Cosmological models with non-minimal coupling and bounce solutions

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based on E. O. Pozdeeva, M. A. Skugoreva, A. V. Toporensky S. Yu. Vernov, arXiv:1608.NNNN [gr-qc]
A. Yu. Kamenshchik, E. O. Pozdeeva, A. Tronconi, G. Venturi, S. Yu. Vernov,Class. Quantum Grav. 33 (2016) 015004, arXiv:1509.00590

Helmholtz - DIAS International Summer School "Cosmology, Strings, New Physics"
28 August- 10 September, 2016 in Dubna, Russia.

Introduction

- The non-monimal coupled model

$$S=\int d^4x\sqrt{-g}\left(U(\varphi)R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V(\varphi)\right),$$

  where $g=\det(g_{ik})$ is the determinant consisting of metric tensor components g_{ik}, R is the Ricci scalar, $U(\varphi)$ and $V(\varphi)$ are differentiable functions of a scalar field φ.

- The flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric

$$ds^2=\,-\,dt^2+a^2(t)d\vec{x}^2$$

- The bounce conditions: $H=0, \quad \dot{H}>0$, where H is a Hubble parameter $H=\dfrac{\dot{a}}{a}$

Introduction

- The bouncing solution to non-monimally coupled model with $U = \frac{\kappa^{-2}}{2} - \frac{\phi^2}{12}$, $V = \frac{\Lambda}{2\kappa^2} - \frac{c\phi^4}{2}$, $\kappa^{-2}, \Lambda, c > 0$
- in flat FLRW universe
- with constant Ricci scalar $R = 6(\dot{H} + 2H^2) = 4\Lambda$
- was considered in *B. Boisseau, H. Giacomini, D. Polarski, and A.A. Starobinsky, Bouncing Universes in Scalar-Tensor Gravity Models admitting Negative Potentials, J. Cosmol. Astropart. Phys.* **1507** *(2015) 002 (arXiv:1504.07927)*
- The bouncing solution has form:

$$a = a_0 \cosh^{\frac{1}{2}}\left[2\sqrt{\frac{\Lambda}{3}}t\right], \quad H = \sqrt{\frac{\Lambda}{3}}\tanh\left[2\sqrt{\frac{\Lambda}{3}}t\right]$$

where a_0 is a constant.

Introduction

In our works we try to generalized the results of the publication, namely, we consider:

- the non-flat FLRW universe
- another forms of the potential
- another form of effective gravitational constant

Constant Ricci scalar R

- The one of interesting point is the constant Ricci scalar R and how the special form of function U and the corresponding potential can be found from the requirement that the Ricci scalar is an integral of motion to the non-minimally coupled model.
- The non-minimal coupled model in spatially FLRW universe leads to the following equations:

$$6UH^2 + 6\dot{U}H - \frac{1}{2}\dot{\varphi}^2 - V = 0, \tag{1}$$

$$2U\left[2\dot{H} + 3H^2\right] + 2U'\left[\ddot{\varphi} + 2H\dot{\varphi}\right] = V - \left[2U'' + \frac{1}{2}\right]\dot{\varphi}^2, \tag{2}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - 6U'\left[\dot{H} + 2H^2\right] + V' = 0, \tag{3}$$

where a "dot" means a derivative with respect to the cosmic time t and a "prime" means a derivative with respect to the scalar field φ.

Constant Ricci scalar R

- In the spatially flat FLRW metric $R = 6(\dot{H} + 2H^2)$.
- From (1)- (3) we get (details are in [1])

$$2R\left(U + 3U'^2\right) + (6U'' + 1)\,\dot{\varphi}^2 = 4V + 6V'U'\,. \qquad (4)$$

- From the structure of Eq. (4) it is easy to see that the simplest way to get a constant R is to choose such $U(\varphi)$ that

$$U + 3U'^2 = U_0, \qquad 6U'' + 1 = 0, \qquad U_0 R = 2V + 3V'U'. \qquad (5)$$

- The solution to the first two equations (5) is $\quad U_c(\varphi) = U_0 - \dfrac{\varphi^2}{12}$
- For such a choice of $U(\varphi)$ Eq. (4) can be simplifyed:

$$2U_0 R = 4V(\varphi) - \varphi V'(\varphi). \qquad (6)$$

and has the following solution:

$$V_{int} = \frac{\Lambda}{K} + C_4\varphi^4, \qquad \Lambda = \frac{R}{4}, \qquad K = \frac{1}{2U_0}. \qquad (7)$$

where C_4 is an integration constant.

[1] E.O. Pozdeeva,M.A. Skugoreva,A.V. Toporensky,S.Yu. Vernov,arXiv:1608.NNNN

Constant Ricci scalar R

- Thus, requiring that the Ricci scalar is a constant one can define both functions $U(\varphi) = U_c$ and $V(\varphi) = V_{int}$. To get a positive G_{eff} for some values of φ we choose $U_0 > 0$.
- Using the explicite forms of U_c and V_{int} we get that the condition $\dot{H}_b > 0$ is equivalent to $\Lambda > 0$, hence, from $V(\varphi_b) < 0$ it follows $C_4 < 0$. This integrable cosmological model has been considered in[2], where the behavior of bounce solutions has been studied in detail.
- Considering the equation $R = 4\Lambda$ such as a differential equation for the Hubble parameter with a positive Λ, the two possible real solutions in dependence of the initial conditions can be obtained:

$$H_1 = \sqrt{\frac{\Lambda}{3}} \tanh\left(\frac{2\sqrt{\Lambda}(t - t_0)}{\sqrt{3}}\right), \quad H_2 = \sqrt{\frac{\Lambda}{3}} \coth\left(\frac{2\sqrt{\Lambda}(t - t_0)}{\sqrt{3}}\right),$$

where t_0 is an integration constant. For convenience we locate the bounce at $t = 0$, so hereafter $t_0 = 0$.

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[2] B. Boisseau, H. Giacomini, D. Polarski, and A.A. Starobinsky, J. Cosmol. Astropart. Phys. **1507** (2015) 002 (arXiv:1504.07927)

Generalization using metric

- In [3] we consider non-minimal coupled model in the Friedmann–Lemaître–Robertson–Walker (FLRW) metric:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} - r^2 d\theta^2 - r^2 \sin^2(\theta) d\varphi^2 \right), \qquad (8)$$

  where $a(t)$ is the scale factor, K is a constant. As usual $K = 0$ describes a flat universe, $K = 1$ a closed universe, and $K = -1$ an open one.
- For all these cases the Friedmann equation can be integrated explicitly, because the Ricci scalar as an integral of motion. It is easy to show that $R = R_0 = 2\Lambda / U_0$ at all values of K.

[3] A. Yu. Kamenshchik, E. O. Pozdeeva, A. Tronconi, G. Venturi, S. Yu. Vernov, Class. Quantum Grav. 33 (2016) 015004, arXiv:1509.00590

Generalization using metric

- We present the a list of the cosmological evolutions for different choices of the curvature K and the radiation constant A.
- For example in the case $K = 1$, $A < 0$ we have a bounce at $a_B = \left(\frac{3U_0}{\Lambda} + \sqrt{\frac{9U_0^2}{\Lambda^2} - \frac{A}{\Lambda}}\right)^{1/2}$.
- For this case we shall write down the explicit solution. The expression for the scale factor is

$$a(t) = \left(\frac{3U_0}{\Lambda} + \sqrt{\frac{9U_0^2}{\Lambda^2} - \frac{A}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{24U_0}}\, t\right)^{1/2}. \qquad (9)$$

The Hubble parameter is

$$H(t) = \frac{\Lambda\sqrt{9U_0^2 - A\Lambda}\sinh\left(\sqrt{\frac{\Lambda}{24U_0}}\, t\right)}{96U_0\left[3U_0 + \sqrt{9U_0^2 - A\Lambda}\cosh\left(\sqrt{\frac{\Lambda}{24U_0}}\, t\right)\right]}. \qquad (10)$$

Generalization of bouncing potential

- The bouncing potential V_{int} is a particle case of potential at $\beta = 1/3$

$$V_c(\phi) = \frac{c_1}{144U_0^2 2^{\frac{2(1-\beta)}{\beta}}} \frac{\left[(\sqrt{12U_0}+\phi)^{3\beta} + (\sqrt{12U_0}-\phi)^{3\beta}\right]^{\frac{2(1-\beta)}{\beta}}}{(12U_0-\phi^2)^{1-3\beta}} + \\ + \frac{c_2}{144U_0^2 2^{\frac{2(1-\beta)}{\beta}}} \frac{\left[(\sqrt{12U_0}+\phi)^{3\beta} - (\sqrt{12U_0}-\phi)^{3\beta}\right]^{\frac{2(1-\beta)}{\beta}}}{(12U_0-\phi^2)^{1-3\beta}} \tag{11}$$

- The model with $U_c(\phi)$ and $V_c(\phi)$ is integrable at any value of β.
- And the bounce is possible for another β, for example, in the case $\beta = 2/3$ at $c_2 = -\frac{289\sqrt{2}}{48}c_1$, $\frac{1}{12}\sqrt{6U_0} < \sigma < \sqrt{12U_0}$.

- Let us introduce the new function[4]

$$V_{eff}(\varphi) = \frac{V(\varphi)}{4K^2 U(\varphi)^2}. \tag{12}$$

- If $V'_{eff} = 0$ then we have de Sitter solution.
- If $V''_{eff} > 0$ then the de Sitter solution is stable
- If $V''_{eff} < 0$ then the de Sitter solution is unstable
- The de Sitter point is a stable node (the scalar field decreases monotonically)

$$\frac{3(U + 3U'^2)}{8U^2} \geqslant \frac{V''_{eff}}{V_{eff}}, \tag{13}$$

and a stable focus (the scalar field oscillations exist) at

$$\frac{3(U + 3U'^2)}{8U^2} < \frac{V''_{eff}}{V_{eff}}. \tag{14}$$

[4] E.O. Pozdeeva, M.A. Skugoreva, A.V. Toporensky, S.Yu. Vernov, arXiv:1608.NNNN

Generalization of bouncing potential

- Let us consider model with

$$U_c(\varphi) = U_0 - \frac{\varphi^2}{12}, \quad U_0 = 1/(2K) \tag{15}$$

and

$$V_c = C_4\varphi^4 + C_2\varphi^2 + C_0, \tag{16}$$

- Such as we consider only gravity regime ($G_{eff} > 0$) $U_c > 0$, we will use restriction $\varphi_b^2 < 6/K$.
- $$V_{eff} = \frac{36(C_4\varphi^4 + C_2\varphi^2 + C_0)}{(K\varphi^2 - 6)^2}. \tag{17}$$

  The even potential V_{eff} has an extremum at $\varphi = 0$ and at points

$$\varphi_m = \pm\sqrt{\frac{-2(3C_2 + KC_0)}{12C_4 + KC_2}}. \tag{18}$$

Generalization of bouncing potential

- For the model with $V_c = C_4\varphi^4 + C_2\varphi^2 + C_0$ we have the following bounce conditions to the constants

$$C_4\varphi_b^4 + C_2\varphi_b^2 + C_0 < 0, \quad C_2\varphi_b^2 + 2C_0 > 0, \quad C_2 + 2C_4\varphi_b^2 < 0.$$

where at least one of the constants C_2 or C_0 should be positive.

- We specify the case $C_4 < 0$. Supposing that ϕ_m are real we get

$$0 > C_2 + 2\varphi_b^2 C_4 > C_2 + \frac{12}{K} C_4.$$

- So, the model with a bounce solution has real φ_m only at

$$3C_2 + KC_0 > 0 \quad \text{and} \quad KC_2 + 12C_4 < 0. \qquad (19)$$

Generalization of bouncing potential

- Using conditions to the model constants, we get

$$V''_{eff}(0) = \frac{\frac{C_0 K}{3} + C_2}{2} > 0,$$

$$V''_{eff}(\varphi_m) = -\frac{36(C_2 K + 12 C_4)^3 (C_0 K + 3 C_2)}{(C_0 K^2 + 6 C_2 K + 36 C_4)^3} < 0.$$

- Thus, the effective potential has a minimum at $\varphi = 0$ and maxima at $\varphi = \varphi_m$.

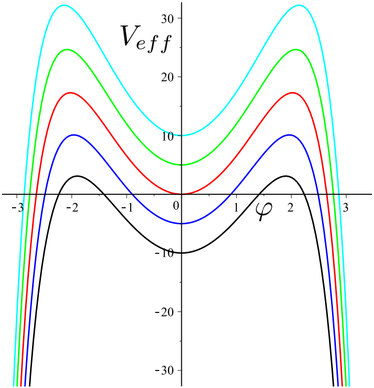

Figure: The effective potential V_{eff} at different values of parameters. In the picture we choose $K = 1/4$. The values of parameters are $C_4 = -1$, $C_2 = 7$ (left picture). The parameter $C_0 = -10$ (black curve), -5 (red curve), 0 (blue curve), 5 (green curve), and 10 (cyan curve).

$$0 < \varphi_m < \varphi_1^+ < \varphi_b < \sqrt{\frac{6}{K}},\ \varphi_1^+ = \sqrt{\frac{1}{2}\left(\sqrt{\left(\frac{C_2}{C_4}\right)^2 - 4\frac{C_0}{C_4}} - \frac{C_2}{C_4}\right)}.$$

For $C_0 > 0$ there exists the stable de Sitter solution $\varphi_{dS} = 0$ and $H_{dS} = \sqrt{\frac{C_0 K}{3}}$. It is a stable node at $KC_0 - 24C_2 \geqslant 0$ and a stable focus in the opposite case $C_0 K - 24C_2 < 0$.

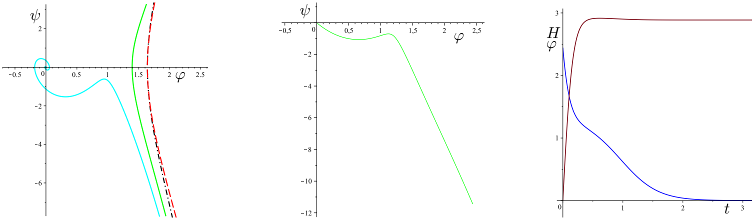

Figure: The phase trajectory, presented in the left picture, corresponds to the following values of parameters: $K = 1/4$, $C_4 = -4$, $C_2 = 7$ and $C_0 = 10$. Initial values are $\varphi_i = 4.88$ and $\psi_i = -64.68078215$ (cyan curve), $\varphi_i = 3.4$ and $\psi_i = -29.78638615$ (green curve). The cyan curve is an example of a stable focus. On the middle and right pictures the example of a stable node at $\varphi = 0$ is presented. The field φ (blue line) and the Hubble parameter (red line) as functions of the cosmic time are presented in the right picture. The values of parameters are $K = 1$, $C_4 = -2.7$, $C_2 = 1$ and $C_0 = 25$. The initial conditions of the bounce solution are $\varphi_i = 2.445$, $\psi_i = -11.44650941$, and $H_i = 0$.

For $U = U_c$ and $V = V_c$ we consider numerically the evolution of the scalar field and the corresponding behaviors of the Hubble parameter. We start to consider the evolution from the positive bounce point with a negative velocity $\dot{\varphi}_b = \sqrt{-2V(\varphi_b)}$. The detailed numerical consideration in [5]

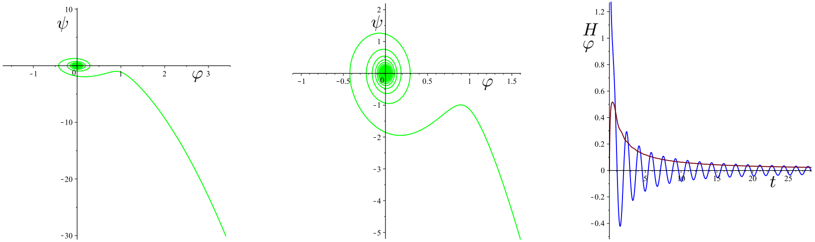

Figure: A phase trajectory for the models with U_c and V_c is presented in the left picture. The values of constants are $K = 1/4,\ C_4 = -4,\ C_2 = 7,\ C_0 = 0$. The initial conditions are $\varphi_i = 3.4$ and $\psi_i = -30.12023904$. A zoom of the central part of phase plane is presented in the middle picture. The Hubble parameter (red) and the scalar field (blue) of functions of cosmic time are presented in the right picture.

[5] E.O. Pozdeeva, M.A. Skugoreva, A.V. Toporensky, S.Yu. Vernov, arXiv:1608.NNNN

Another effective gravitational constant

We change effective gravitational constant changing the parameter ξ, namely choosing $\xi = 20$.

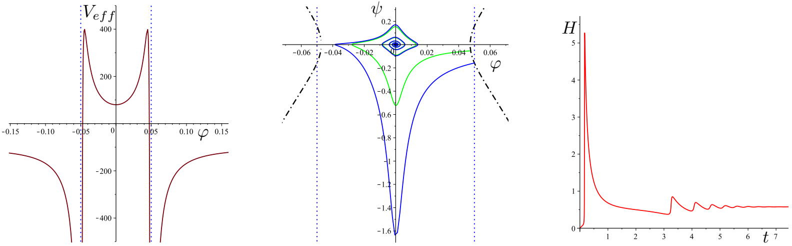

Figure: The effective potential (left picture), phase trajectories (middle picture) and the Hubble parameter and the scalar field as function of time for $V = C_4\varphi^4 + C_0$, $U = U_0 - \xi\varphi^2/2$. The parameters are $\xi = 20$, $K = 20$, $C_4 = -10000$, $C_0 = 0.05$. The initial conditions are $\varphi_i = 1/21$ and $\psi_i = -0.5327109254e - 1$ (green line), $\varphi_i = 0.04999750012$ and $\psi_i = -0.1580348161$ (blue line). The black curves are the lines of the points that correspond to $H = 0$. The blue point lines correspond to $U = 0$. The Hubble parameter $H(t)$ is presented in the right picture.

Conclusion

- Thus, the modification of the FLRW metric allows to obtain the bounce solution with constant Ricci scalar.
- Also the bounce solution can be obtained to the models with modified potential or effective gravity constant, without constant Ricci scalar.
- Moreover the Hubble parameter at first increases, after that decreases and go to constant.
- Thus, we suppose that the generalization of bounce potentials and effective gravity constant can leads to the interesting behaviors of the Hubble parameter.

Thank you for attention