$ ←→ $\sqrt{s_{NN}} \geq 14.5\ GeV$

Thermal conditions at chemical freeze-out / hadronization characterized by lines of constant pressure, energy and entropy densities?

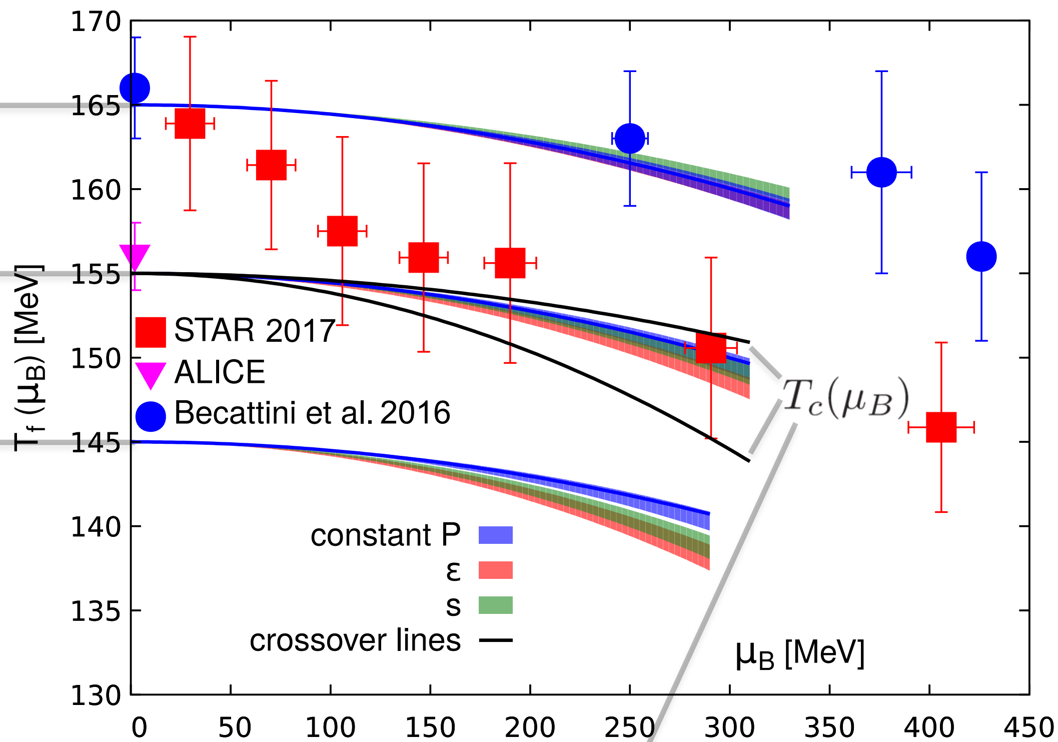
$$T_f(\mu_B) = T_0\left(1 - \kappa_2^f\left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f\left(\frac{\mu_B}{T_0}\right)^4\right)$$

$$0.0064 \le \kappa_2^P \le 0.0101$$
$$0.0087 \le \kappa_2^\epsilon \le 0.012$$
$$0.0074 \le \kappa_2^s \le 0.011$$

T=165 MeV
ϵ = 0.556(57) GeV/fm³

T=155 MeV
ϵ = 0.346(41) GeV/fm³

T=145 MeV
ϵ = 0.203(27) GeV/fm³

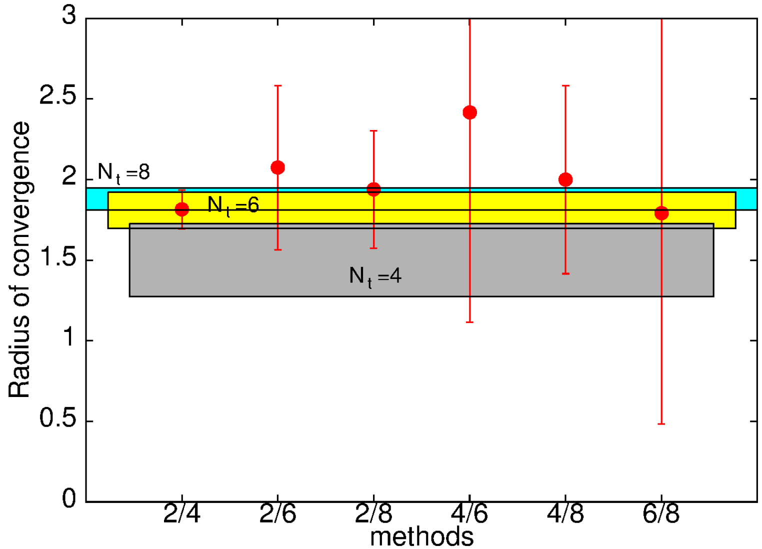


compare well with estimates of the crossover line: $T_c(\mu_B) = T_c(0)\left(1 - \kappa_2^c\left(\frac{\mu_B}{T_c(0)}\right)^2\right)$

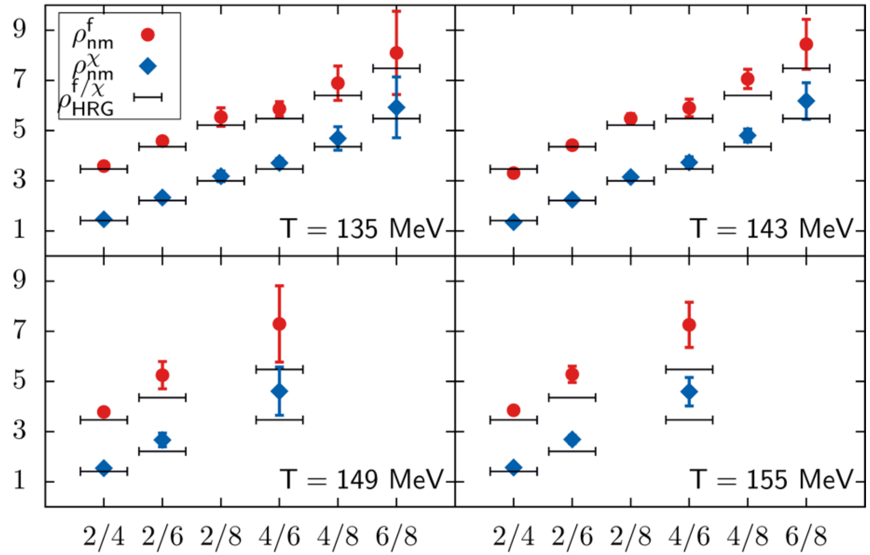
[Bielefeld-BNL-CCNU (2011), Endrodi et a. (2011), Bellwied et al. (2015), Cea et al. (2016), Bonati et al. (2015)]

$$0.0066 \le \kappa_2^c \le 0.020$$

estimated from Taylor expansion of pressure or baryon-number susceptibilty

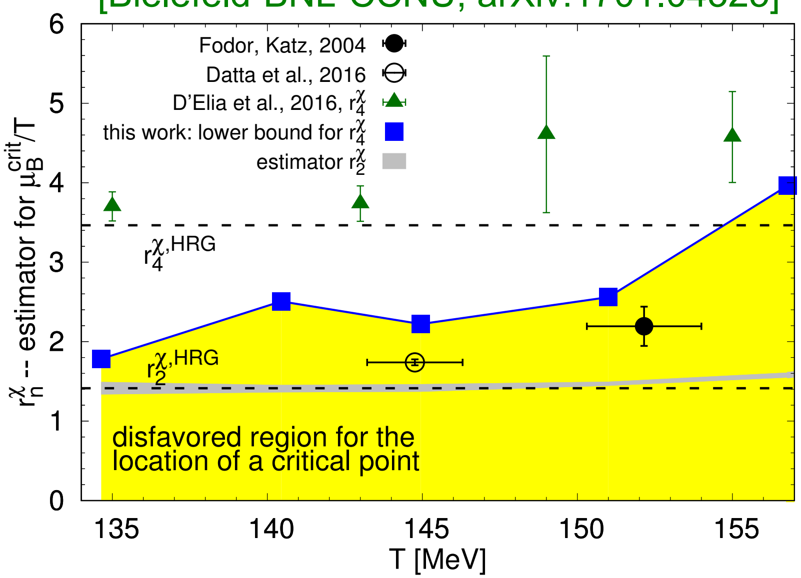


[S.Datta, R.V. Gavai, S. Gupta, arXiv:1612.06673]



[M.D'Elia, G.Gagliardi, F.Sanfilippo, arXiv:1611.08285]

[Bielefeld-BNL-CCNU, arXiv:1701.04325]



$$r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}$$

A critical point at $\mu_B < 2T$ is disfavored for 135 MeV $\leq T \leq$ 155 MeV

and seems to be ruled out at T>155 MeV

Higher order cumulants required for the search of a critical point

Cumulants of net-charge fluctuations

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{(i+j+k)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\vec{\mu}=0}$

with $\hat{\mu}_X = \mu_X/T$

generalized susceptibilities calculated at zero μ | cumulants of net-charge fluctuations measured at the freeze out

Lattice QCD | **Experiment**

$$\begin{aligned}
VT^3\chi_2^X &= \langle(\delta N_X)^2\rangle \\
VT^3\chi_4^X &= \langle(\delta N_X)^4\rangle - 3\langle(\delta N_X)^2\rangle^2 \\
VT^3\chi_6^X &= \langle(\delta N_X)^6\rangle \\
&\quad -15\langle(\delta N_X)^4\rangle\langle(\delta N_X)^2\rangle \\
&\quad +30\langle(\delta N_X)^2\rangle^3
\end{aligned}$$

$$\delta N_X \equiv N_X - \langle N_x\rangle$$

Cumulants of net-charge fluctuations

higher order cumulants characterize the shape of conserved charge distributions

$$S_q \sigma_q = \frac{\chi_3^q}{\chi_2^q}$$

$$q = B, Q, S$$

$$\kappa_q \sigma_q^2 = \frac{\chi_4^q}{\chi_2^q}$$

mean: $\langle \delta N_q \rangle \equiv \langle N_q - N_{\bar{q}} \rangle$

variance: $\sigma_q^2 \equiv \langle (\delta N_q)^2 \rangle - \langle \delta N_q \rangle^2$

skewness: $S_q \equiv \langle (\delta N_q)^3 \rangle / \sigma_q^3$

kurtosis: $\kappa_q \equiv \langle (\delta N_q)^4 \rangle / \sigma_q^4 - 3$

generalized susceptibilities calculated at zero μ

cumulants of net-charge fluctuations measured at the freeze out

Lattice QCD

$$\boxed{\begin{aligned} VT^3\chi_2^X \\ VT^3\chi_4^X \\ VT^3\chi_6^X \end{aligned}} = \boxed{\begin{aligned} &\langle (\delta N_X)^2 \rangle \\ &\langle (\delta N_X)^4 \rangle - 3\langle (\delta N_X)^2 \rangle^2 \\ &\langle (\delta N_X)^6 \rangle \\ &-15\langle (\delta N_X)^4 \rangle \langle (\delta N_X)^2 \rangle \\ &+30\langle (\delta N_X)^2 \rangle^3 \end{aligned}}$$

Experiment

$$\delta N_X \equiv N_X - \langle N_x \rangle$$

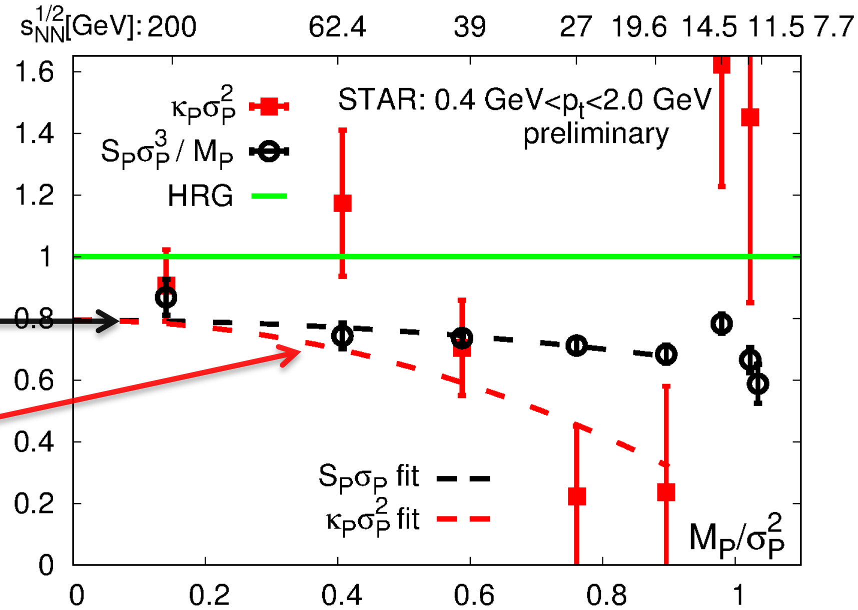
Cumulants of net-baryon number and freeze-out

(M_B=mean σ_B=variance S_B=skewness κ_B=kurtosis)

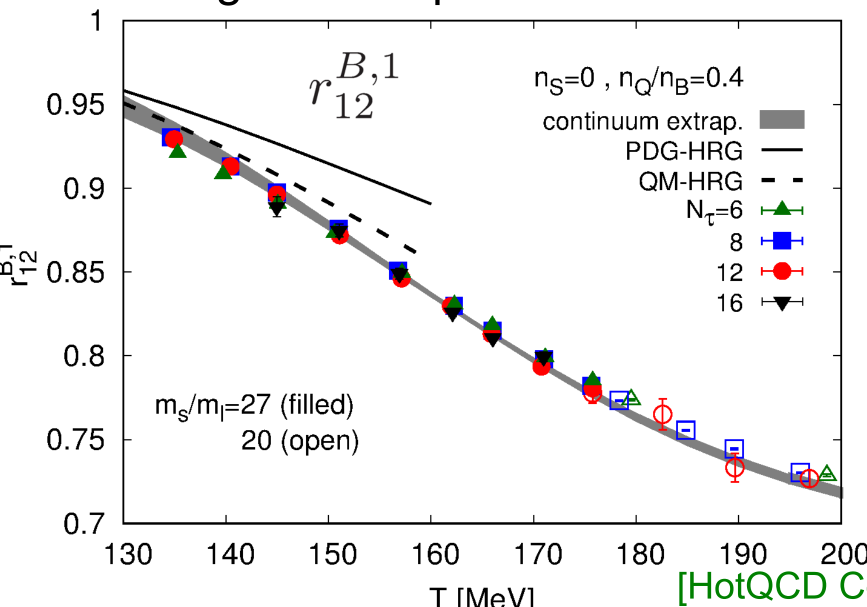
$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{M_B}{\sigma_B^2}$$

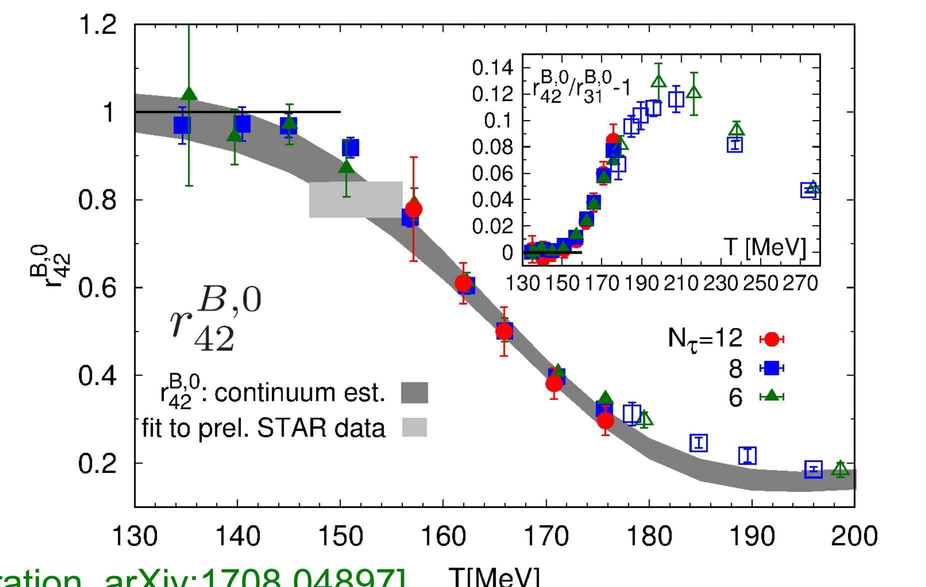
$$R_{31}^B(T,\mu_B) = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{S_B\sigma_B^3}{M_B}$$

$$R_{42}^B(T,\mu_B) = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \kappa_B\sigma_B^2$$



Leading order expansion coefficients:





[HotQCD Collaboration, arXiv:1708.04897]

Cumulants of net-baryon number and freeze-out

(M_B=mean σ_B=variance S_B=skewness κ_B=kurtosis)

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{M_B}{\sigma_B^2}$$

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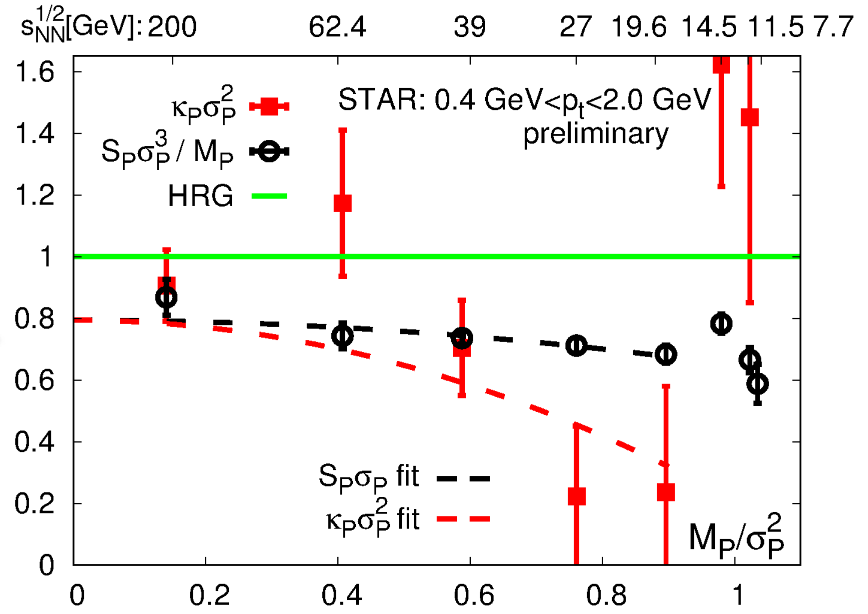
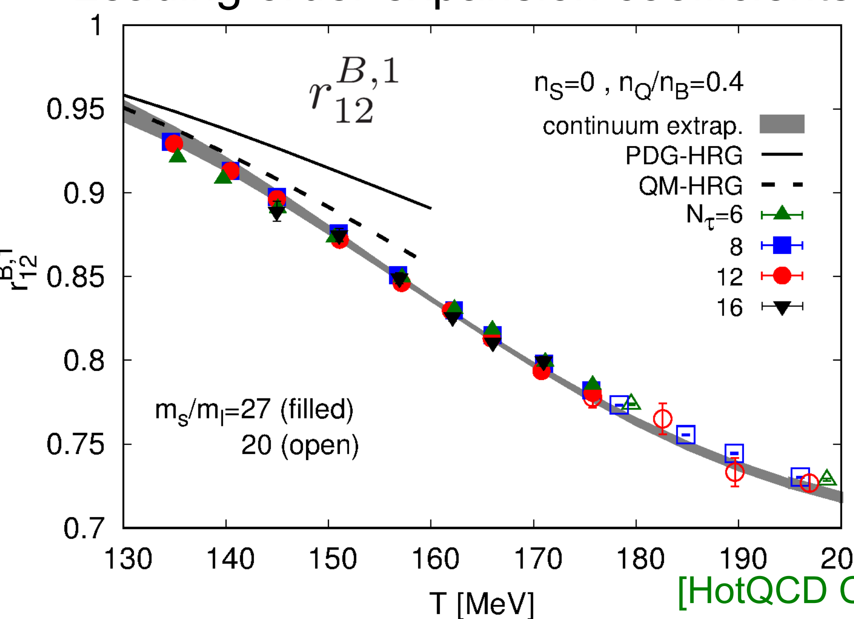
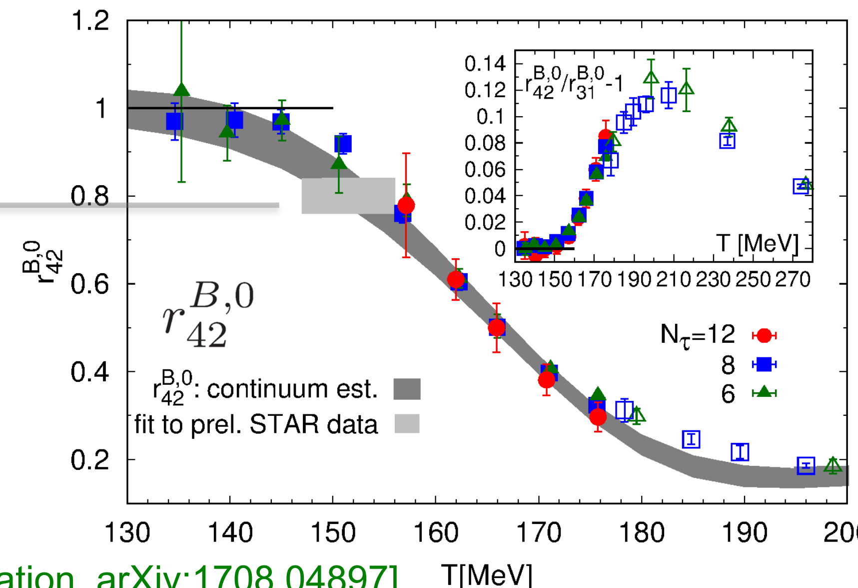
$$R_{42}^B(T,\mu_B) = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \kappa_B\sigma_B^2$$



for $\mu/T \rightarrow 0$ → $T_{f,0}$ = 153(5) MeV

Leading order expansion coefficients:





[HotQCD Collaboration, arXiv:1708.04897]

Cumulants of net-baryon number and freeze-out

NLO expansion of ratios of cumulants

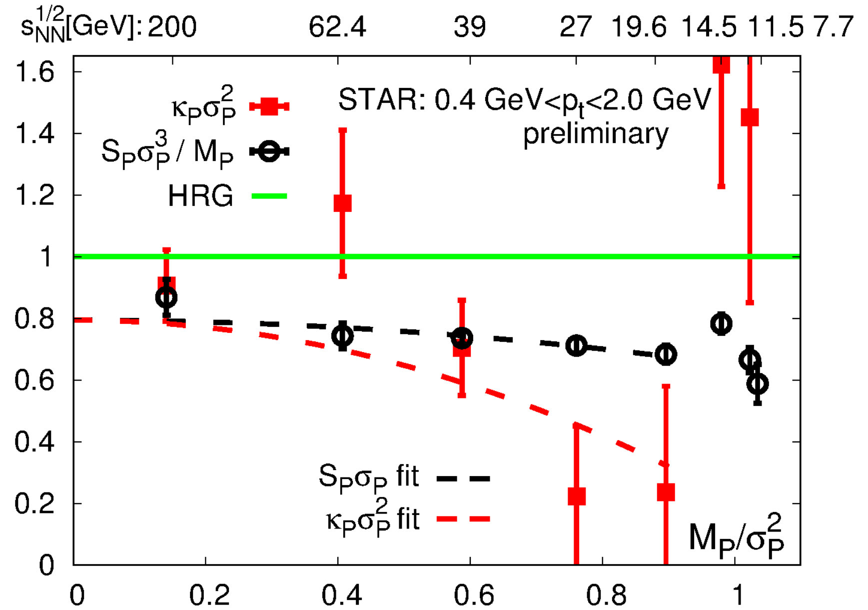
on lines of constant physics:

$$R_{12}^{B}(T,\mu_B) = r_{12}^{B,1}\hat{\mu}_B + r_{12,f}^{B,3}\hat{\mu}_B^3$$

$$R_{31}^{B}(T,\mu_B) = r_{31}^{B,0} + r_{31,f}^{B,2}\hat{\mu}_B^2$$

$$R_{42}^{B}(T,\mu_B) = r_{42}^{B,0} + r_{42,f}^{B,2}\hat{\mu}_B^2$$

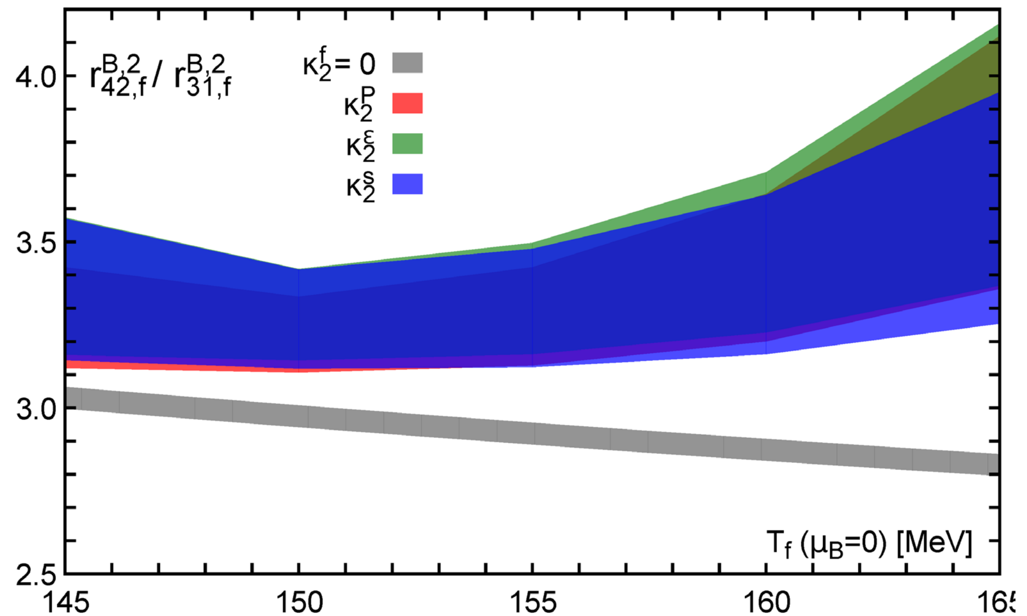
Ratio of slopes from STAR = 3.9(2.1)



compares well with QCD predictions

$$\frac{r_{42,f}^{B,2}}{r_{31,f}^{B,2}} = 3.1 \ - \ 4.1$$

on lines of constant physics



[HotQCD Collaboration, arXiv:1708.04897]

Conclusions – part I

- Equation of state well controlled for $\mu_B/T\lesssim 2$ using expansion up to 6th-order

- A critical point at $\mu_B < 2T$ is disfavored for 135 MeV $\leq T \leq$ 155 MeV and seems to be ruled out at T>155 MeV

- Higher order cumulants required for the search of a critical point

- Decrease of skewness and kurtosis for $\sqrt{s_{NN}} > 19.6\ GeV$ in accordance with QCD

- Physics above 160MeV much different from a hadron gas

Part II: Spectral and transport properties in the QGP

Thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega\mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\ \rho_{\mathbf{V}}(\omega,\mathbf{T})$$

Thermal photon rate

$$\omega\frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\rho_V(\omega=|\vec{k}|,T)$$

Transport coefficients are encoded in the same spectral function

→ Kubo formulae

Diffusion coefficients:

$$DT = \frac{T}{2\chi_q}\lim_{\omega\to 0}\frac{\rho_{\mathrm{ii}}(\omega)}{\omega}$$

Need to determine vector-meson spectral functions

On the lattice only correlation functions can be calculated

→ spectral reconstruction required

Dilepton rates

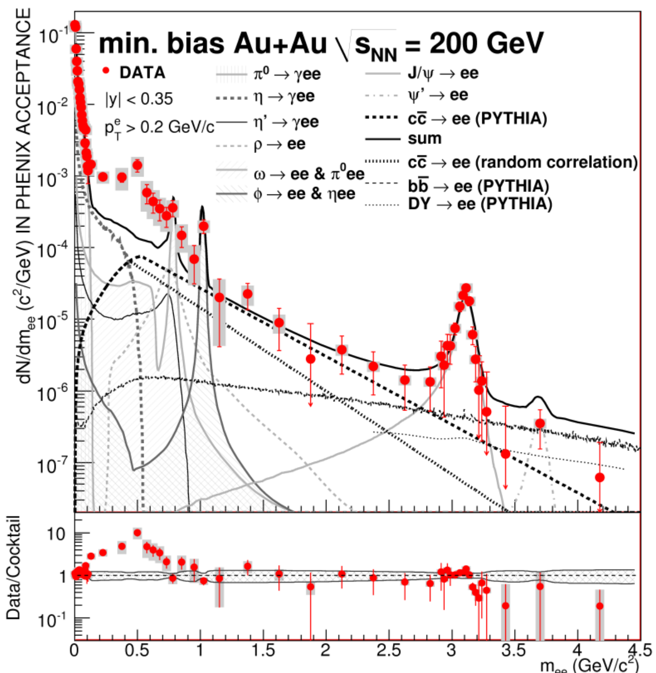
Photon rates

large enhancement between 150-750 MeV

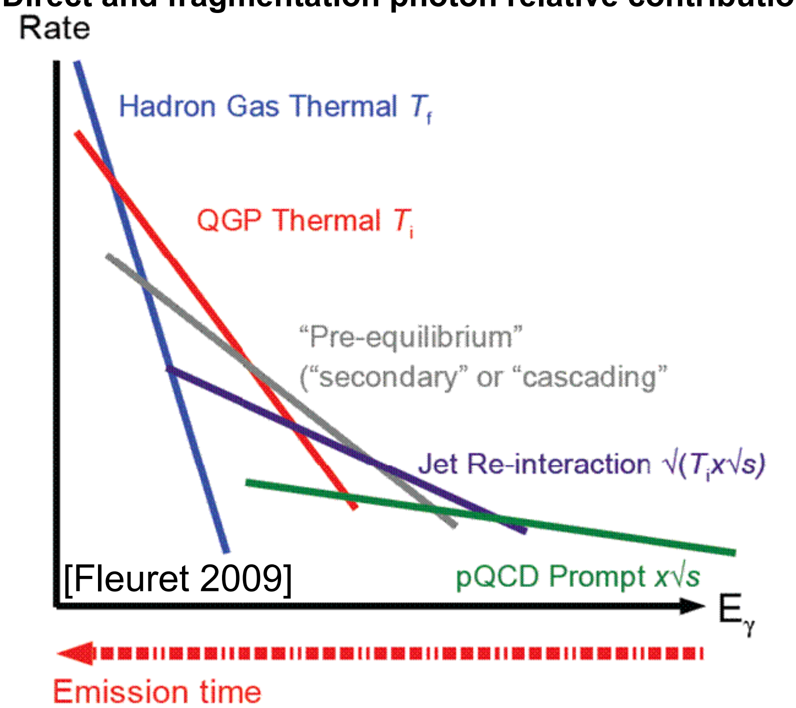
possible window for photons from QGP

indications for thermal effects!?

Need to understand the contribution from QGP



[PHENIX PRC81, 034911 (2010)]



both directly related to vector-meson spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega\mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{(\omega^2-\vec{p}^2)(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\vec{\mathbf{p}},\mathbf{T})$$

$$\omega\frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\sigma_V(\omega=|\vec{p}|,T)$$

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

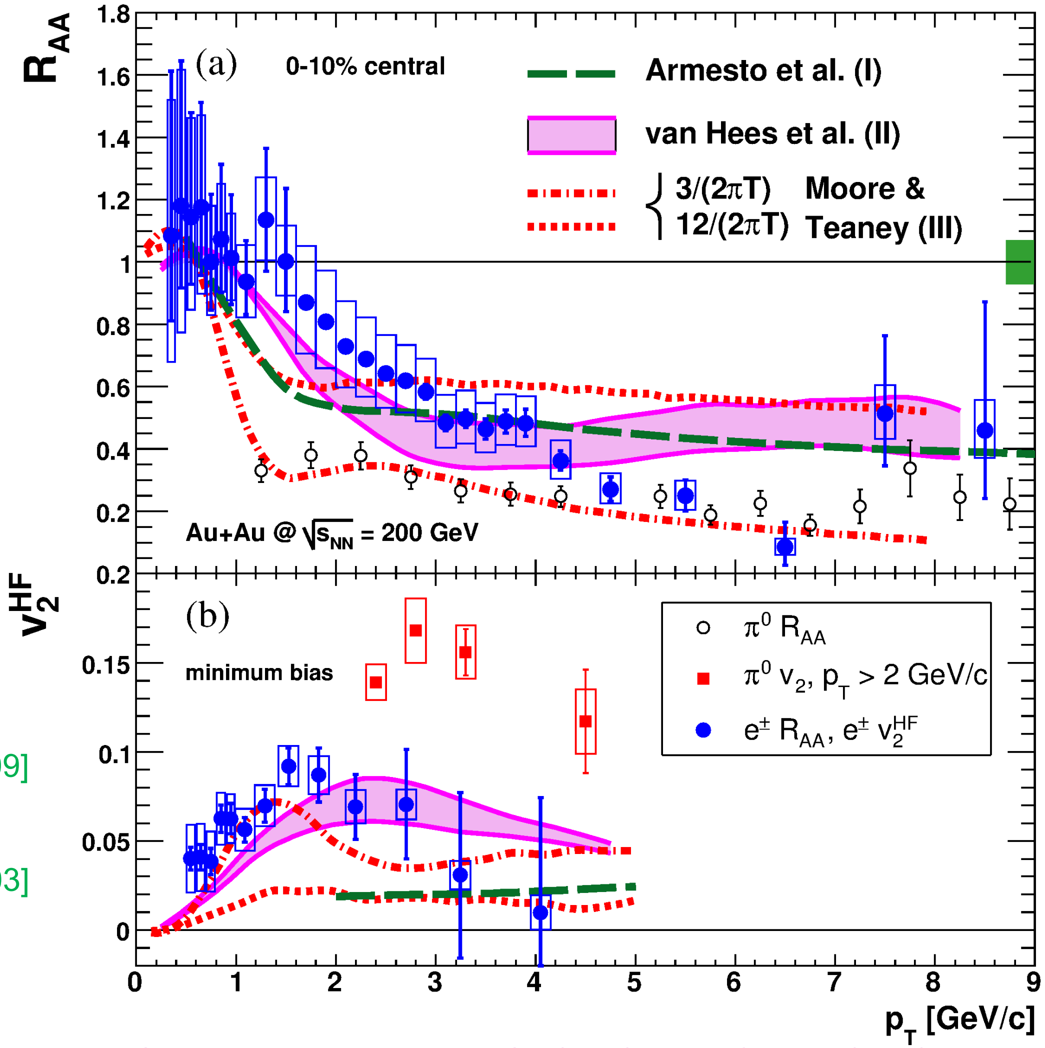
Need to be determined from QCD using first principle lattice calculations!

for heavy flavour:

- Heavy Quark Diffusion Constant D
  [H.T.Ding, OK et al., PRD86(2012)014509]
- Heavy Quark Momentum Diffusion κ
  [A.Francis, OK, et al., PRD92(2015)116003]

for light quarks:

- Light quark flavour diffusion / Electrical conductivity
  [A.Francis, OK et al., PRD83(2011)034504
  H-T.Ding, F.Meyer, OK, PRD94 (2016) 034504]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

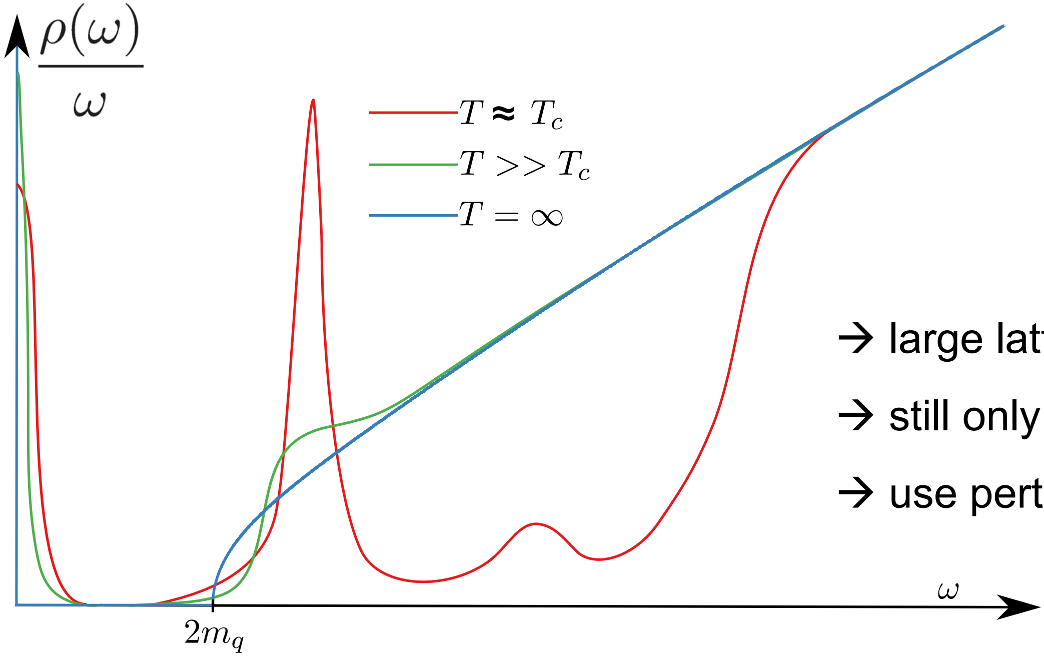
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

Vector-meson correlation function for light quarks on large & fine lattices

[H-T.Ding, F.Meyer, OK, PRD94(2016)034504, H.T.Ding, A.Francis, OK et al., PRD83(2011)034504]

quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

non-perturbative renormalization constants and quark masses close to the chiral limit

	N_τ	N_σ	β	κ	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$	confs
	32	96	7.192	0.13440	0.2796	1.12	0.8164	1.09	314
1.1 T_c	48	144	7.544	0.13383	0.2843	1.14	0.8169	1.10	358
	64	192	7.793	0.13345	0.2862	1.15	0.8127	1.09	242
	28	96	7.192	0.13440	0.3195	1.28	0.9330	1.25	232
1.3 T_c	42	144	7.544	0.13383	0.3249	1.31	0.9336	1.25	417
	56	192	7.793	0.13345	0.3271	1.31	0.9288	1.25	273
	24	128	7.192	0.13440	0.3728	1.50	1.0886	1.46	340
1.5 T_c	32	128	7.457	0.13390	0.3846	1.55	1.1093	1.49	255
	48	128	7.793	0.13340	0.3817	1.53	1.0836	1.45	456

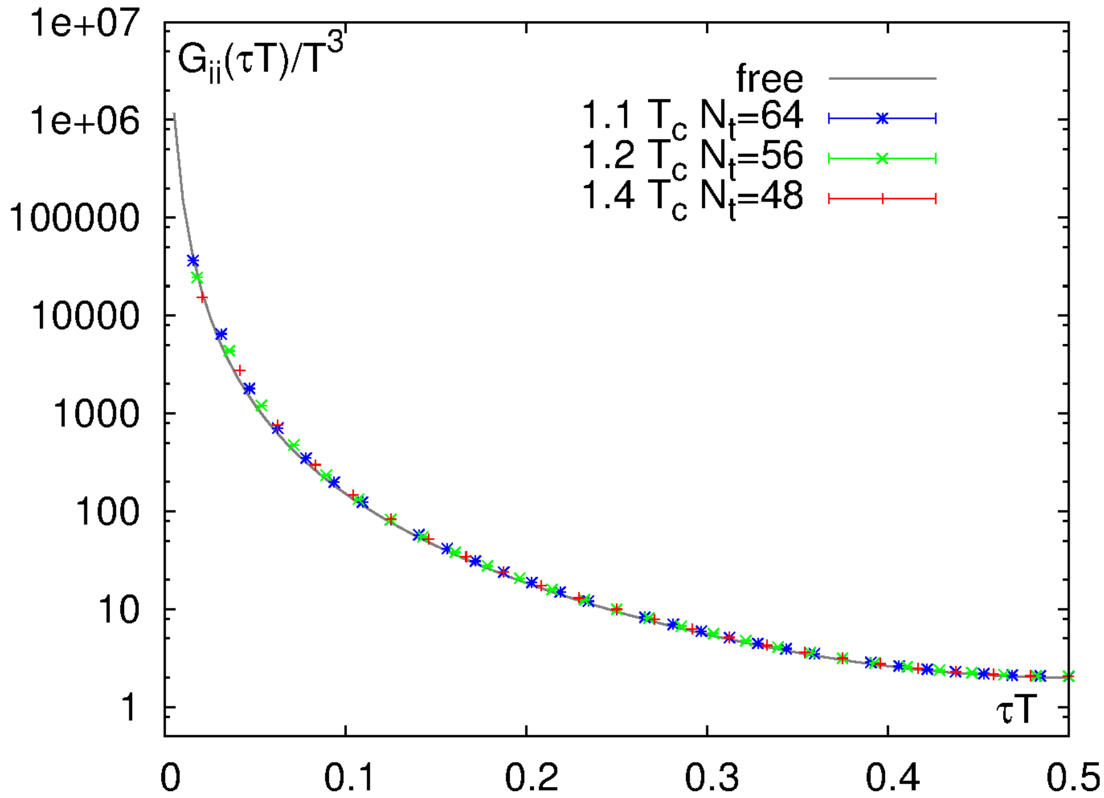
Scale setting using r_0 and t_0 [A.Francis, M.Laine, T.Neuhaus, H.Ohno PRD92(2015)116003]

fixed aspect ratio N_σ/N_τ = 3 and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

Vector-meson correlation function

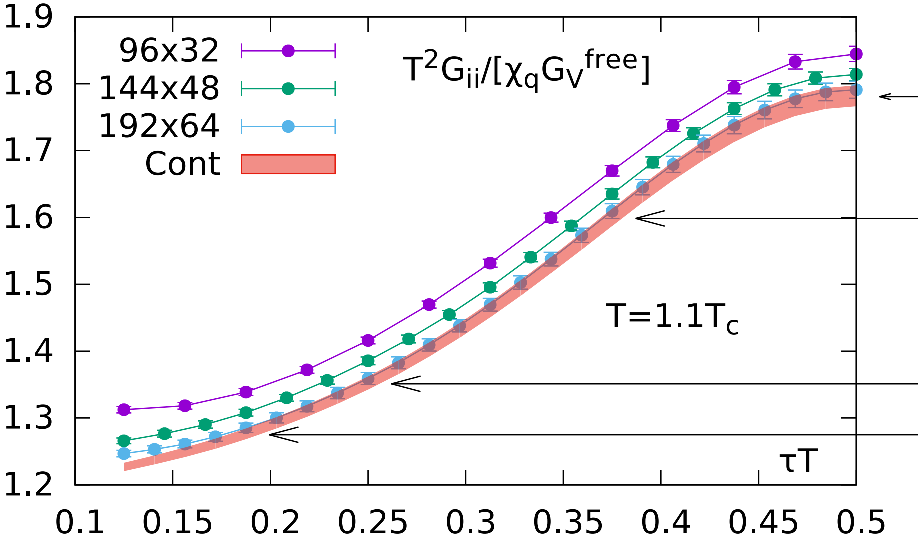


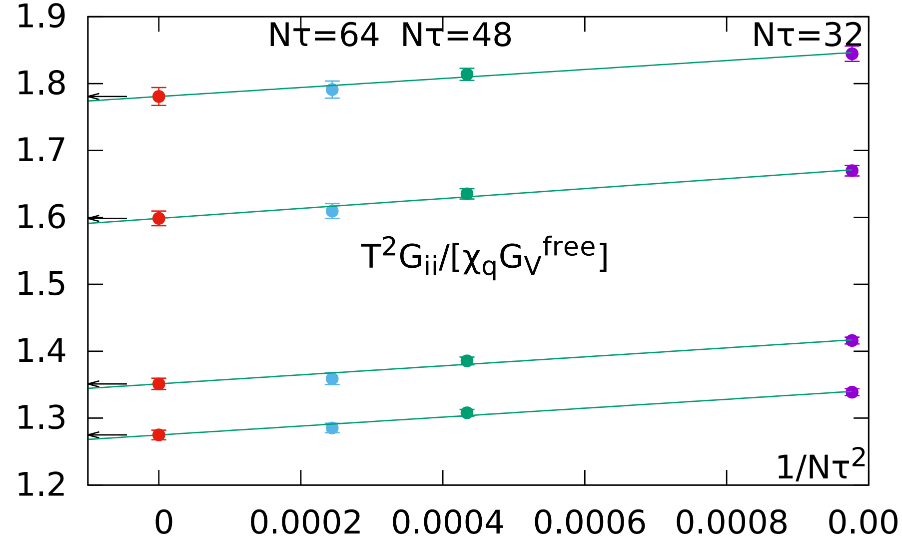
compared to free (non-interacting) correlator:

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

hard to distinguish differences due to different orders of magnitude in the correlator

→ in the following we will use $G_V^{free}(\tau)$ as a normalization





correlators normalized by quark number susceptibility χ_q independent of renormalization

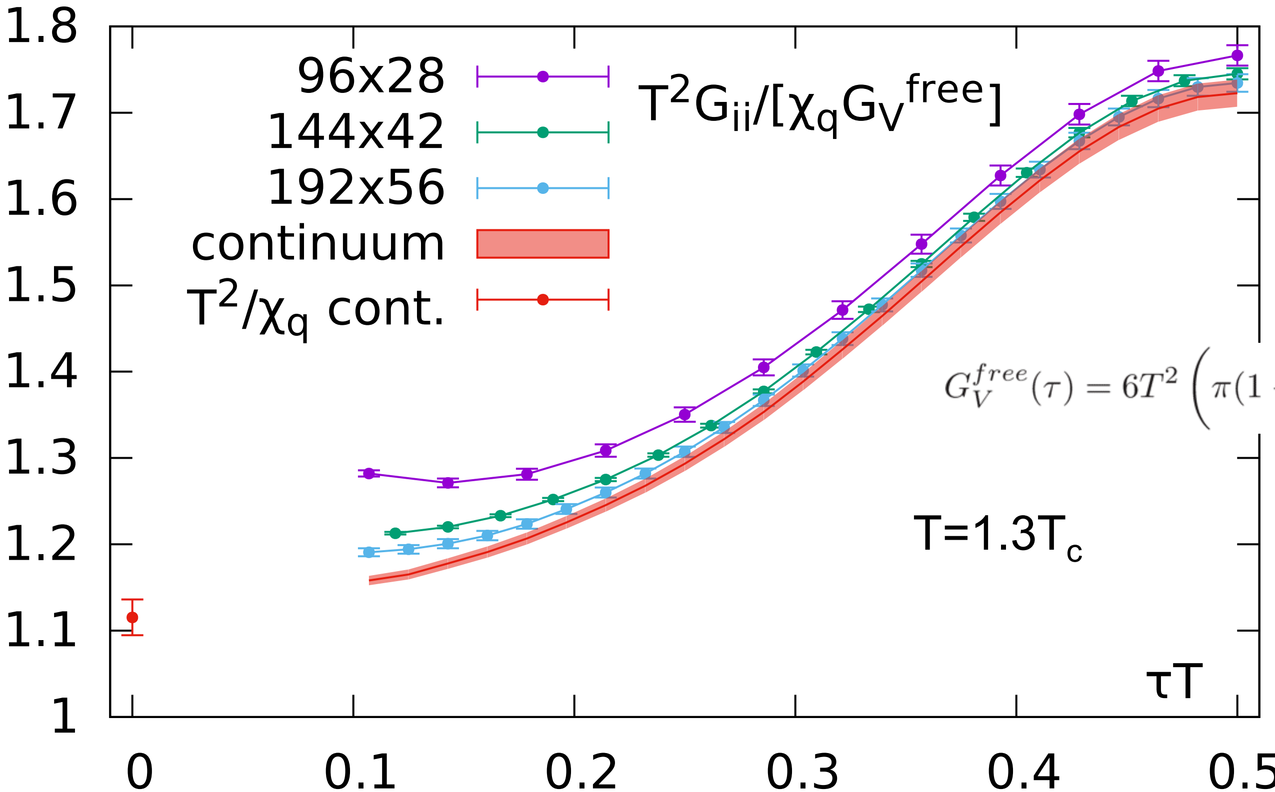
and by the free non-interacting correlator $G_V^{free}(\tau)$

we interpolate the correlator for each lattice spacing

and perform the continuum limit a→0 at each distance τT

cut-off effects are visible at all distances on finite lattices

Continuum extrapolation



normalized by free correlator

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T) \frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

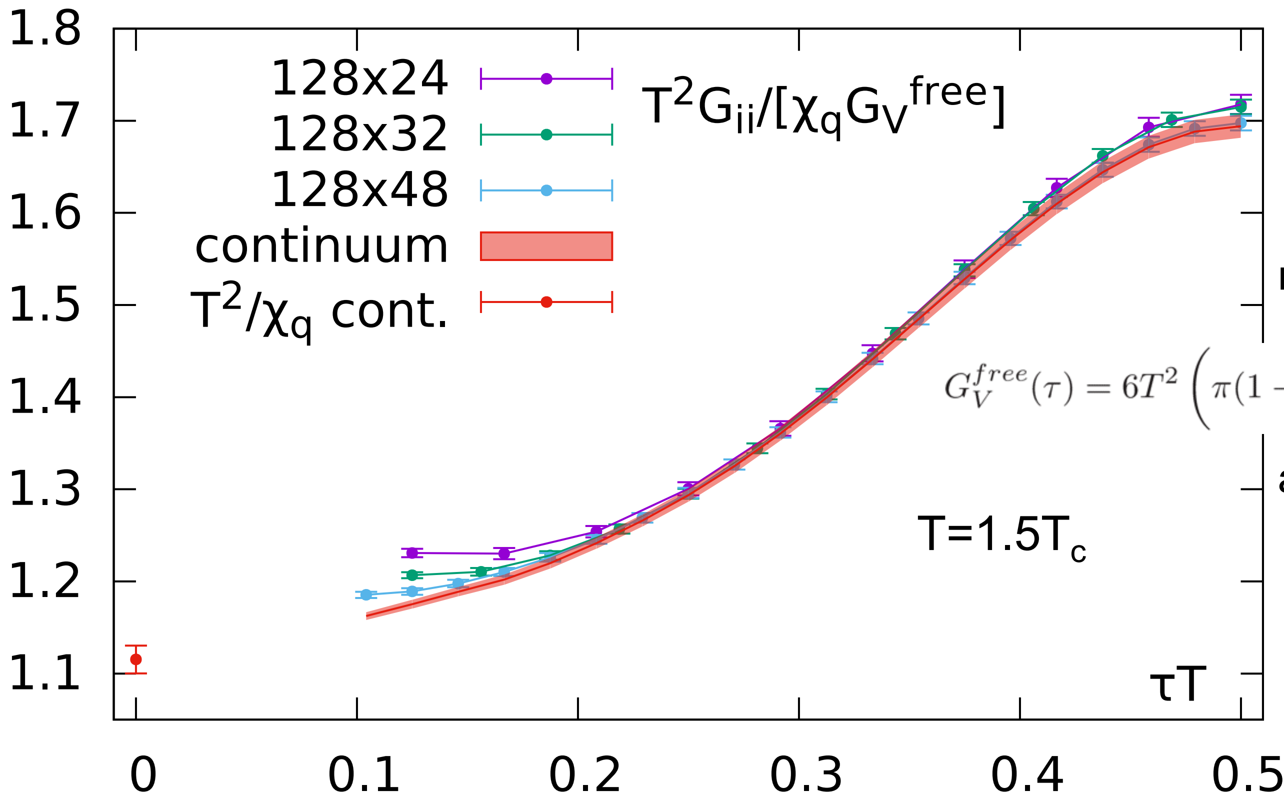
and quark number susceptibility

cut-off effects are visible at all distances on finite lattices but

well defined continuum limit on the correlator level

well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$



normalized by free correlator

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

and quark number susceptibility

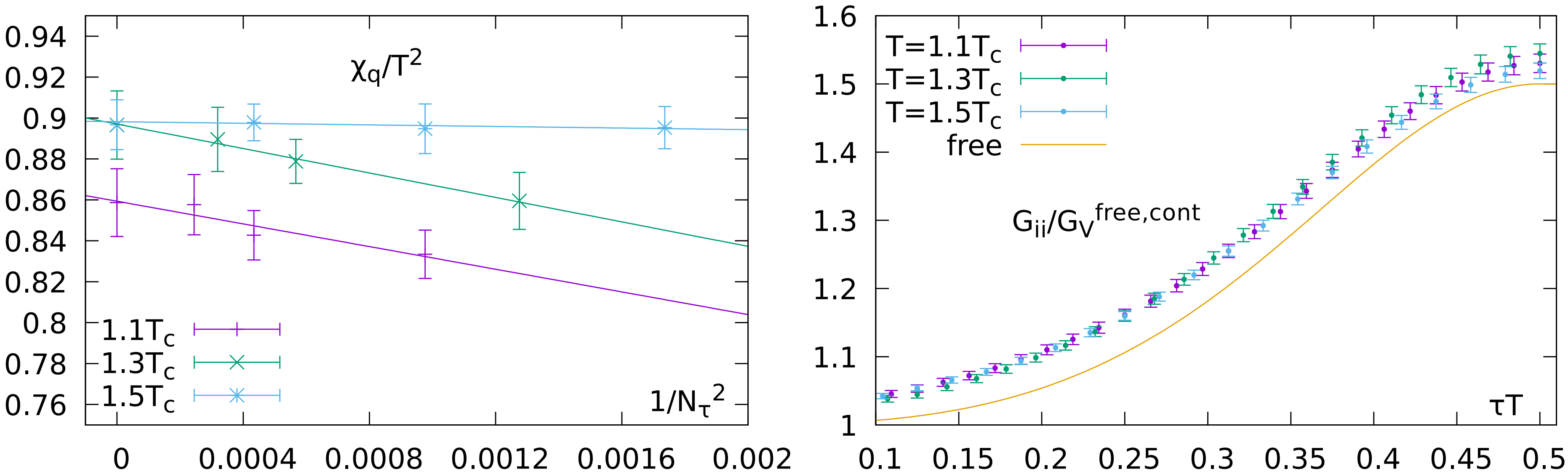
cut-off effects are visible at all distances on finite lattices but

well defined continuum limit on the correlator level

well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

continuum extrapolated results available for three temperatures in the QGP



similar behavior in this temperature region

main difference due to different quark number susceptibility χ_q/T^2

→ indications for a weak T-dependence of the temperature scaled

electrical conductivity and thermal dilepton rates

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\rho_{00}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega)$$
$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)$$

δ-functions exactly cancel in $\rho_V(\omega)=-\rho_{00}(\omega)+\rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{00} is protected, the δ-funtion in ρ_{ii} gets smeared → transport peak:

Ansatz:

$$\kappa = \frac{\alpha_s}{\pi}$$ at leading order

$$\rho_{00}(\omega) = 2\pi\chi_q \omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2 \tanh(\omega/4T)$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

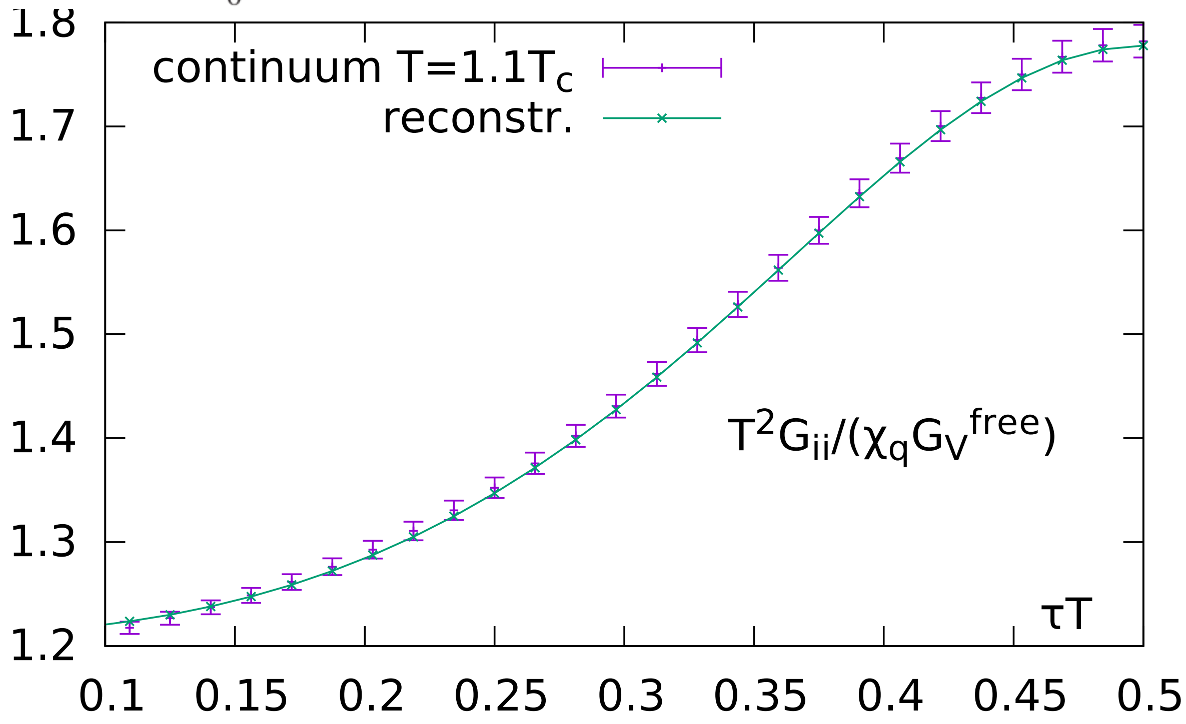
→ electrical conductivity: $$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T} \qquad \frac{\sigma}{T} = \frac{2}{3}\frac{\chi_q}{T^2}\frac{T}{\Gamma}\,c_{BW}\,C_{em}$$

use our Ansatz for the spectral function and fit to the continuum extrapolated correlators

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$



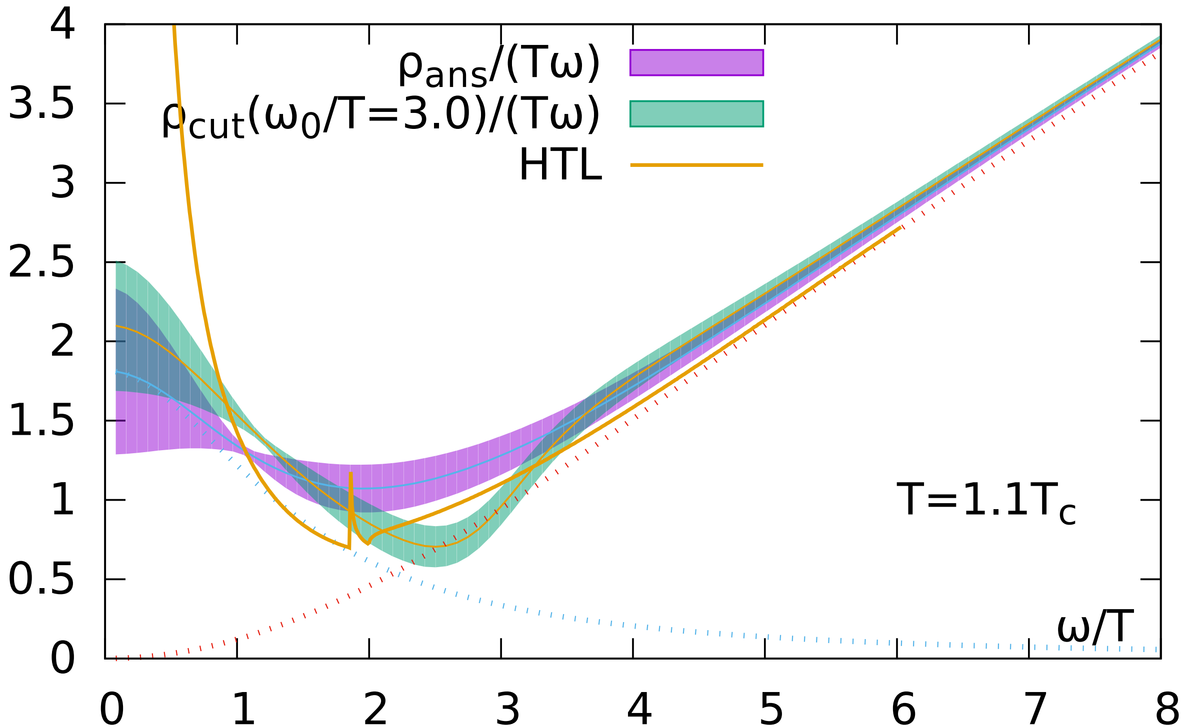
all three temperatures well described by this rather simple Ansatz

T	$\sigma/(C_{\rm em}T)$	Γ/T	$c_{BW}T/\Gamma$	k	χ^2/dof
$1.1T_c$	0.302(88)	2.86(1.16)	0.528(154)	0.038(8)	1.15
$1.3T_c$	0.254(51)	3.91(1.25)	0.425(85)	0.029(9)	0.52
$1.5T_c$	0.266(48)	3.33(89)	0.445(80)	0.040(7)	1.13

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

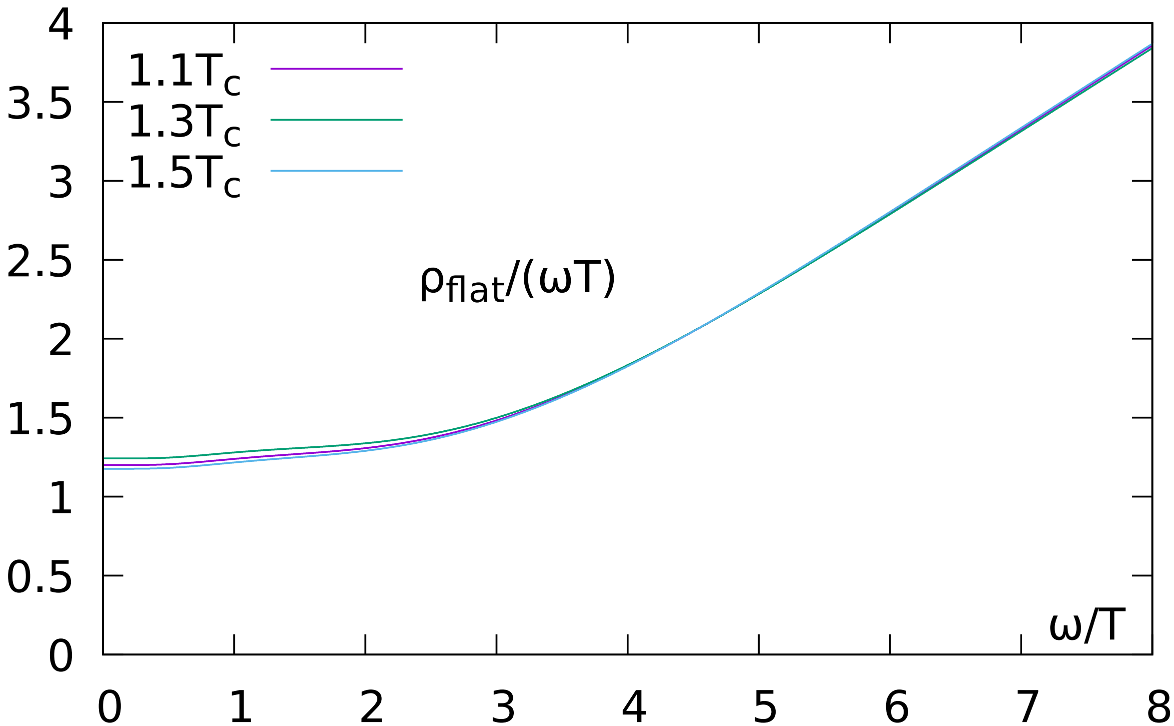
systematic uncertainties (within this Ansatz) estimated by varying the truncation

with similar χ^2/dof ~ 0.5-1.1

Use a flat transport Ansatz for the spectral function

$$\rho_{\text{flat}}(\omega) \quad = \quad a\chi_q\omega\left(1-\widetilde{\Theta}(\omega_0,\Delta_0)\right)+(1+k)\rho_{\text{free}}(\omega)\widetilde{\Theta}(\omega_1,\Delta_1)$$

$$\widetilde{\Theta}(\omega,\omega_i,\Delta_i)=\left(1+\exp\left(\frac{\omega_i^2-\omega^2}{\omega\Delta_i}\right)\right)^{-1}$$



Analysis of the systematic errors

using a flat behavior at small ω

$$\frac{\sigma}{T}=\frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

systematic uncertainties (within this Ansatz) estimated by varying the truncation

with similar χ^2/dof ~ 0.5-1.1 still consistent with our data → lower limit for σ/T

Improve the UV behavior of the spectral function using perturbation theory:

At very high energies, due to asymptotic freedom

→ perturbation should be working

→ thermal effects should be suppressed

→ “vacuum physics”

5-loop vacuum spectral function:

$$\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)$$

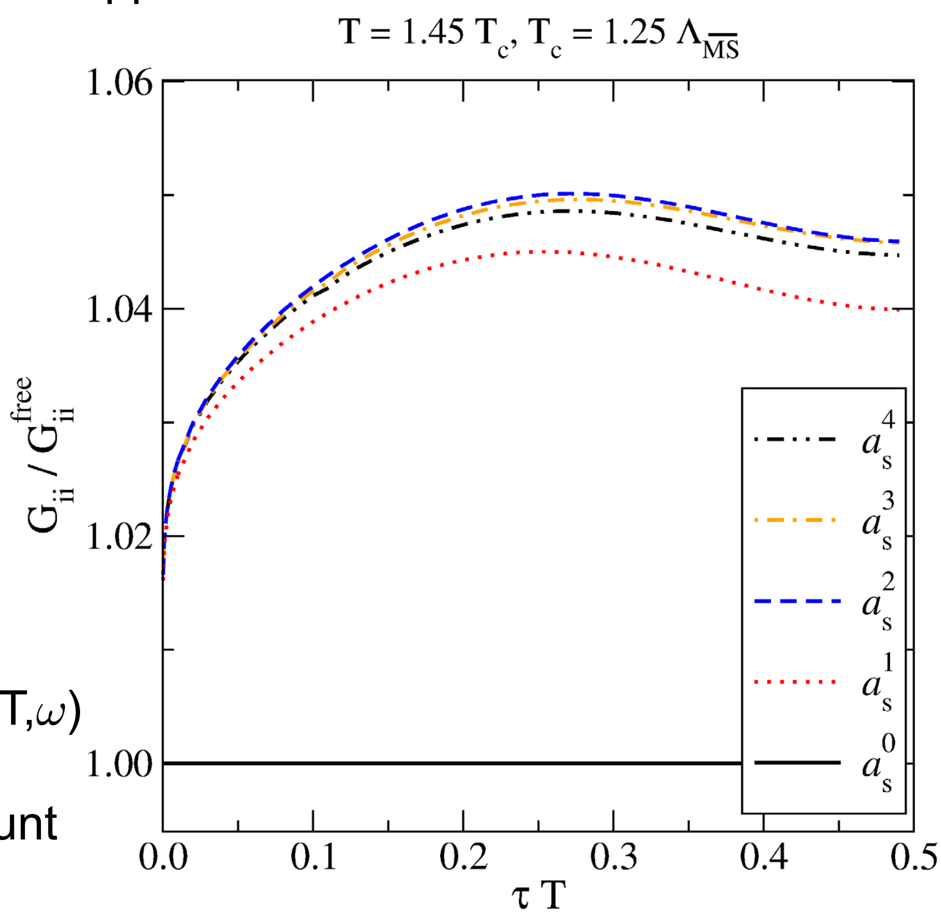
$$\begin{aligned} R(\omega^2) &= r_{0,0} + r_{1,0}\, a_s + \left(r_{2,0} + r_{2,1}\,\ell\right) a_s^2 \\ &+ \left(r_{3,0} + r_{3,1}\,\ell + r_{3,2}\,\ell^2\right) a_s^3 \\ &+ \left(r_{4,0} + r_{4,1}\,\ell + r_{4,2}\,\ell^2 + r_{4,3}\,\ell^3\right) a_s^4 + \mathcal{O}(a_s^5) \end{aligned}$$

using 3-loop α_s and l=log(μ^2/ω^2)

using a renormalization scale μ=(1..5)max(πT,ω)

taking leading order thermal effect into account

$$\rho_{ii}^{(T)}(\omega) \equiv \frac{3\omega^2}{4\pi}\left[1 - 2n_{\mathrm{F}}\left(\frac{\omega}{2}\right)\right] R(\omega^2) + \pi \chi_{\mathrm{q}}^{\mathrm{free}} \omega \delta(\omega)$$



[Y.Burnier and M.Laine, arXiv 1201.1994]

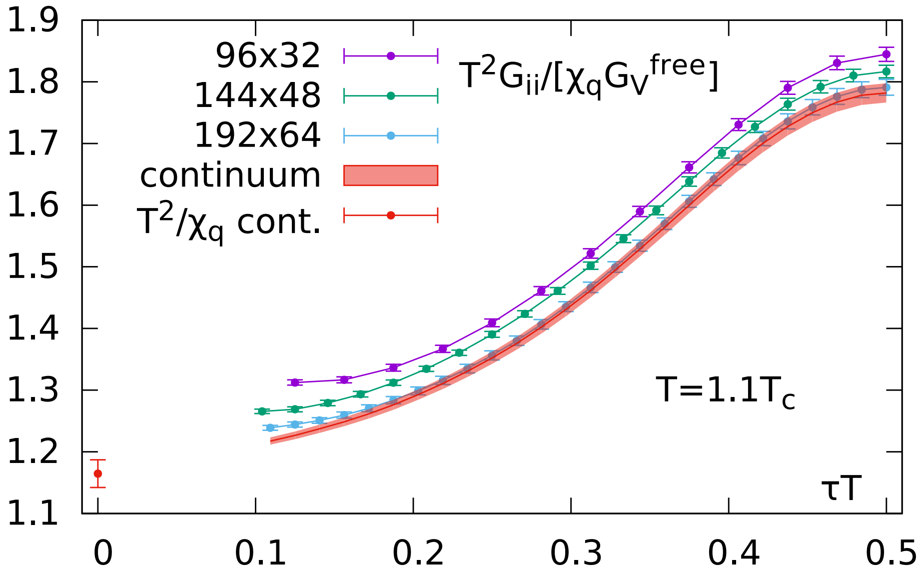
$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T) \qquad K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

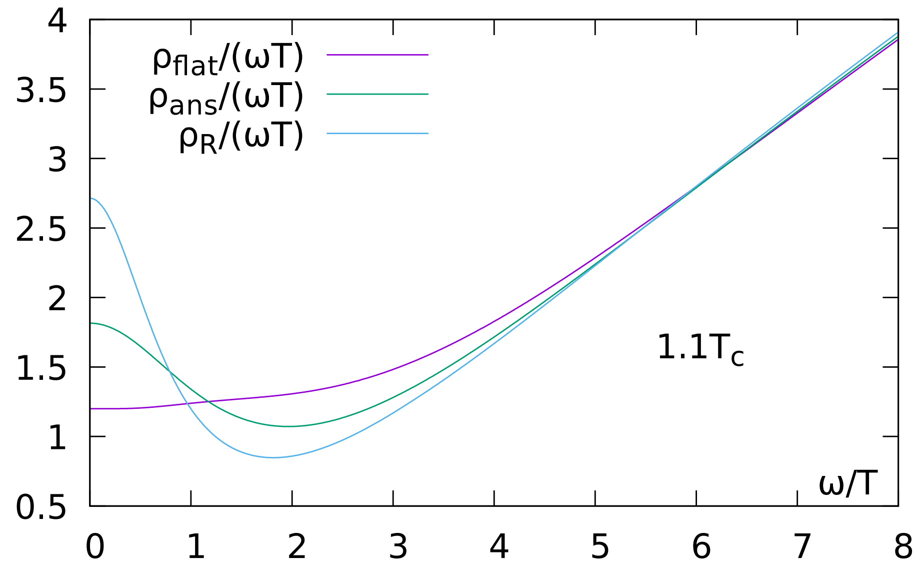
Ansatz for the (non-perturbative) transport contribution: $\rho_{BW}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2}$

and perturbative constraints for the UV part of the spectral function

$$\rho_R(\omega) = \rho_{BW}(\omega) + \frac{3\omega^2}{4\pi}[1-2n_F(\frac{\omega}{2})]R(\omega^2)$$ (5-loop vacuum + LO thermal correction)

Fit to continuum extrapolated vector-meson correlation function $G_{ii}(\tau,T)$

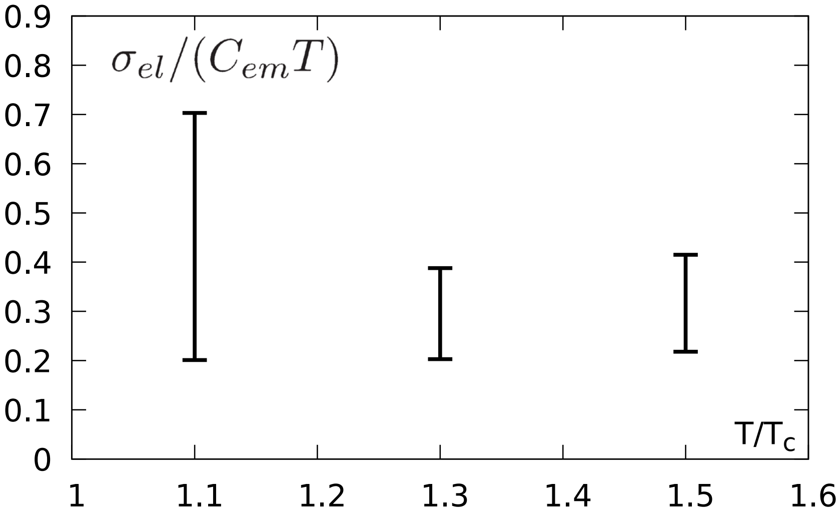




continuum estimate for the of the electrical conductivity

lower and upper limits from analysis of different classes of spectral functions:

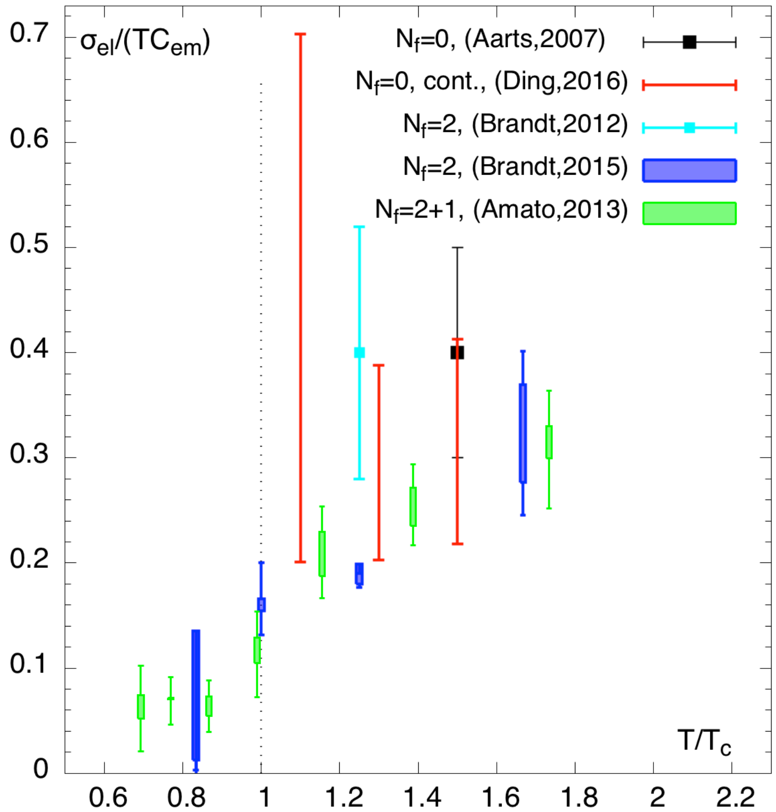
$$\frac{\sigma_{el}}{C_{em}T} = \frac{1}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$



[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

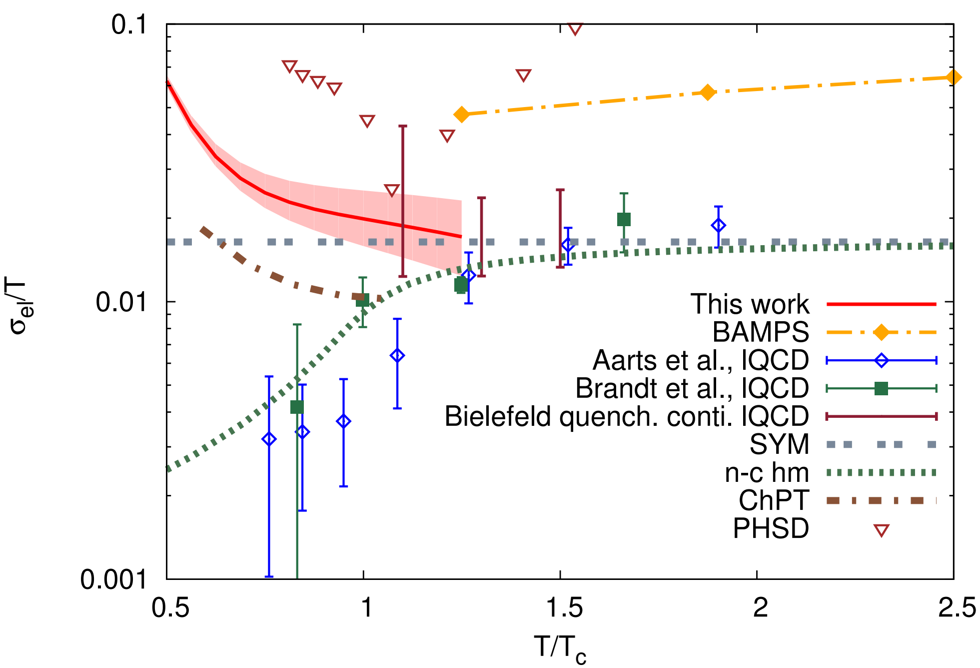
comparison of different lattice results

(Plot courtesy of A.Francis)



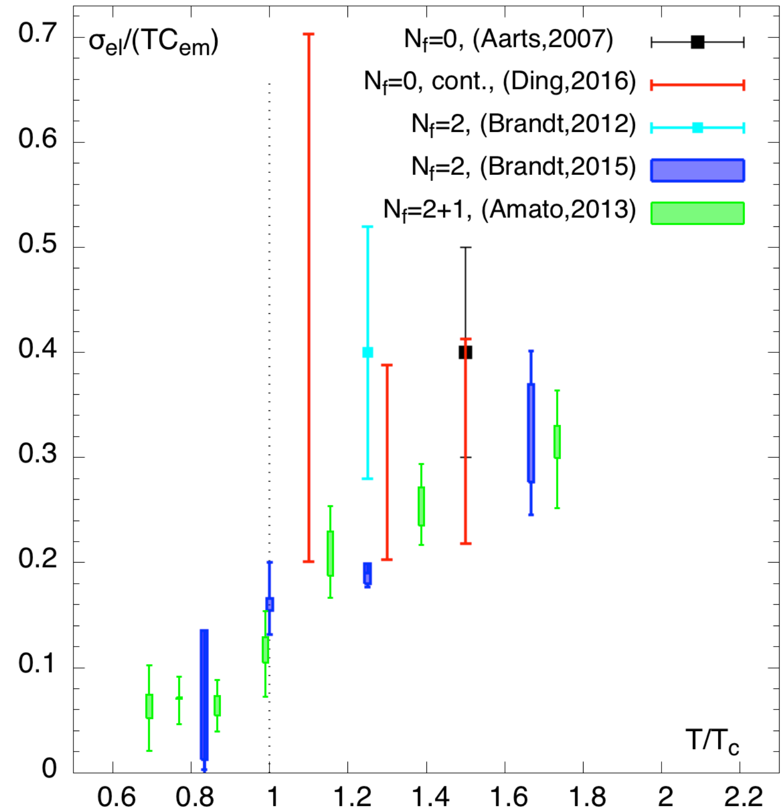
[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

compared to calculations in partonic transport approaches



[M.Greif, C.Greiner, G.Denicol, PRD93 (2016) 096012]

comparison of different lattice results
(Plot courtesy of A.Francis)

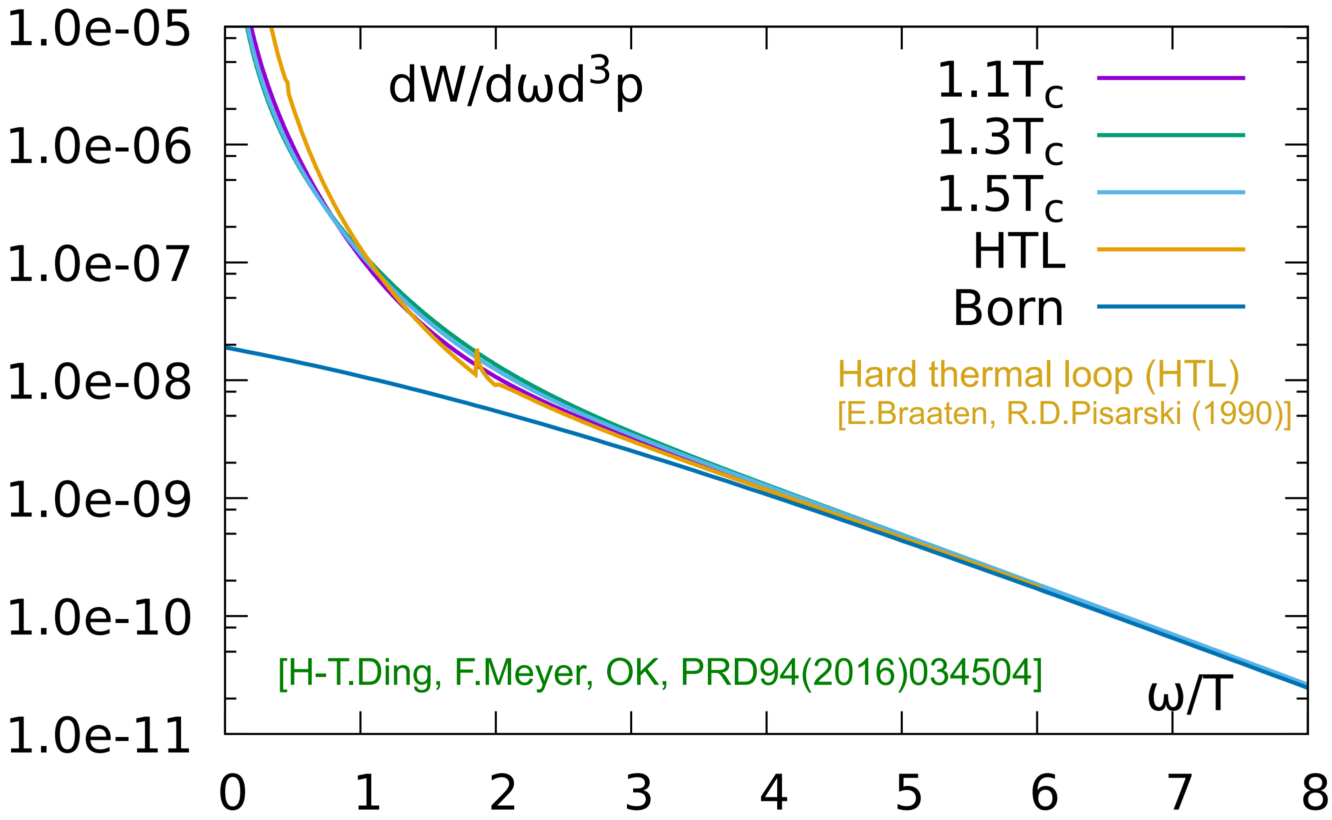


[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

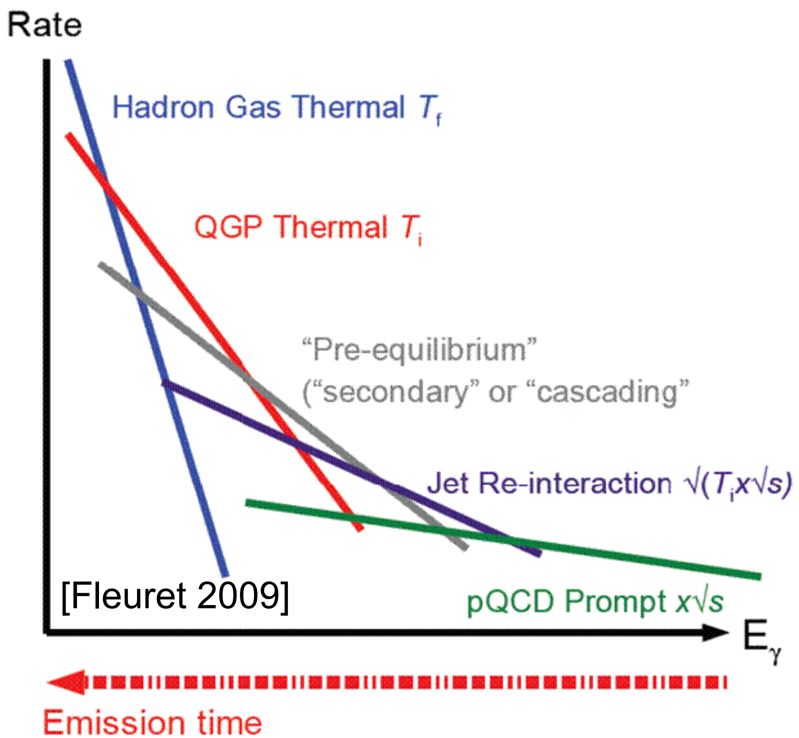
Progress in determining transport coefficients, although systematic uncertainties still need to be reduced in the future.

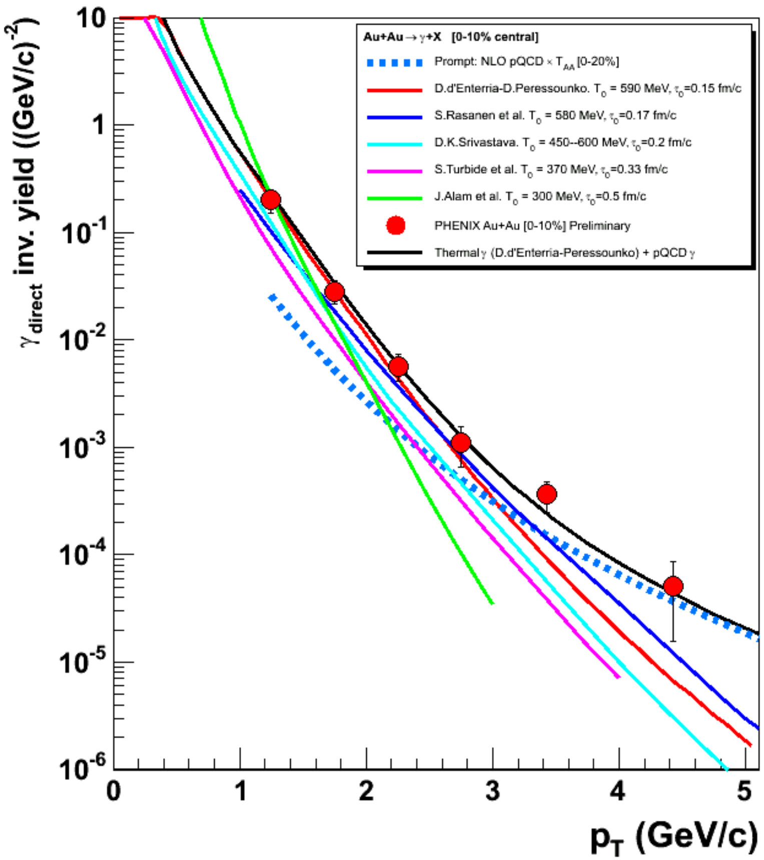
Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$



Direct and fragmentation photon relative contribution





Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x\, d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \sigma_V(\omega = |\vec{k}|, T)$$

Perturbative knowledge on the vector spectral function

Non-interacting limit, “Born rate” for large invariant mass M>>πT, with $M^2=\omega^2+k^2$

$$\rho_V(\omega,\mathbf{k}) = \frac{N_c T M^2}{2\pi k}\left\{\ln\left[\frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})}\right] - \frac{\omega\,\theta(k-\omega)}{2T}\right\},$$

[G. Aarts and J.M. Martinez Resco, NPB 726 (2005) 93]

Leading-log order for invariant mass M=0: [J.I. Kapusta et al.,PRD44 (1991) 2774, R. Baier et al. Z.Phys.C53 (1992) 433]

$$\rho_V(k,\mathbf{k}) = \frac{\alpha_s N_c C_F T^2}{4}\ln\left(\frac{1}{\alpha_s}\right)\left[1-2n_F(k)\right] + \mathcal{O}(\alpha_s T^2),$$

Complete leading order for invariant mass M=0:

[Arnold, Moore, Yaffe, JHEP11(2001)57 and JHEP12(2001)9]

NLO at M = 0: [J.Ghiglieri et al., JHEP 1305 (2013) 010]

NLO at M $\sim$ gT: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

NLO at M $\sim$ πT: [M.Laine, JHEP 1311 (2013) 120]

N^4LO at M >> πT: [S. Caron-Huot, PRD79 (2009) 125009, P.A.Baikov et al. PRL101 (2008) 012002]

Vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$:

$$\frac{\rho_{\mathrm{V}}(\omega,\mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2\right)\chi_{\mathrm{q}} D$$

with the quark number susceptibility: $\chi_{\mathrm{q}} \equiv \int_0^\beta \mathrm{d}\tau \int_{\mathbf{x}} \langle V^0(\tau,\mathbf{x}) V^0(0)\rangle$

and the diffusion coefficient: $D \equiv \frac{1}{3\chi_{\mathrm{q}}} \lim_{\omega\to 0^+} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega,\mathbf{0})}{\omega}$

which relate to the electric conductivity: $\sigma = e^2 \sum_{f=1}^{N_{\mathrm{f}}} Q_f^2 \chi_{\mathrm{q}} D$

In this limit the (soft) photon rate becomes: $\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} \overset{k \lesssim \alpha_{\mathrm{s}}^2 T}{\approx} \frac{2T\sigma}{(2\pi)^3 k}$

In the AdS/CFT framework the vector spectral function has the same infrared structure

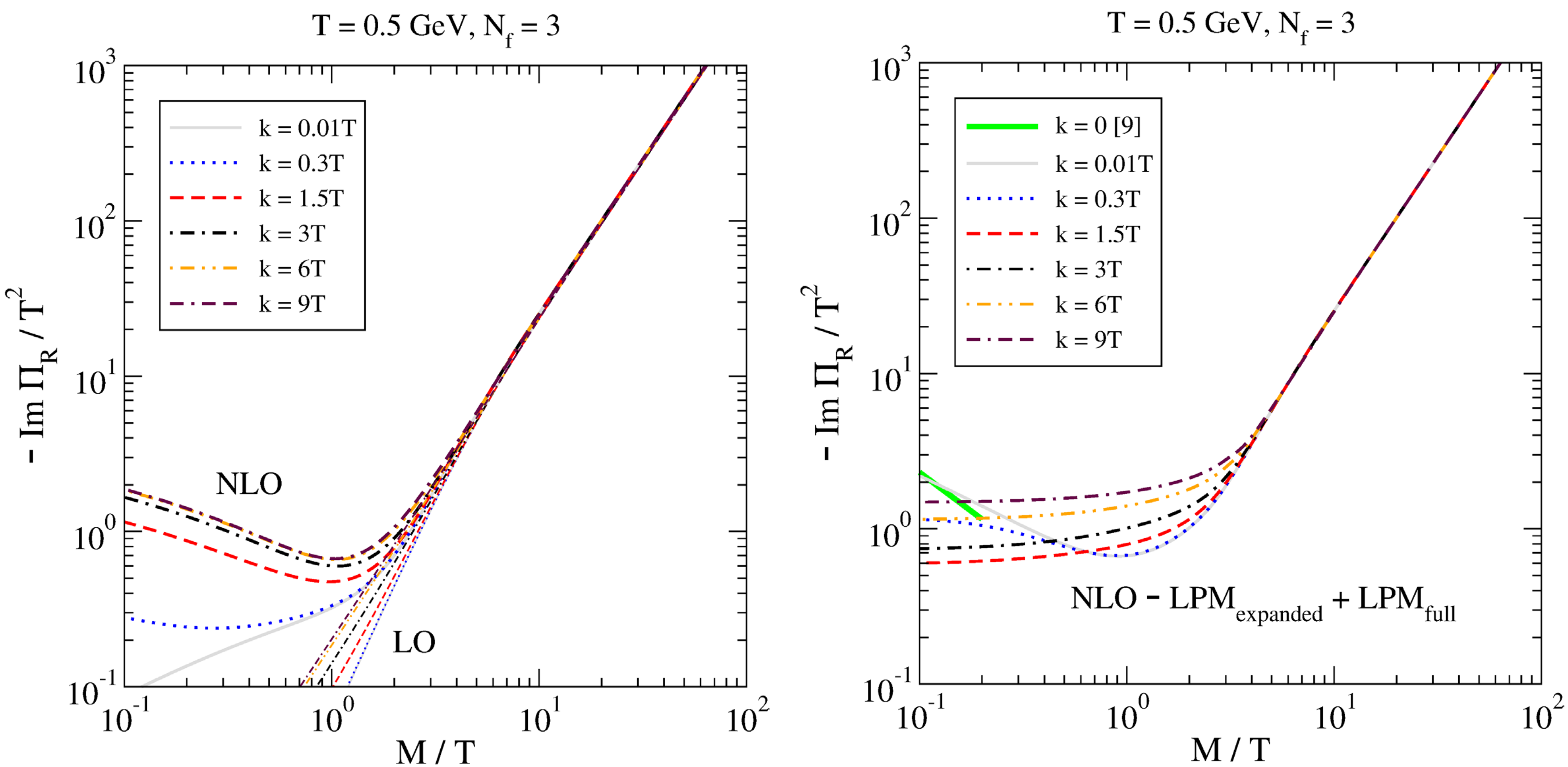
and here numerical result can make predictions beyond the hydro regime

[S.Caron-Huot, JHEP12 (2006) 015]

Although not QCD, it may give qualitative insight into the structures at small ω and k

Thermal corrections in the intermediate frequency regime required [Altherr+Aurenche 1989]

and proper treatment of the small frequency regime [Ghiglieri+Moore 2014] [Moore+Robert 2006]



interpolation between different regimes [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

progress in perturbation theory in the past years → compare to lattice QCD results

Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \rho_V(\omega = |\vec{k}|, T)$$

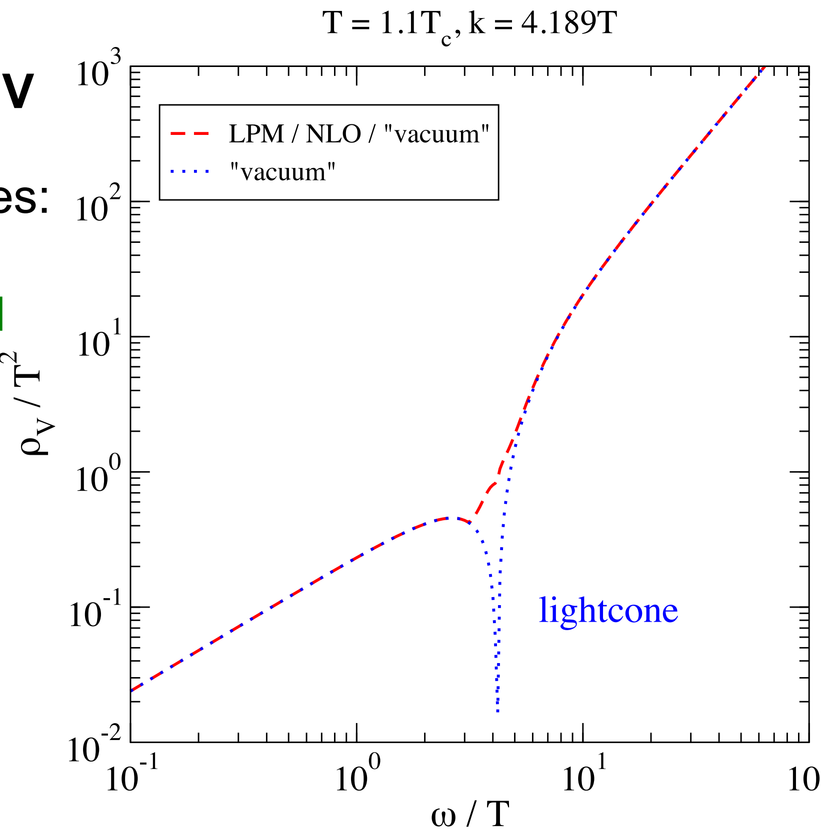
pQCD spectral function used to constrain the UV

interpolation between different (perturbative) regimes:

3T < ω < 10T: [J.Ghiglieri, G.D.Moore, JHEP 1412 (2014) 029]

ω > 10T: [I. Ghisoiu, M.Laine, JHEP 10 (2014) 84]

ω >> 10T: [M.Laine, JHEP 1311 (2013) 120]



to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators

Modeling the spectral function

$(5+2\,n_{max})^{th}$ order polynomial Ansatz at small ω:

$$\rho_{\rm fit} \equiv \frac{\beta\,\omega^3}{2\omega_0^3}\left(5-\frac{3\omega^2}{\omega_0^2}\right)-\frac{\gamma\,\omega^3}{2\omega_0^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)+\sum_{n=0}^{n_{\rm max}}\frac{\delta_n\omega^{1+2n}}{\omega_0^{1+2n}}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2$$

with the constraints to match smoothly with pQCD at ω_o

$$\rho_{\rm V}(\omega_0,\mathbf{k}) \equiv \beta\,, \quad \rho'_{\rm V}(\omega_0,\mathbf{k}) \equiv \gamma\,,$$

and n_{max}+1 free parameters

starting with a linear behavior at $\omega \ll$ T

smoothly matched to the perturbative spectral function at $\omega_0 \simeq \sqrt{k^2+(\pi T)^2}$

In the following we will use n_{max} = 0 and n_{max} = 1 for the fits to the lattice data

and to estimate the systematic uncertainties

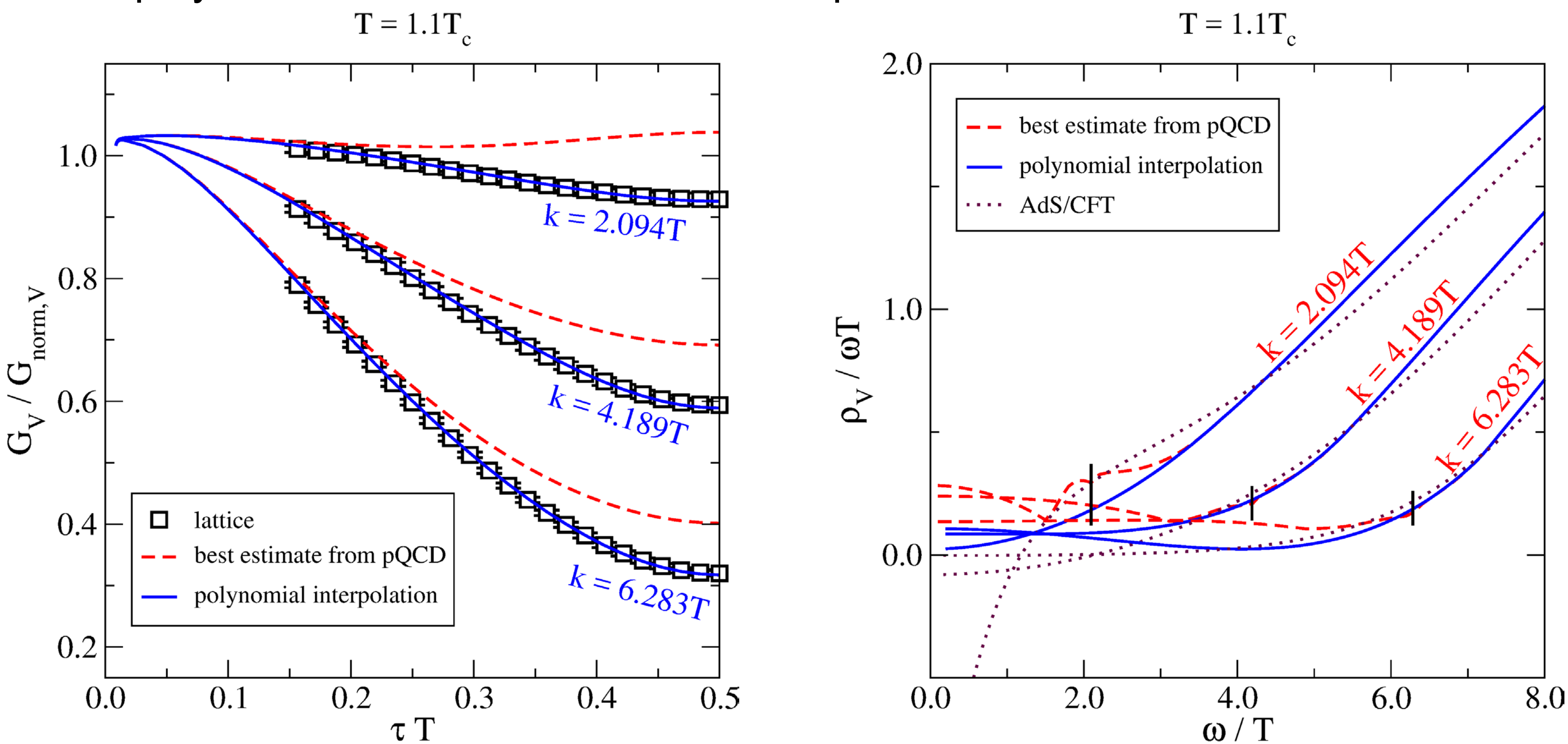
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

Fixed aspect ratio used to perform continuum extrapolation at finite p

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$
7.192	$96^3 \times 32$	314	0.2796	1.12	0.816	1.09
7.544	$144^3 \times 48$	358	0.2843	1.14	0.817	1.10
7.793	$192^3 \times 64$	242	0.2862	1.15	0.813	1.09
7.192	$96^3 \times 28$	232	0.3195	1.28	0.933	1.25
7.544	$144^3 \times 42$	417	0.3249	1.31	0.934	1.25
7.793	$192^3 \times 56$	273	0.3271	1.31	0.929	1.25

use perturbation theory at large ω and fit a polynomial at small ω to extract the spectral function



[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

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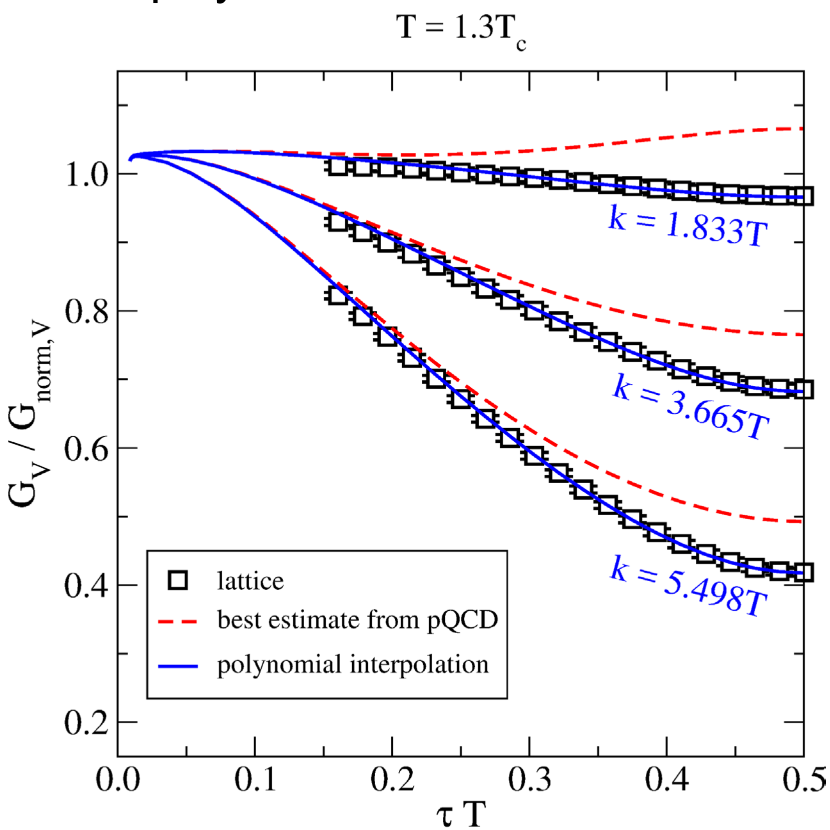
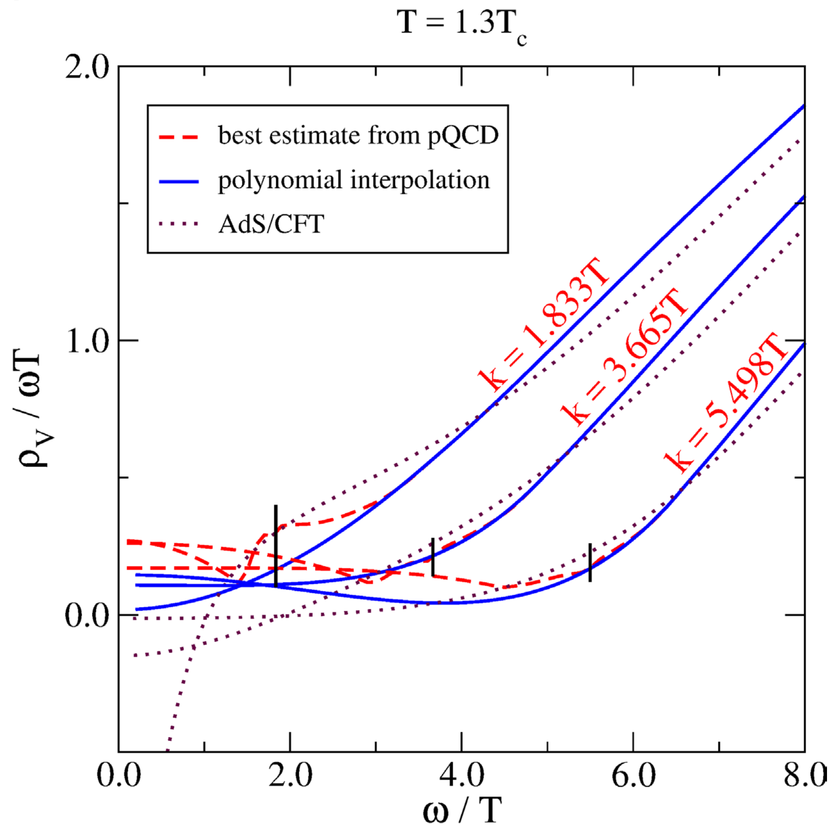




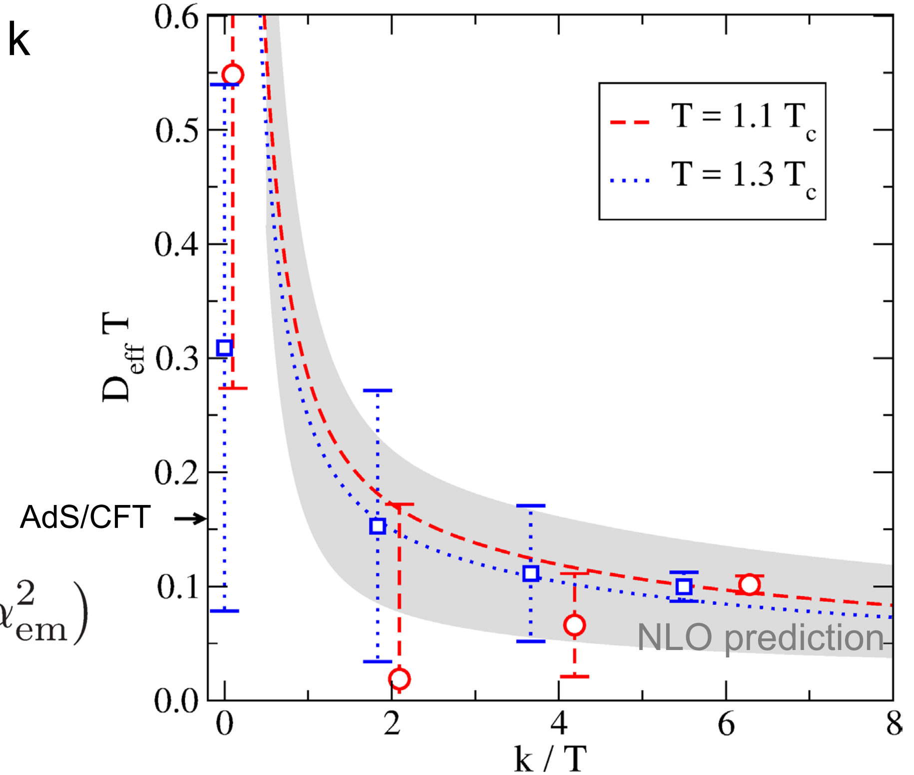
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

The spectral function at the photon point ω = k

$$D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q}k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases} .$$

can be used to calculate the photon rate

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} = \frac{2\alpha_{\rm em}\chi_{\rm q}}{3\pi^2} n_{\rm B}(k) D_{\rm eff}(k) + \mathcal{O}\left(\alpha_{\rm em}^2\right)$$



becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)

[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

Electrical conductivity obtained in the limit k→0 between the results from

AdS/CFT: $DT = \dfrac{1}{2\pi}$

[G.Policastro, D.T.Son,A.O.Starinets, JHEP09(2002)043]

LO perturbation theory [Arnold, Moore Yaffe, JHEP 05 (2003)]

using lattice value for χ_q/T^2 : $DT = 2.9 - 3.1$

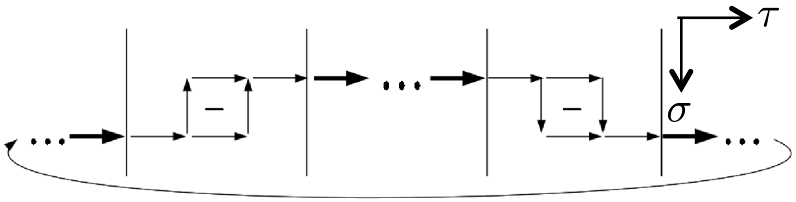
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

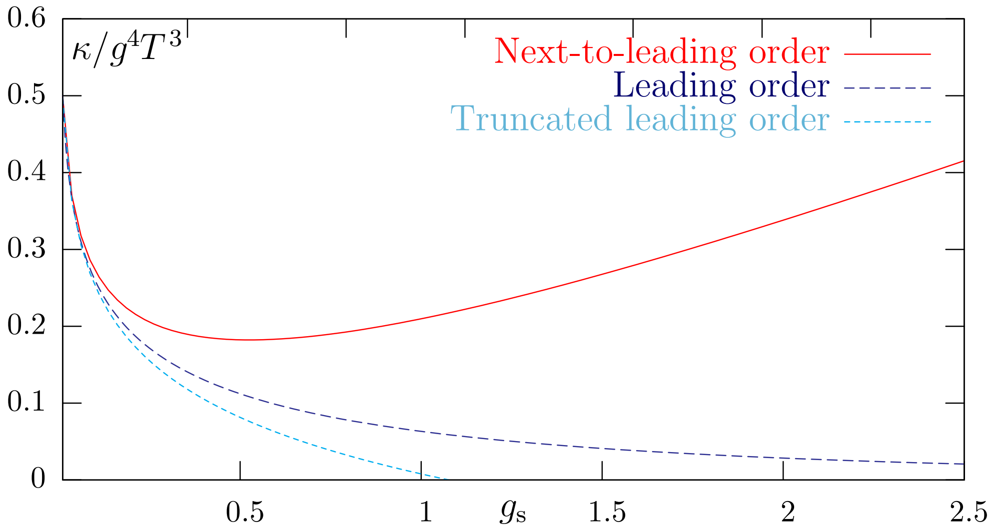
$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

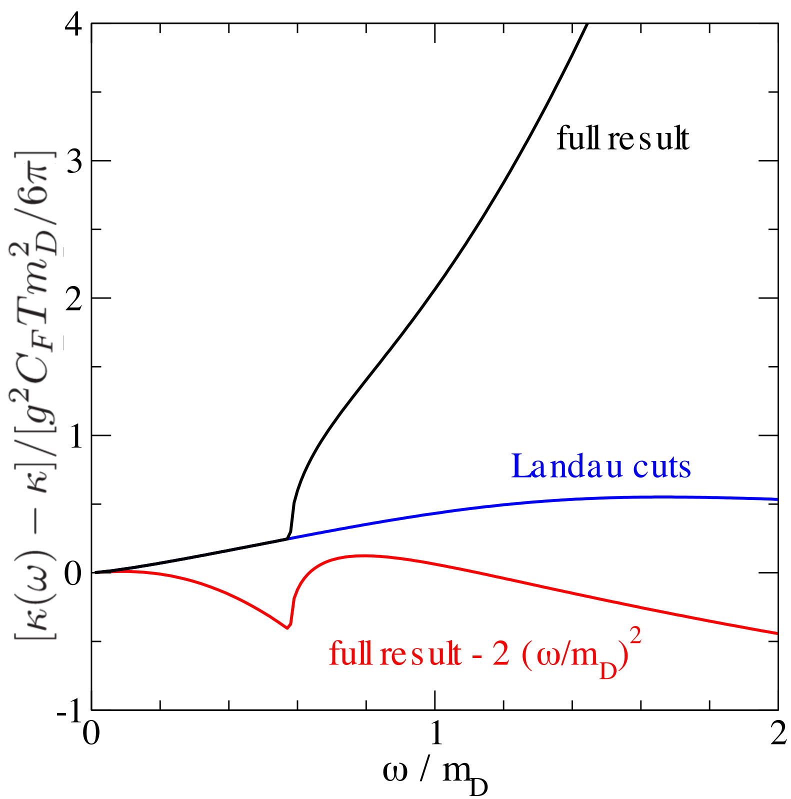
[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



→ large correction towards strong interactions

→ non-perturbative lattice methods required

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



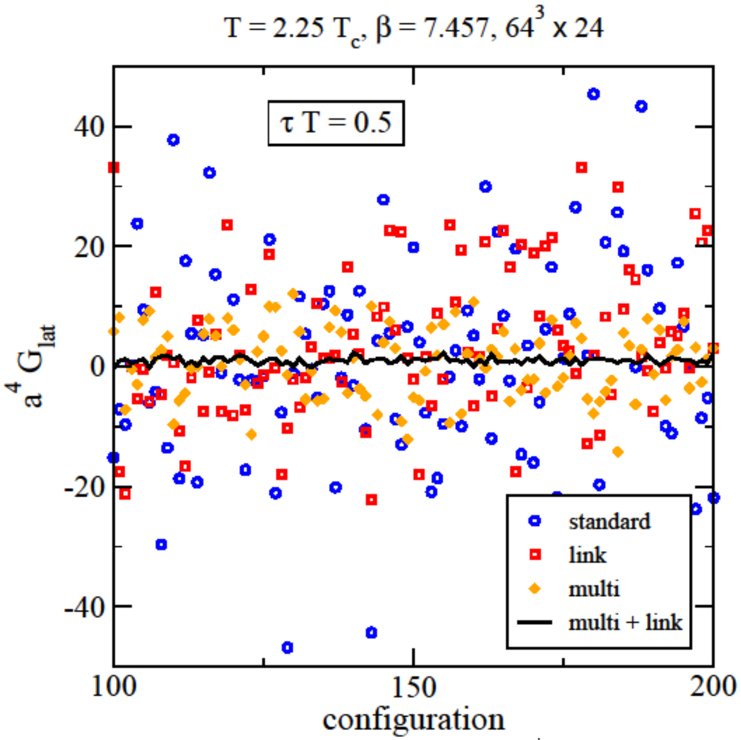
in contrast to a narrow transport peak, from this a smooth limit

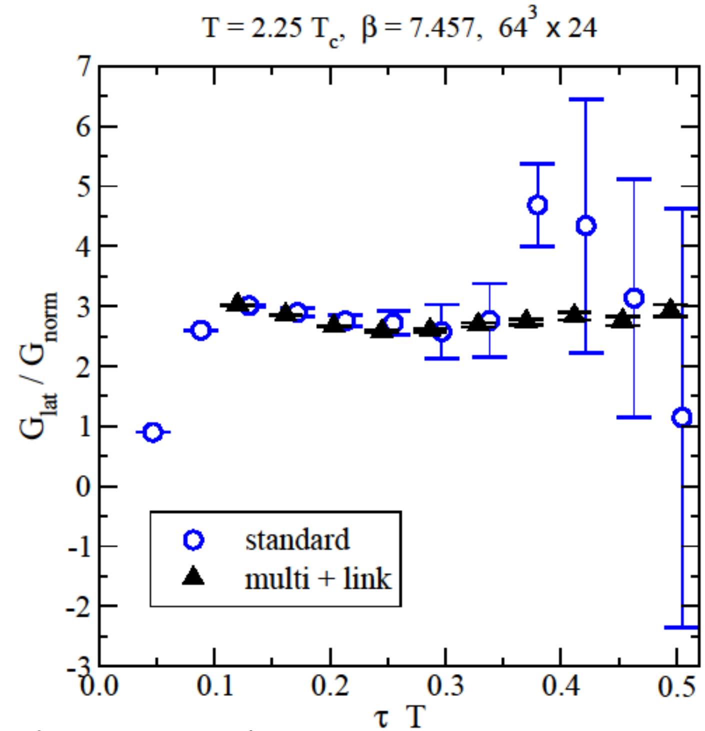
$$\boxed{\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}}$$

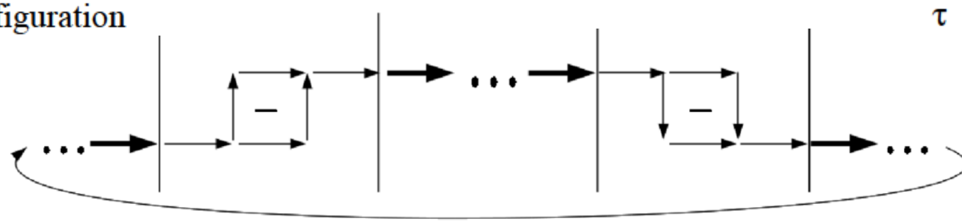
is expected

qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]







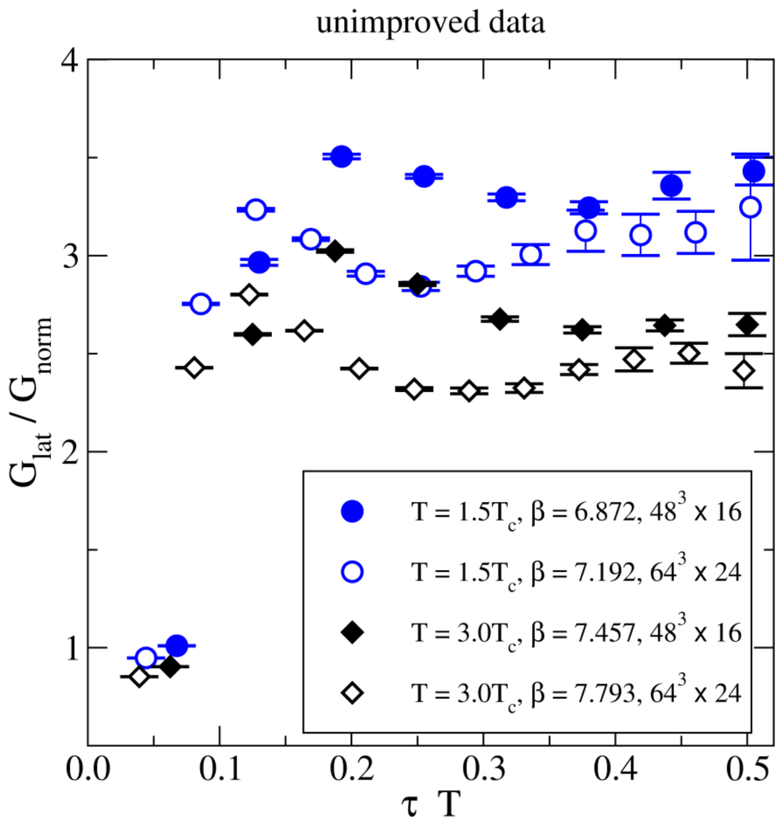
due to the gluonic nature of the operator, signal is extremely noisy

→ multilevel combined with link-integration techniques to improve the signal

[Lüscher,Weisz JHEP 0109 (2001)010
and H.B.Meyer PRD (2007) 101701]

[Parisi,Petronzio,Rapuano PLB 128 (1983) 418,
and de Forcrand PLB 151 (1985) 77]

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]

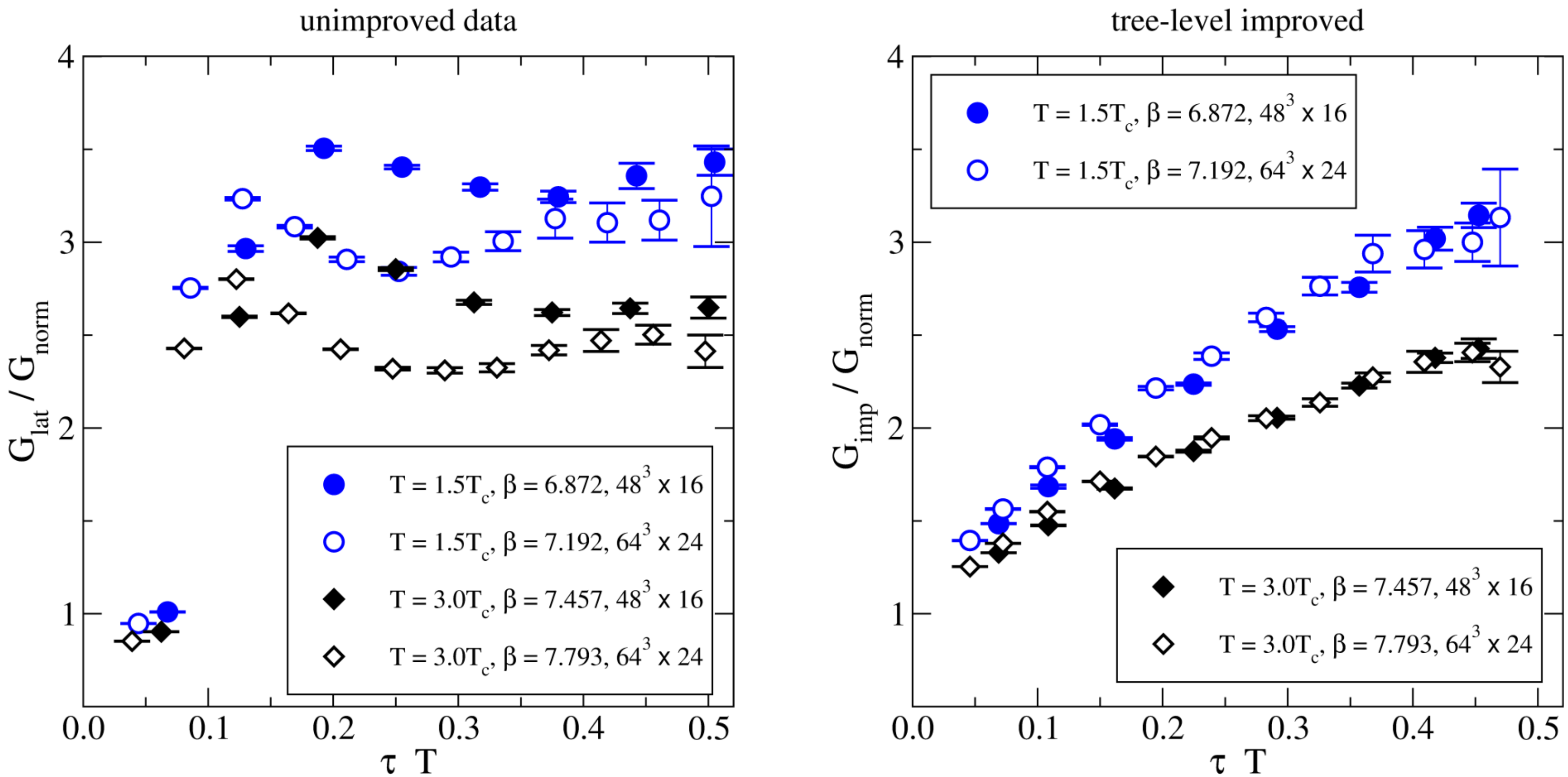


normalized by the LO-perturbative correlation function:

$$G_{\rm norm}(\tau T) \equiv \frac{G^{\rm LO}_{\rm cont}(\tau T)}{g^2 C_F} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right] \qquad C_F \equiv \frac{N_c^2-1}{2N_c}$$

and renormalized using NLO renormalization constants $Z(g^2)$

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G^{\mathrm{LO}}_{\mathrm{cont}}(\overline{\tau T}) = G^{\mathrm{LO}}_{\mathrm{lat}}(\tau T)$$

leads to an effective reduction of cut-off effect for all τT

[A.Francis, OK, M.Laine, T.Neuhaus, H.Ohno, PRD92(2015)116003]

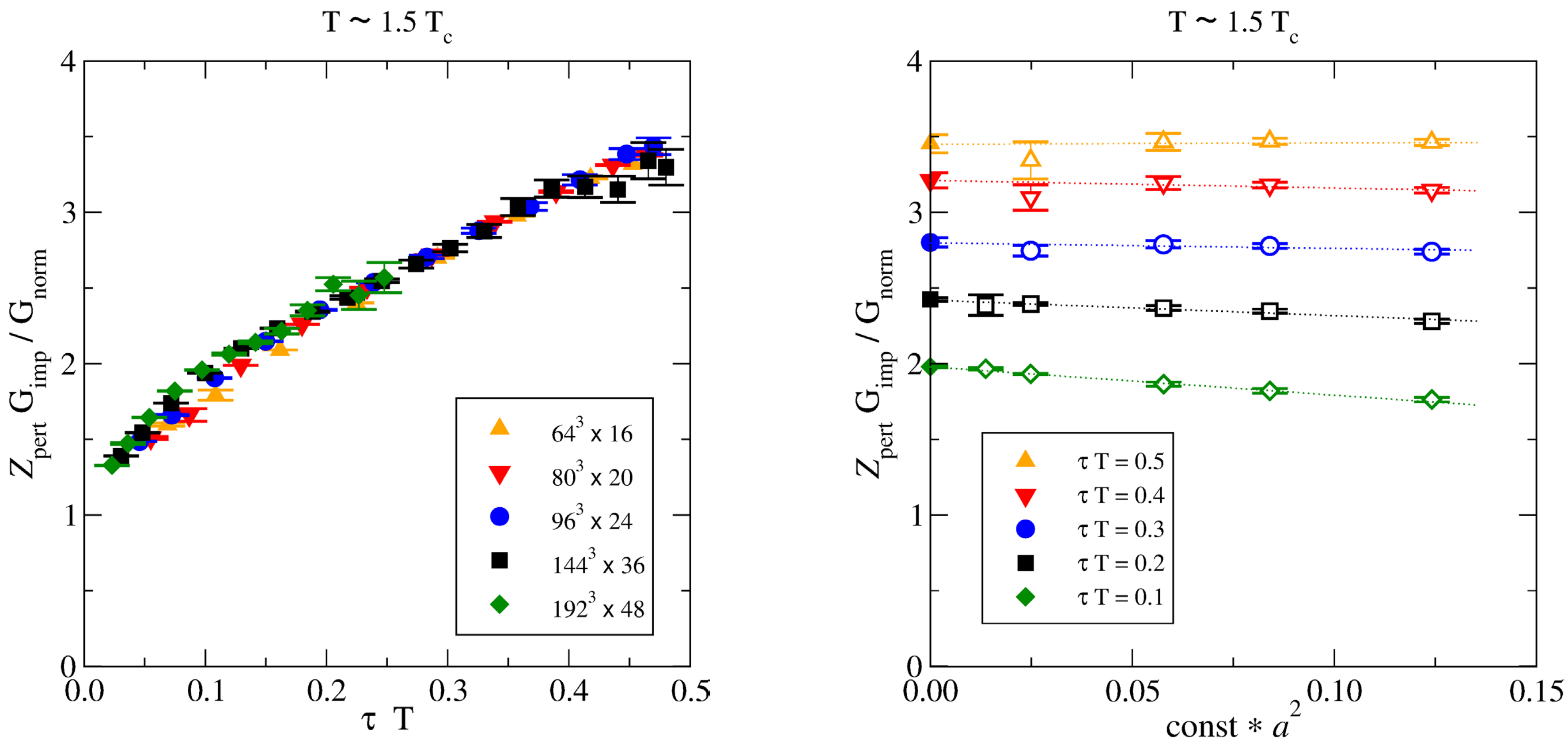
Quenched Lattice QCD on large and fine isotropic lattices at T$\simeq$ 1.5 T_c

- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration N_s/N_t = 4, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, a$\rightarrow$ 0 $\leftrightarrow$ $N_t \rightarrow \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$
- scale setting using r_0 and t_0 scale [A.Francis,OK,M.Laine, T.Neuhaus, H.Ohno, PRD91(2015)096002]

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}^{(\text{imp})}$	$T/T_c\|_{t_0}^{(\text{imp})}$	$T\sqrt{t_0}^{(\text{clov})}$	$T/T_c\|_{t_0}^{(\text{clov})}$	Tr_0	$T/T_c\|_{r_0}$
6.872	$64^3 \times 16$	172	0.3770	1.52	0.3805	1.53	1.116	1.50
7.035	$80^3 \times 20$	180	0.3693	1.48	0.3739	1.50	1.086	1.46
7.192	$96^3 \times 24$	160	0.3728	1.50	0.3790	1.52	1.089	1.46
7.544	$144^3 \times 36$	693	0.3791	1.52	0.3896	1.57	1.089	1.46
7.793	$192^3 \times 48$	223	0.3816	1.53	0.3955	1.59	1.084	1.45

similar studies by [Banerjee,Datta,Gavai,Majumdar, PRD85(2012)014510]
and [H.B.Meyer, New J.Phys.13(2011)035008]

we performed a continuum extrapolation, a$\rightarrow$ 0 $\leftrightarrow$ $N_t \rightarrow \infty$, at fixed $T=1/a\, N_t$



well behaved continuum extrapolation for 0.05 $\leq \tau T \leq$ 0.5

finest lattice already close to the continuum

coarser lattices at larger τT show almost no cut-off effects

how to extract the spectral function from the correlator?

$\omega \ll$T: linear behavior motivated at small frequencies

$$\rho_{\rm IR}(\omega) = \frac{\kappa\omega}{2T}$$

$\omega \gg$T: vacuum perturbative results and leading order thermal correction:

$$\rho_{\rm UV}(\omega) = [\rho_{\rm UV}(\omega)]_{T=0} + \mathcal{O}\left(\frac{g^4T^4}{\omega}\right)$$

using a renormalization scale $\bar{\mu}_\omega = \omega$ for $\omega \gg \Lambda_{\overline{MS}}$ leading order becomes

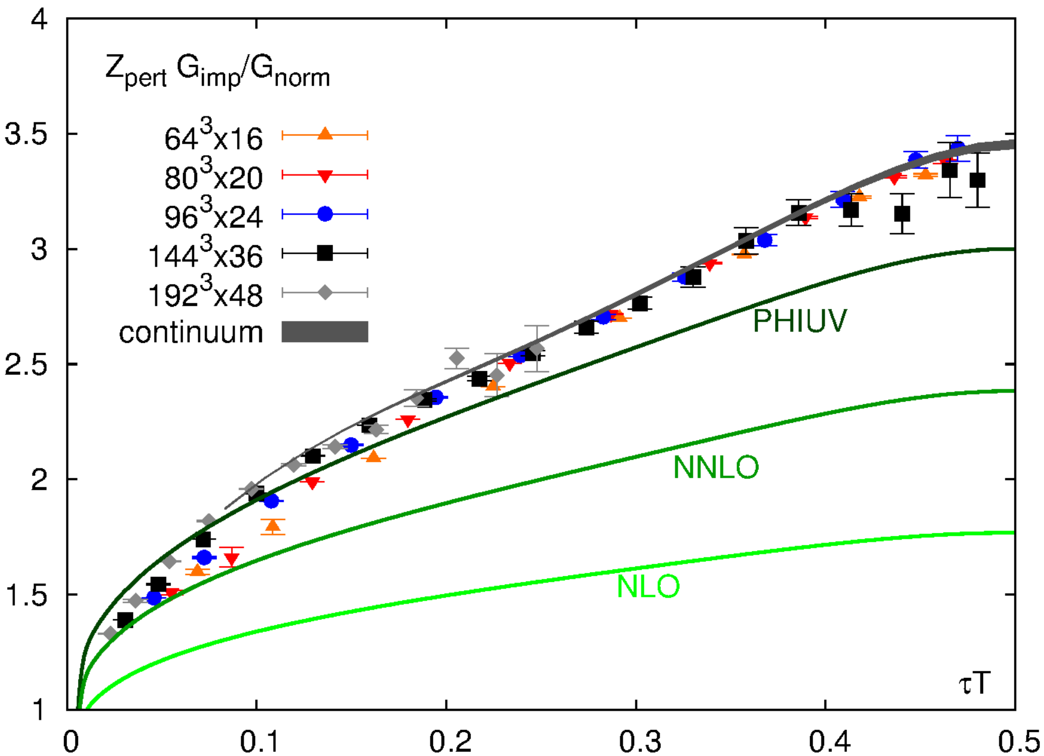
$$\rho_{\rm UV}(\omega) = \Phi_{UV}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{MS})}\right)\right]$$

$$\Phi_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega)C_F\omega^3}{6\pi} \quad , \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

model the spectral function using these asymptotics with two free parameters

$$\rho_{\rm model}(\omega) \equiv \max\left\{A\Phi_{\rm UV}(\omega), \frac{\omega\kappa}{2T}\right\}$$



$$\rho_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

already closer to the data

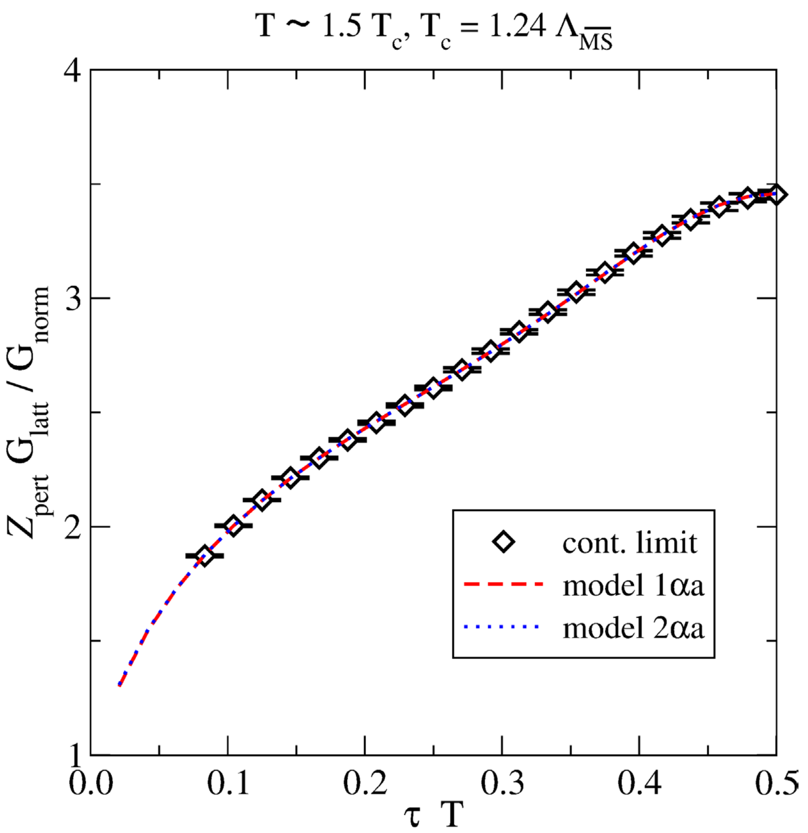
Model spectral function: transport contribution + UV-asymptotics

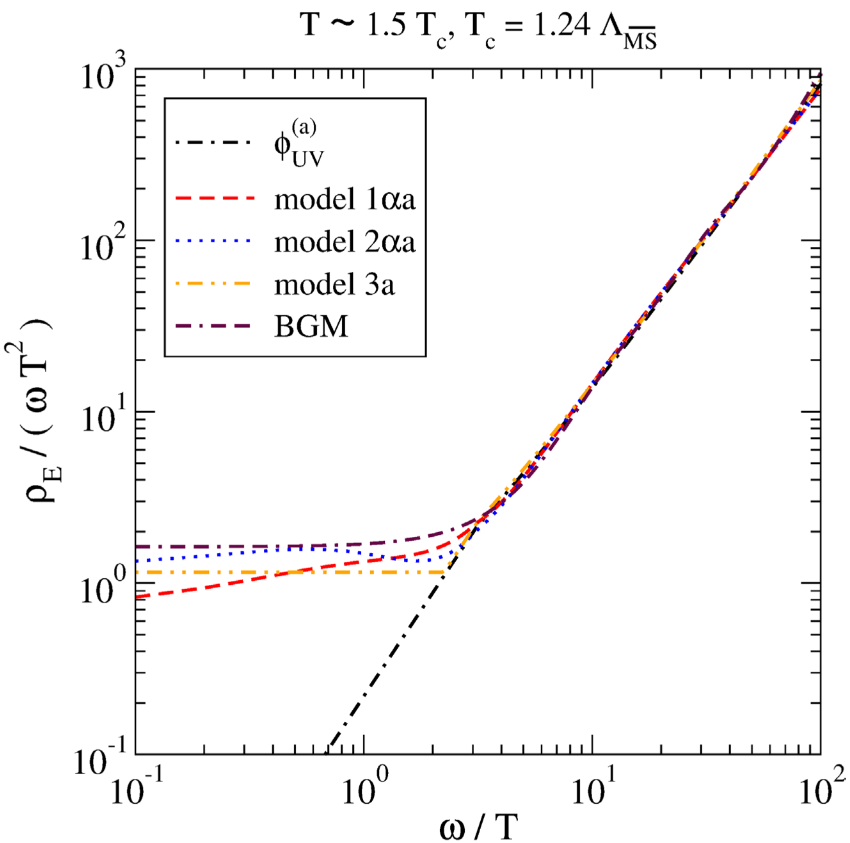
$$\boxed{\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm UV}(\omega), \frac{\omega\kappa}{2T} \right\}}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

analysis of the systematic uncertainties by

modeling corrections to ρ_{IR} by a power series in ω





$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

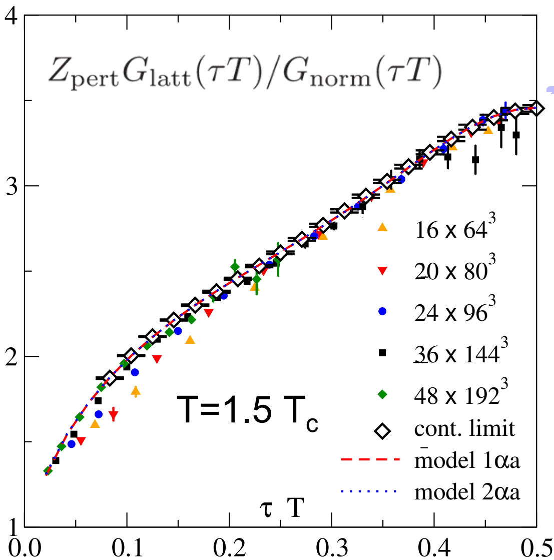
$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

Heavy-quark momentum diffusion κ

$$G_E(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_E(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}} \qquad \kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

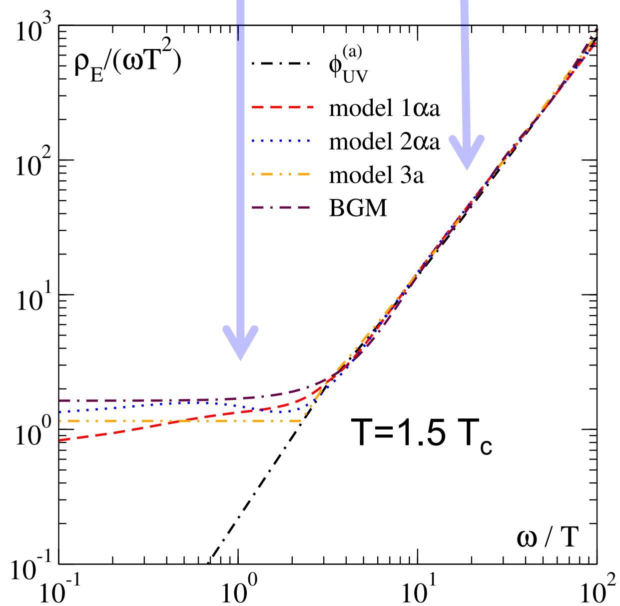
Combine continuum-extrapolated lattice results and perturbative input at large ω and model corrections in the non-perturbative low-ω region by power series in ω

continuum extrapolated lattice correlator

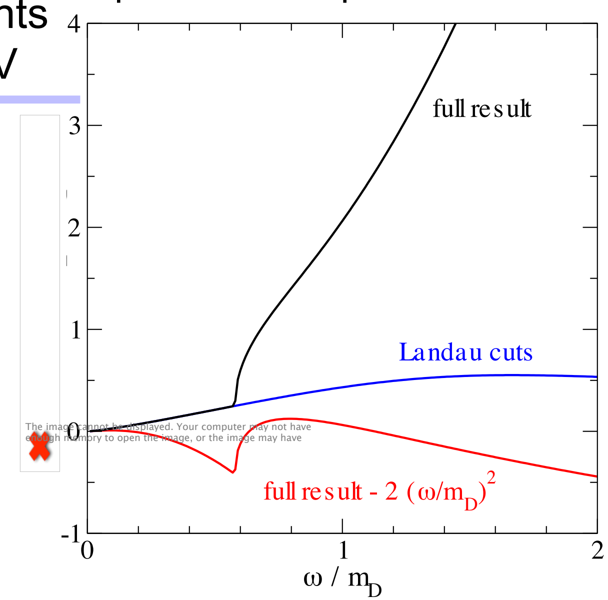


non-perturbative input from lattice QCD

perturbative constraints in the UV



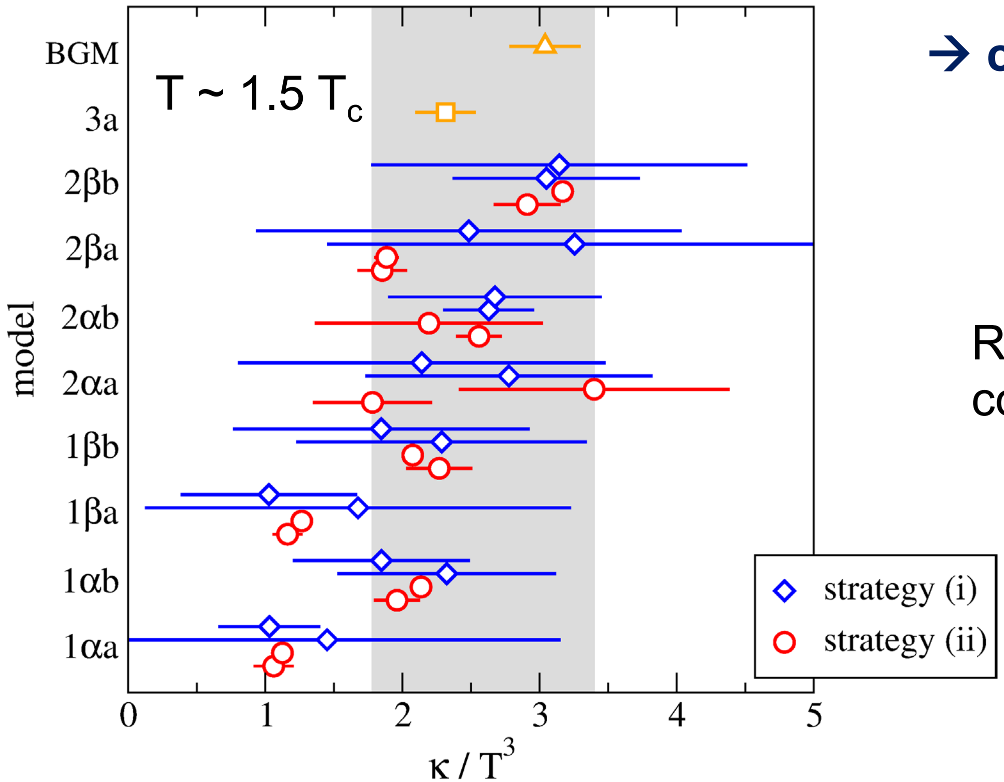
perturbative spectral function



[A.Francis, OK et al., PRD92(2015)116003]

[Caron-Huot, Laine, Moore, JHEP04 (2009) 053, Burnier, Laine et al, JHEP08 (2010) 094]

[A.Francis, OK et al., PRD92(2015)116003]



Detailed analysis of systematic uncertainties

→ continuum estimate of κ :

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega} = 1.8 \ldots 3.4$$

Related to diffusion coefficient D and drag coefficient η_D (in the non-relativistic limit)

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7 \ldots 7.0$$

$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

time scale associated with the kinetic equilibration of heavy quarks:

$$\tau_{\rm kin} = \frac{1}{\eta_D} = (1.8 \ldots 3.4)\left(\frac{T_{\rm c}}{T}\right)^2\left(\frac{M}{1.5\ \rm GeV}\right) \rm fm/c$$

→ close to T$_c$, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.

Conclusions – part II

using **continuum extrapolated correlation functions** from Lattice QCD and

using phenomenologically inspired and **perturbatively constrained Ansätze**

allows to extracted transport properties and spectral properties

we obtained continuum estimates for

→ Electrical conductivity / Diffusion coefficients

→ Thermal dilepton rates

→ Thermal photon rates

next goals: continuum extrapolation for charm and bottom correlators

→ quark mass dependence of diffusion coefficient + sequential melting of quarkonia

The methodology developed in this studies within the quenched approximation

shall be extended to full QCD calculations for a realistic QGP medium

as close to T_c dynamical fermion degrees of freedom will become important