Quantum correlations in bipartite systems

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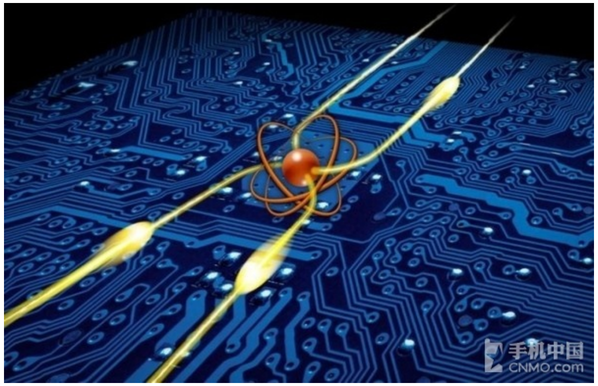
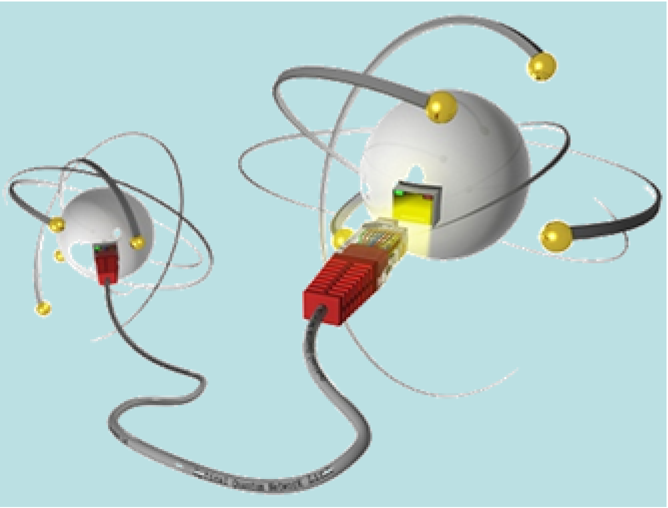
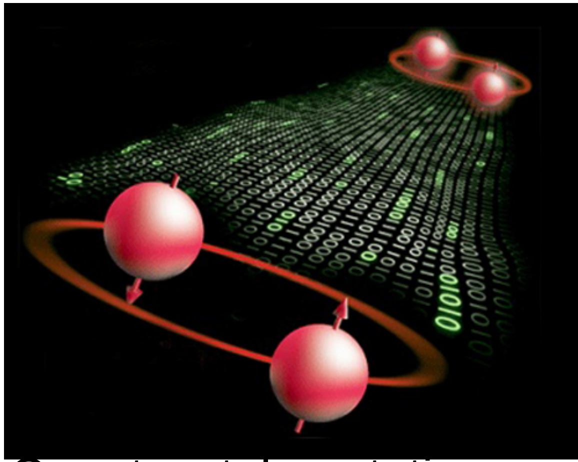
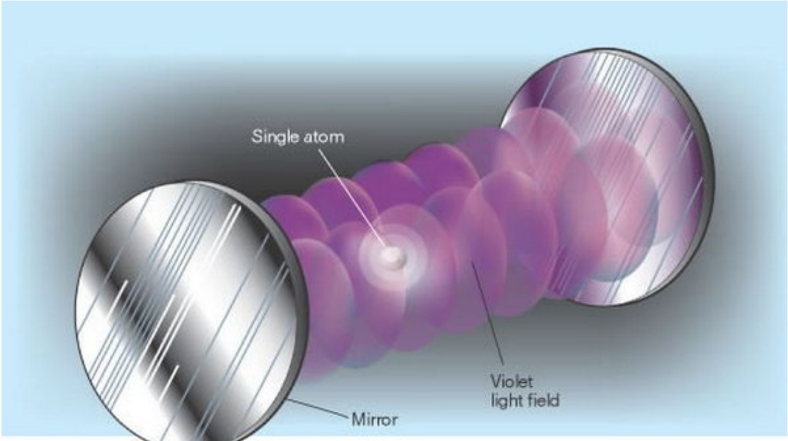
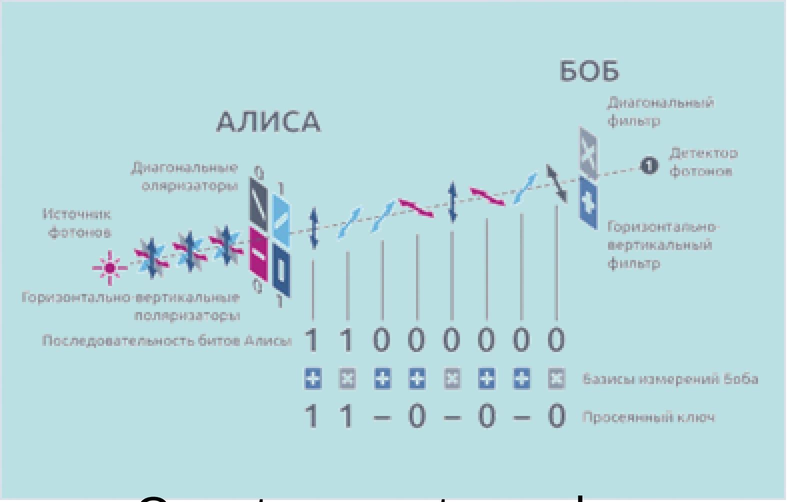

Quantum calculations


Quantum networks


Quantum teleportation



Quantum metrology

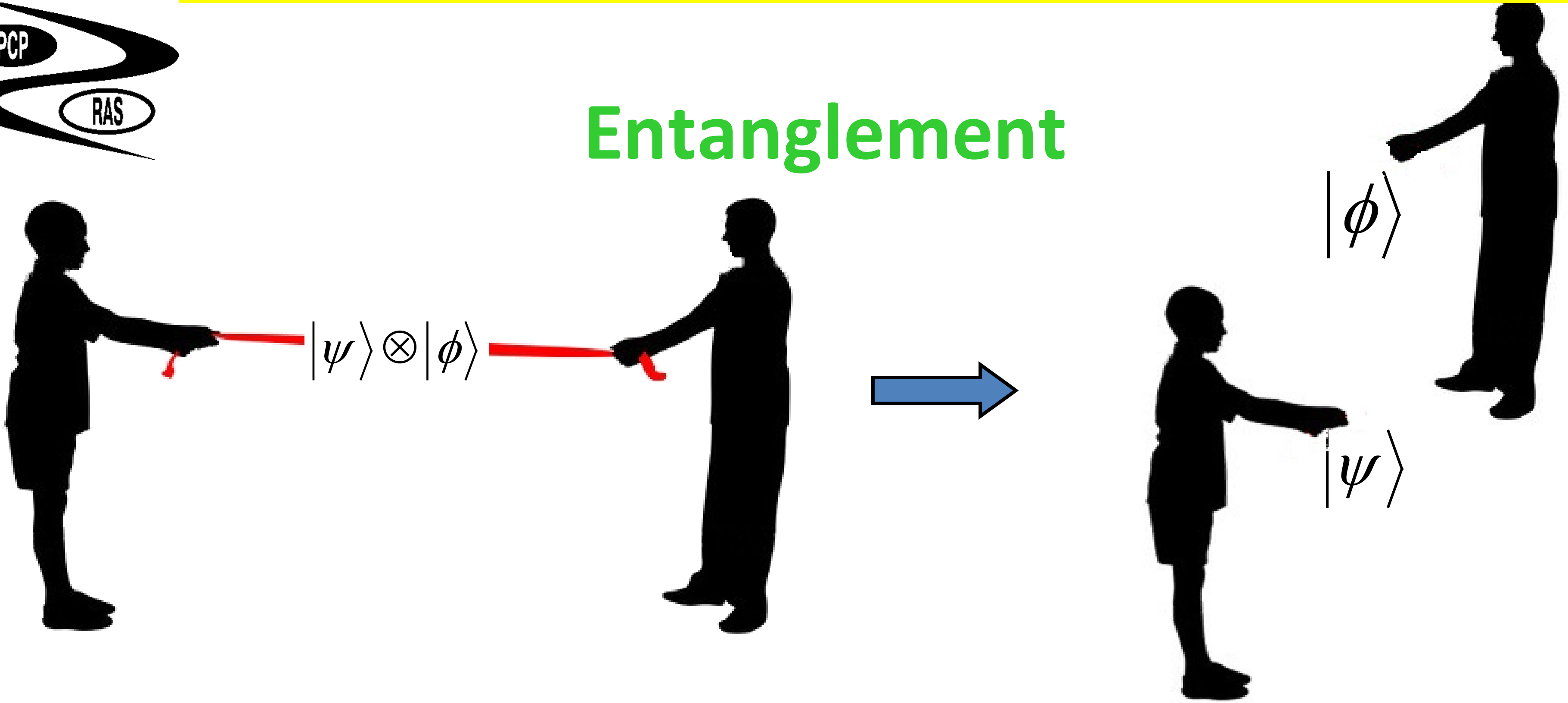


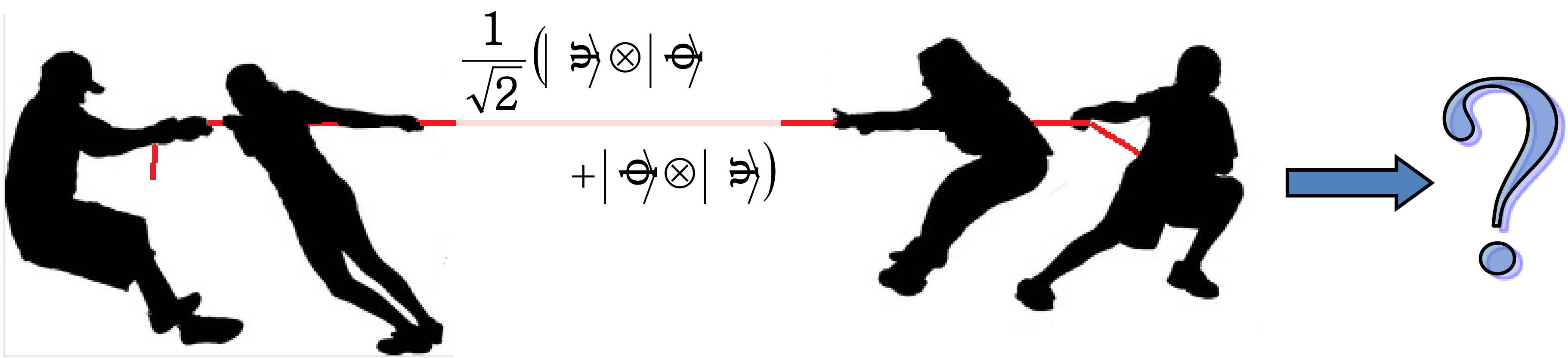
Quantum cryptography

Outline

- Quantum correlations in pure states . Entanglement as a measure of these correlations .
- Multiple quantum (MQ) NMR spectroscopy in solids. Emergence of entangled states in MQ NMR spectroscopy.
- The quantum discord as a measure of quantum correlations in pure and mixed states.
- A comparison of the quantum discord with entanglement in the nitrosyl iron complexes.
- The ring model of interacting spins for an investigation of quantum correlations.
- Contributions of different parts of spin-spin interactions to quantum correlations in a spin ring model in an external magnetic field.
- Quantum correlations in the multi-pulse spin locking

Entanglement



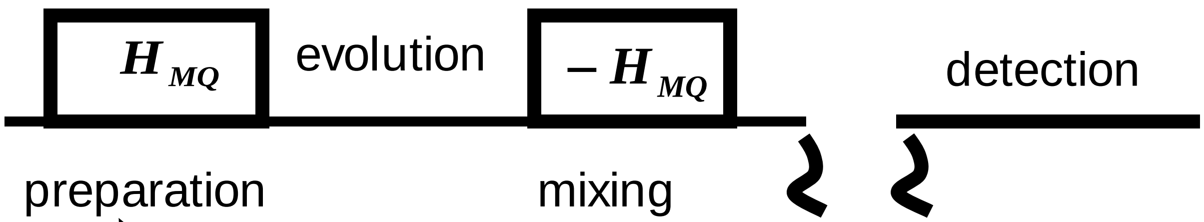


Classical and quantum correlations at measurements

$$|\psi\rangle = |\uparrow\uparrow\rangle \xrightarrow[measurement]{} |\uparrow\uparrow\rangle$$

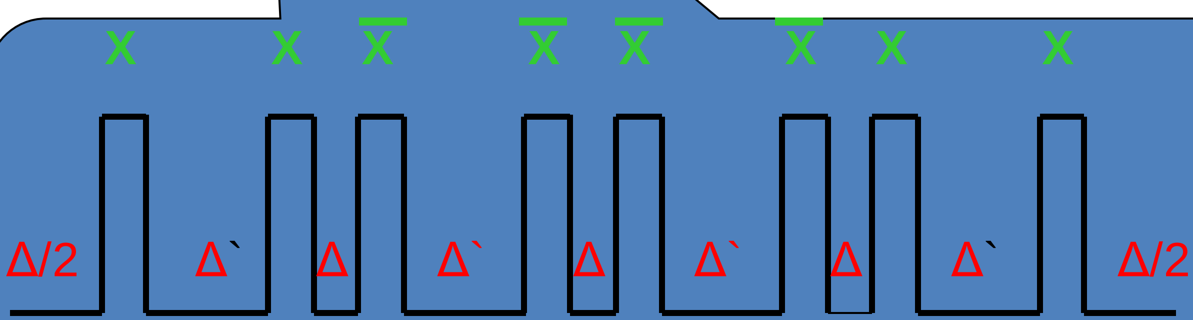
$$|\psi\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \xrightarrow[measurement]{} |\uparrow\downarrow\rangle$$

Multiple quantum NMR in solids



$$\overline{H}_{MQ} = -\frac{1}{2}\sum_{i<j} D_{ij}(I_i^+ I_j^+ + I_i^- I_j^-) =$$

$$= H^{(2)} + H^{(-2)}$$



$$\Delta^{`} = 2\Delta + t_p$$

Emergence of entanglement

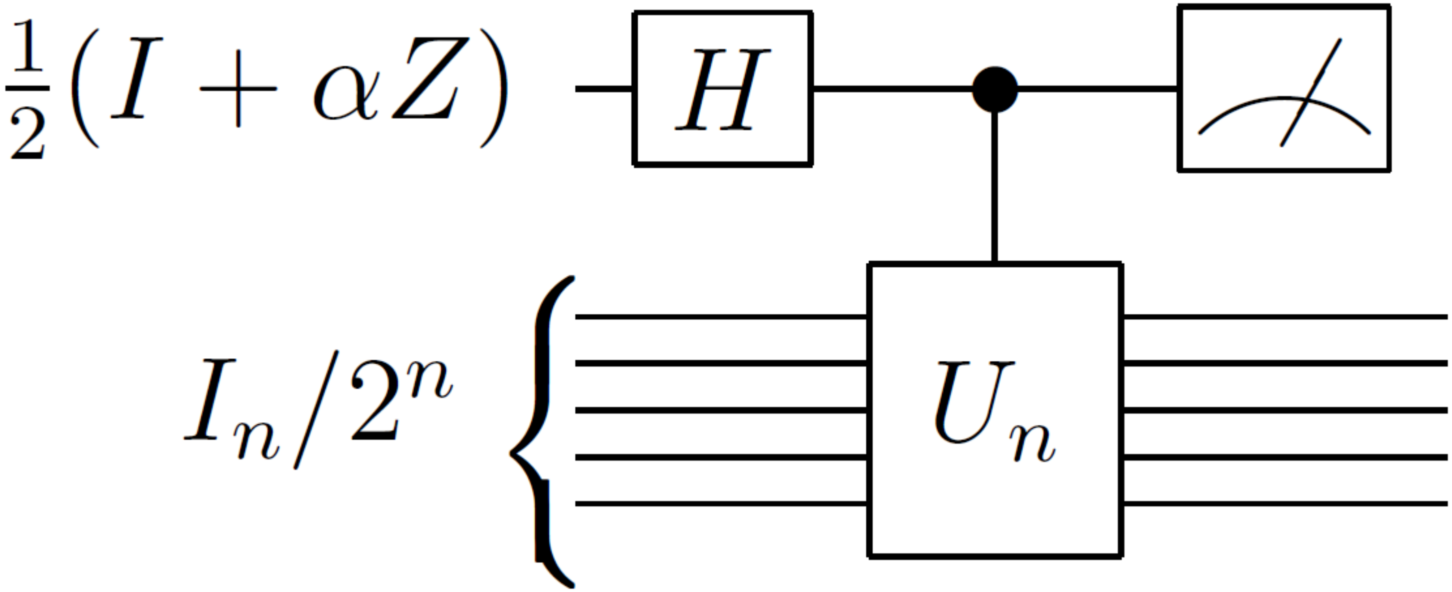
$$T \le T_E = \frac{\hbar\omega_0}{k\ln(1+\sqrt{2})}$$

$$\omega_0 = 2\pi\cdot 500\times 10^6 s^{-1}, \qquad T_E = 27\ \text{mK}.$$

$$T < T_E = \frac{\hbar\omega_0}{k\ln\left(\dfrac{\sqrt{2+a}}{2-\sqrt{2+a}}\right)} \qquad a = e^{\tau/T_{MQ}} - 1$$

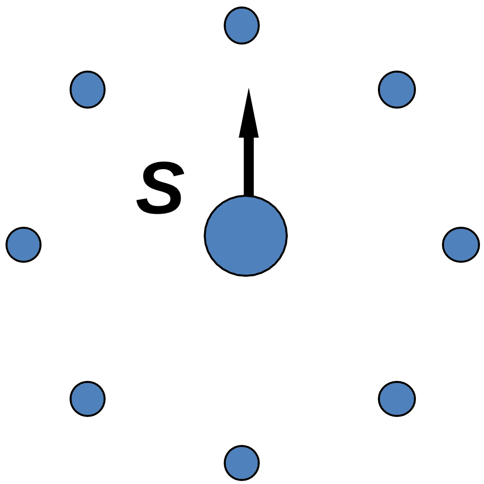
E.B.Fel'dman, A.N.Pyrkov, JETP Letters **88**, 398 (2008)

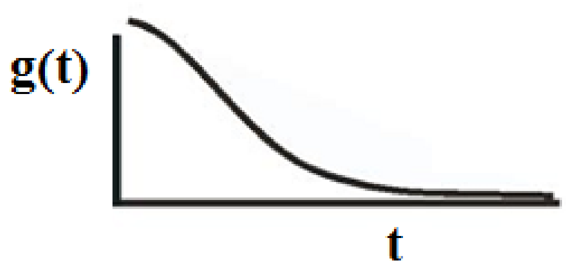
The DQC1 algorithm for computing the trace of a unitary operator



Datta et al., Phys. Rev. Lett. 100, 050502 (2008)

Quantum calculations with a free induction decay in electron-nuclear system





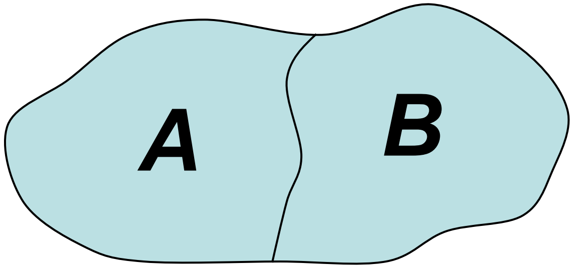
$$H = \hat{S}_z \hat{B} + \omega_0 \hat{S}_z$$

$$\hat{B} = \sum \left(A_{jz} I_{jz} + A_{jy} I_{jy} + A_{jx} I_{jx} \right)$$

$$U(t) = \exp\left\{ -i \hat{B} t \right\}$$

$$g(t) = \frac{Tr\left\{ U(t) S^{+} U^{+}(t) S^{-} \right\}}{Tr\left\{ S^{+} S^{-} \right\}} = Tr\left\{ U(t) \right\} = \sum_{k=1}^{2^N} e^{-i\lambda_k t}$$

Mutual information and correlations

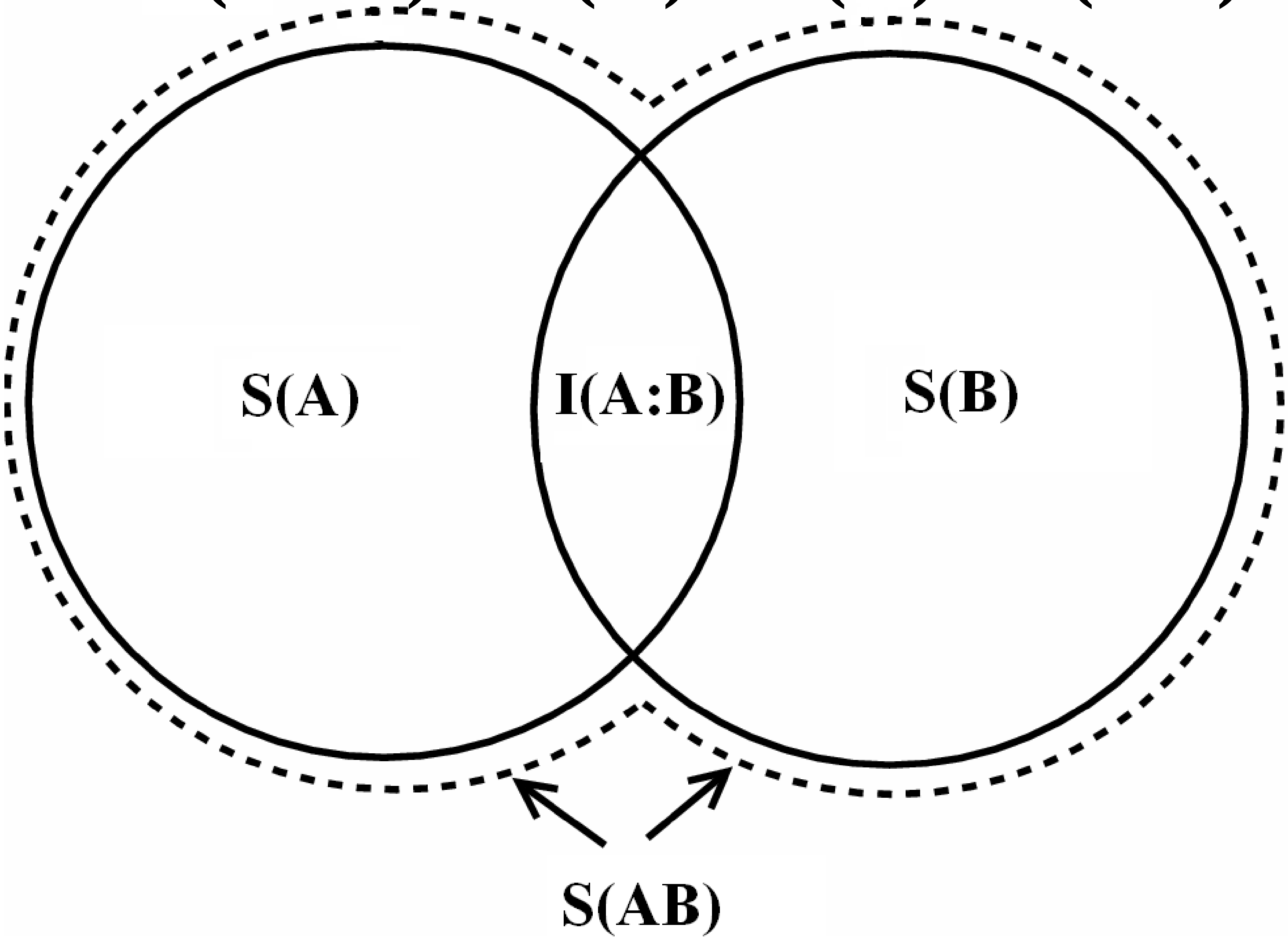


$$I(A:B) = S(A) + S(B) - S(AB)$$

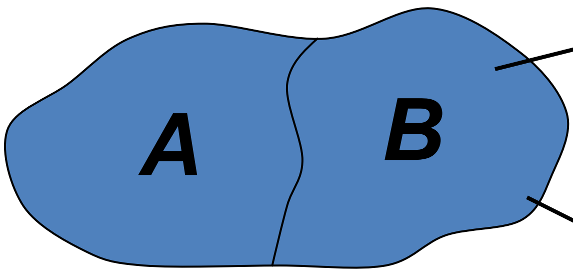
Mutual information is a measure of total correlations in the system

Mutual information

$$I(A:B) = S(A) + S(B) - S(AB)$$



Quantum and classical correlations, quantum discord



$$\Pi_1 = |1\rangle\langle 1|$$

$$\Pi_0 = |0\rangle\langle 0|$$

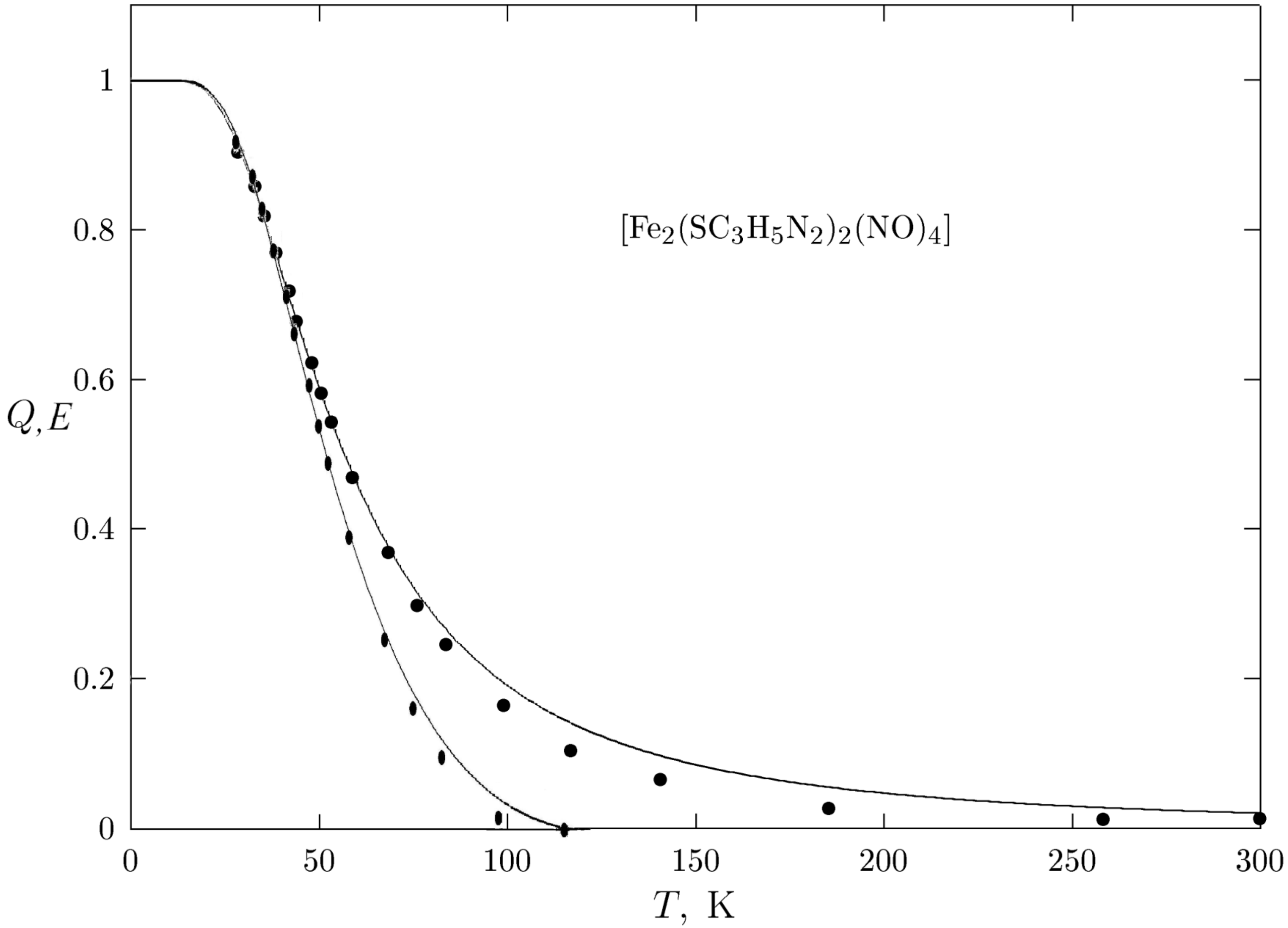
$$\rho \to \eta$$

$$K(\rho) = \max_{\{\Pi_j^B\}} I(\eta)$$

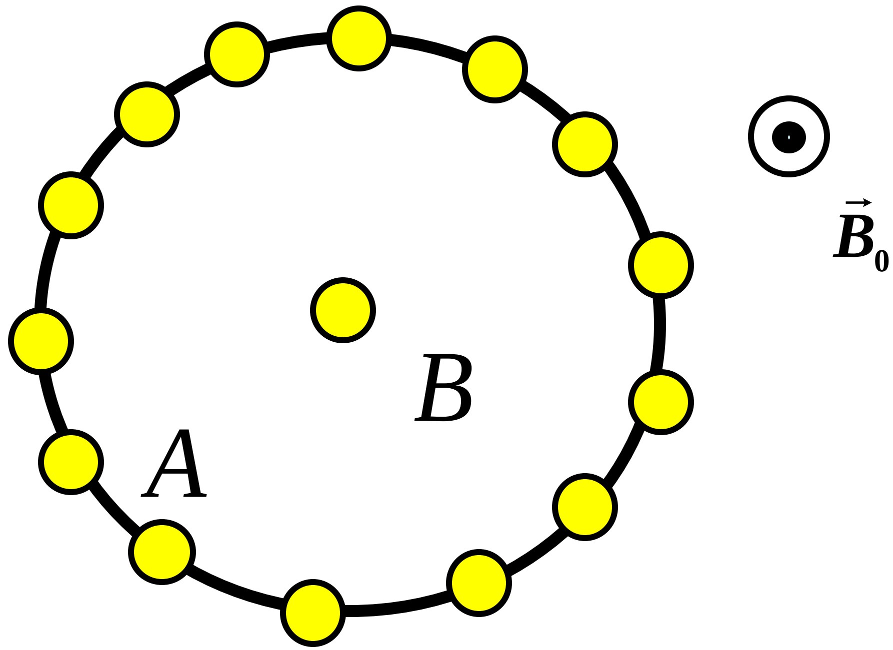
Quantum discord:

$$\boxed{D(\rho) = I(\rho) - K(\rho)}$$

Entanglement and quantum discord in nitrosyl iron complexes



The spin model for an investigation of the quantum correlation (ring model)



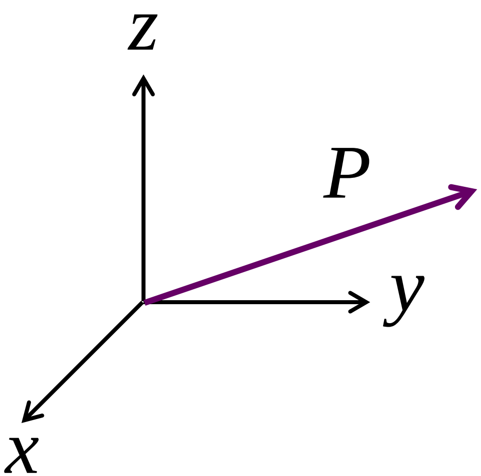
$$H_{dz} = \sum_{i<j} d_{ij} \left(3 I_{iz} I_{jz} - \vec{I}_i \vec{I}_j\right)$$

$$H_{zz} = g \sum_i I_{iz} S_z = g I_z S_z$$

$$\left[H_{dz}, H_{zz}\right] = 0$$

S.I. Doronin, E.B. Fel'dman, E.I. Kuznetsova, Physica Scripta **90**, 074016 (2015)

A spin approach to calculations of the quantum discord



$$P_0 = \frac{1}{2} + \left(n_x S_x + n_y S_y + n_z S_z\right)$$

$$P_1 = \frac{1}{2} - \left(n_x S_x + n_y S_y + n_z S_z\right)$$

$$P_k^2 = P_k, \quad k = 0,1.$$

$$P_k S_\alpha P_k = \frac{(-1)^k}{2} n_\alpha P_k, \;\; \alpha = x, y, z; \;\; k = 0,1.$$

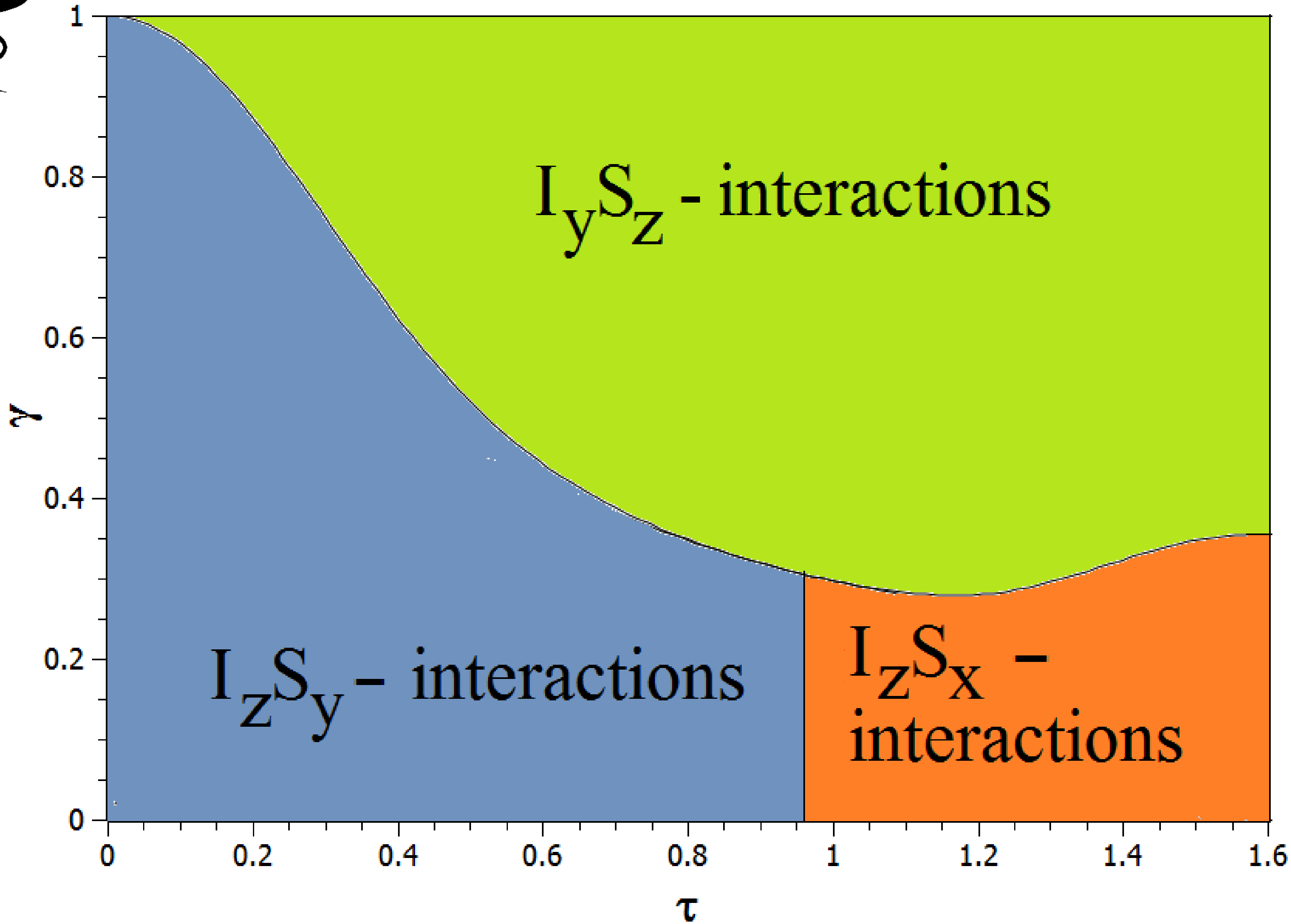
Conditional entropy and discord

$$S_{cond} = -\frac{1}{2\ln 2}\Big\{n_x^2\Big[u^2\frac{1+\cos^{N-1}(2\tau)-2\cos^{2(N-1)}(\tau)}{2} - (N-1)v^2\sin^2(\tau)\Big] + n_y^2\Big[u^2\frac{1-\cos^{N-1}(2\tau)}{2} - (N-1)v^2\sin^2(\tau)\Big] + a(u,v)\Big\}$$

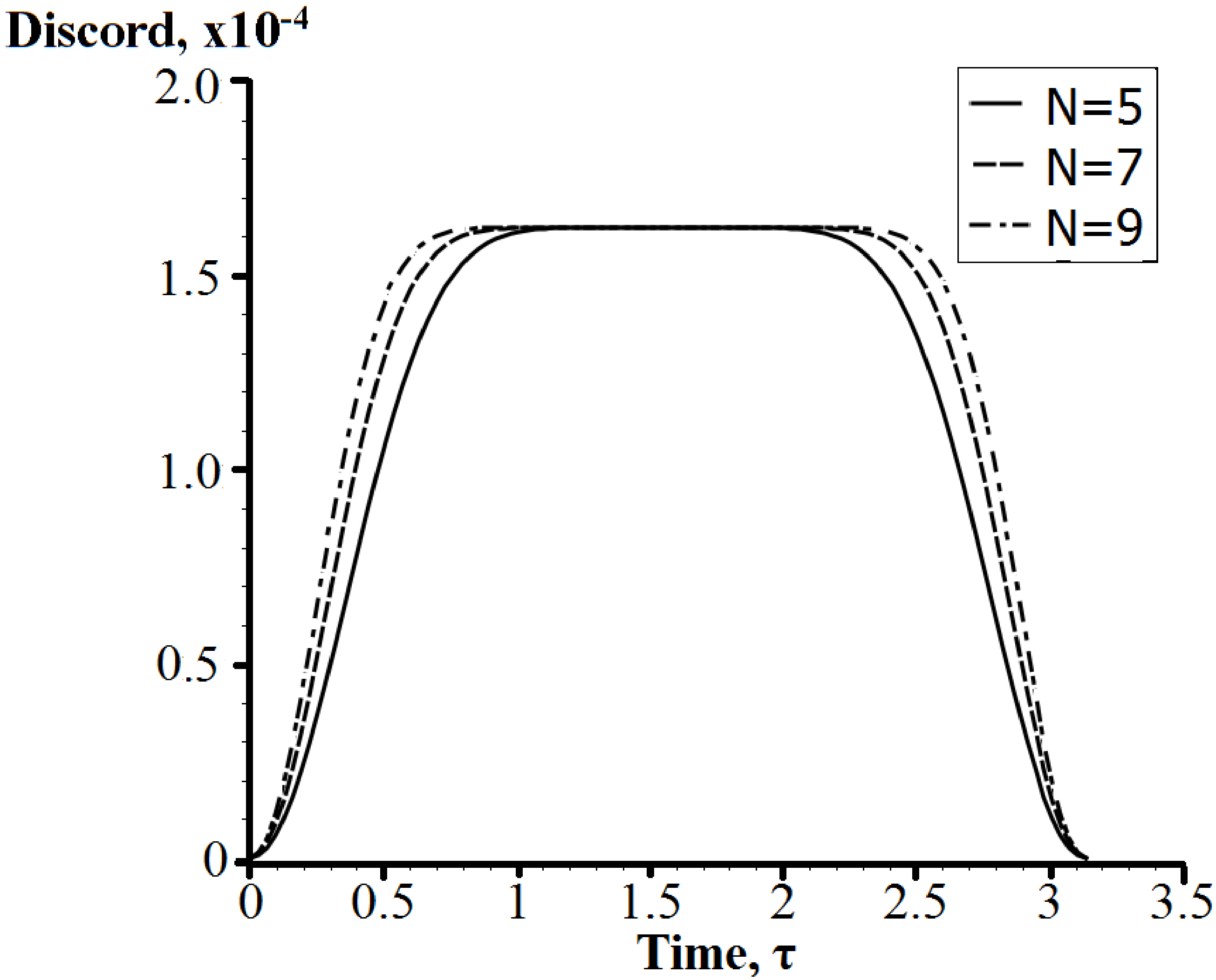
$$u = \frac{1}{2}(N-1)\beta\omega_B < 1, \;\; v = \frac{1}{2}(N-1)\beta\omega_A < 1 \qquad n_{x,y} \in [-1,1]$$

$$\frac{v}{u} = \frac{\omega_A}{\omega_B} > 1: \qquad D = \frac{\beta^2\omega_B^2}{8\ln 2}\left(1-\cos^{2(N-1)}(\tau)\right)$$

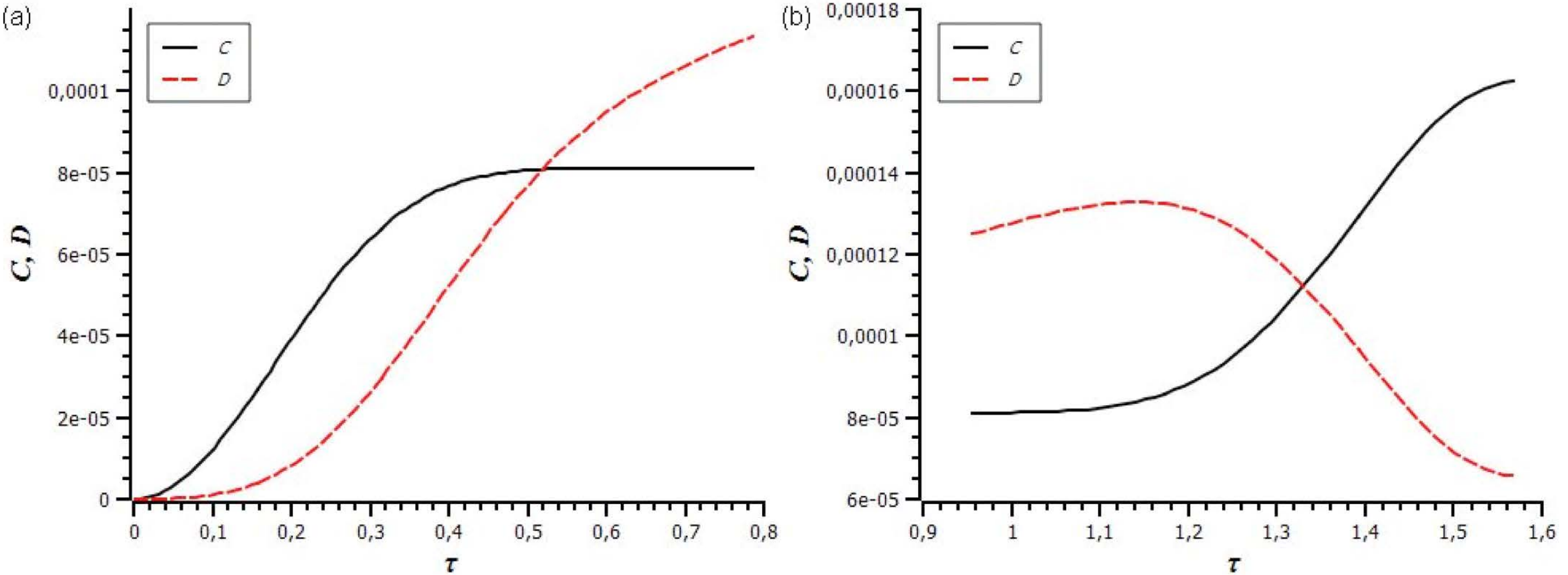
N=9



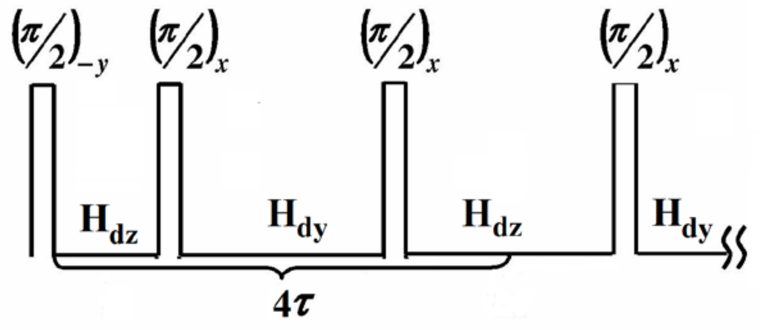
S.I. Doronin, E.B. Fel'dman, E.I. Kuznetsova,
Quantum Inf. Process. (2015) **14**:2929–2943.



$$u = 0.015,\ \ n_x = n_y = 0,\ \ |n_z| = 1.$$


(a)
C
D
0,0001
8e-05
6e-05
4e-05
2e-05
0
C, D
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
τ
(b)
C
D
0,00018
0,00016
0,00014
0,00012
0,0001
8e-05
6e-05
C, D
0,9
1
1,1
1,2
1,3
1,4
1,5
1,6
τ

Entanglement in a two-spin system in the multi-pulse spin lockimg



The initial density matyrix:

$$\rho = \frac{1}{Z} e^{\beta I_x}, \ \beta = \frac{\hbar\omega_0}{kT}, \ Z = 4\cos^2\frac{\beta}{2}.$$

$$\boxed{U(4\tau) = e^{-i\tau H_{dz}} e^{-i2\tau H_{dy}} e^{-i\tau H_{dz}}}$$

$$U(4\tau) = \frac{1}{2}\begin{pmatrix} e^{-2it}+e^{it} & 0 & 0 & -e^{-2it}+e^{it} \\ 0 & 1+e^{it} & e^{it}-1 & 0 \\ 0 & e^{it}-1 & 1+e^{it} & 0 \\ -e^{-2it}+e^{it} & 0 & 0 & e^{-2it}+e^{it} \end{pmatrix}$$

$(U(4\tau)-$ centrosymmetric matrix

$t = \tau d_{12}$, dimensionless time)

$$G = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

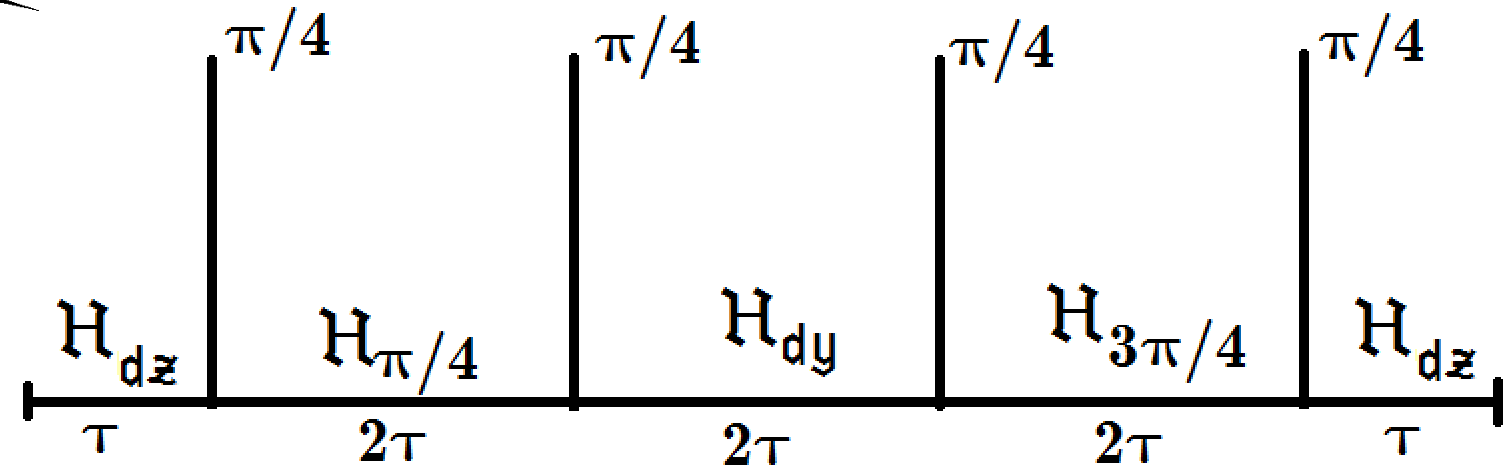
Entanglement is absent in the multi-pulse spin-locking at φ=π/2

$$\rho(4M\tau)=U(4M\tau)\rho_0 U(4M\tau)$$
$$(M \text{ is integer})$$

$$=\frac{1}{8\mathrm{ch}^2\frac{\beta}{2}}\begin{pmatrix} 2\mathrm{ch}^2\frac{\beta}{2} & \mathrm{sh}\frac{\beta}{2} & \mathrm{sh}\frac{\beta}{2} & 2\mathrm{sh}^2\frac{\beta}{2} \\ \mathrm{sh}\frac{\beta}{2} & 2\mathrm{ch}^2\frac{\beta}{2} & 2\mathrm{sh}^2\frac{\beta}{2} & \mathrm{sh}\frac{\beta}{2} \\ \mathrm{sh}\frac{\beta}{2} & 2\mathrm{sh}^2\frac{\beta}{2} & 2\mathrm{ch}^2\frac{\beta}{2} & \mathrm{sh}\frac{\beta}{2} \\ 2\mathrm{sh}^2\frac{\beta}{2} & \mathrm{sh}\frac{\beta}{2} & \mathrm{sh}\frac{\beta}{2} & 2\mathrm{ch}^2\frac{\beta}{2} \end{pmatrix}=\rho_0$$

$$\boxed{H=-\frac{1}{2}H_{dx}}$$

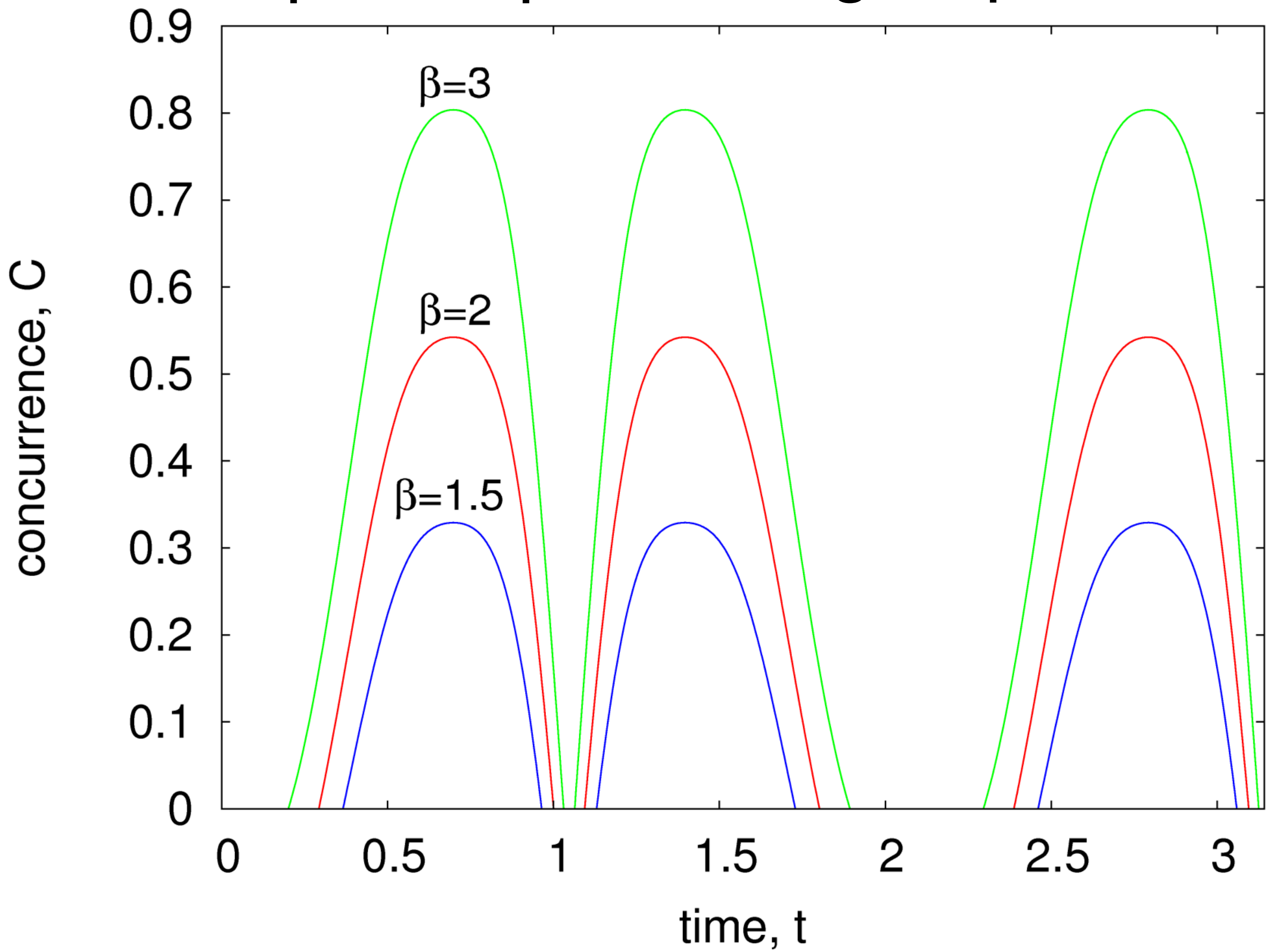
Entanglement in the multi-pulse locking at $\varphi=\pi/4$



$$H_{\pi/4} = d_{12} e^{i\pi I_x/4}\left(3I_{1z}I_{2z} - \vec{I}_1\vec{I}_2\right)e^{-i\pi I_x/4} = \frac{d_{12}}{8}\begin{pmatrix} 1 & -3i & -3i & -3 \\ 3i & -1 & -1 & 3i \\ 3i & -1 & -1 & 3i \\ -3 & -3i & -3i & 1 \end{pmatrix}, \qquad \left[H_{dz}, H_{\pi/4}\right] \neq 0,$$

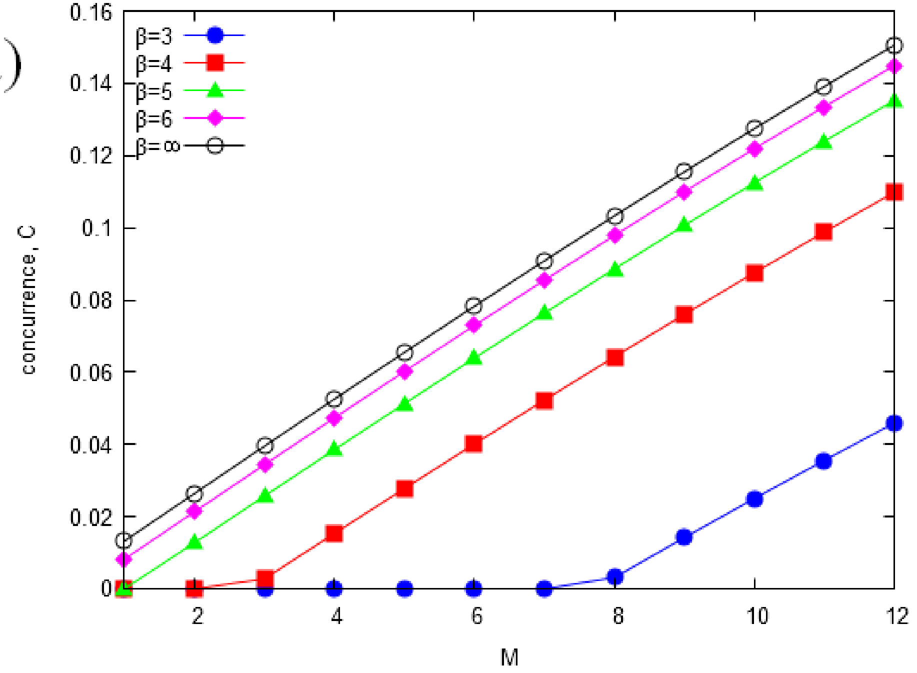
$$H_{3\pi/4} = d_{12} e^{i3\pi I_x/4}\left(3I_{1z}I_{2z} - \vec{I}_1\vec{I}_2\right)e^{-i3\pi I_x/4} = \frac{d_{12}}{8}\begin{pmatrix} 1 & 3i & 3i & -3 \\ -3i & -1 & -1 & -3i \\ -3i & -1 & -1 & -3i \\ -3 & 3i & 3i & 1 \end{pmatrix} \qquad \left[H_{dz}, H_{3\pi/4}\right] \neq 0.$$

Entanglement in a two-spin system in the multi-pulse spin locking at φ=π/4

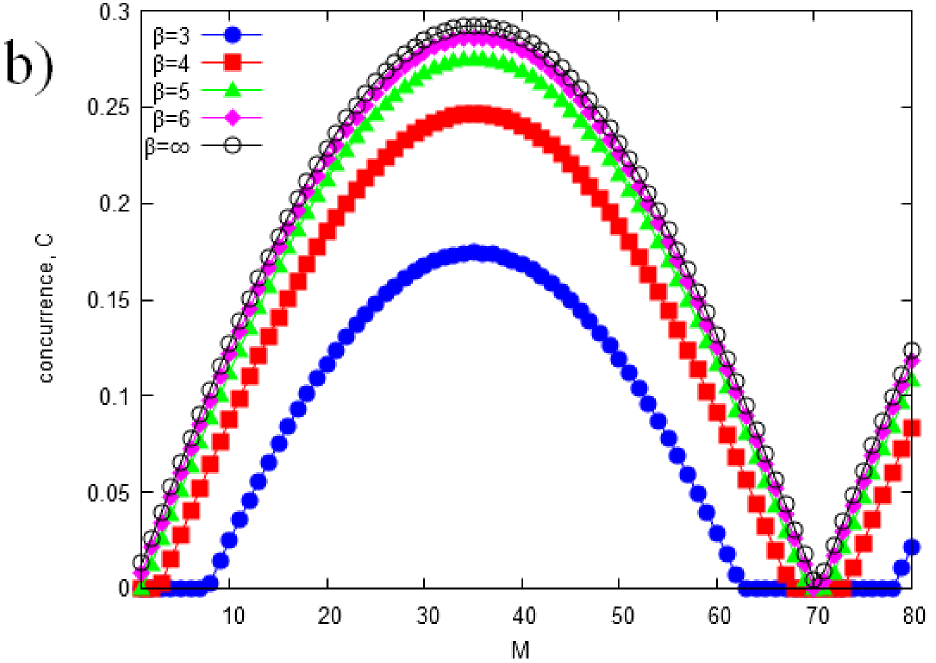


The dependence of entanglement on time in the multi-pulse spin locking

a)



b)



CONCLUSIONS

1. Entanglement and the quantum discord are considered as the measures of quantum correlations.

2. Quantum calculations in electron-nuclear systems are discussed.

3. A comparison of the quantum discord and entanglement in the nitrosyl iron complexes is given.

4. The ring spin model for an investigation of quantum correlations is supposed.

5. The contributions of different parts of spin-spin interactions to quantum correlations are considered in the ring spin model.

6. Quantum correlations are considered in multi-pulse spin locking experiments.

Laboratory of spin dynamics and spin computing

Thank You for attention !

Entanglement

A two-qubit state is described by four complex numbers. A
A three-qubit state is described by eight complex numbers.
An N qubit-state is described by 2^N complex numbers.
A register, consisting of N qubits, is described by $2^{N+1} – 2$ real parameters.

If entanglement would not be in Nature then 2N real numbers were sufficient to describe a quantum register. For example, without entanglement, only 2000 parameters would be enough for a 1000 qubit register. However, the actual number of such parameters is more than 10^{300} when entanglement is taken into account. (We note for comparison that the number of protons and neutrons in the Universe is only of order 10^{78}).

Speedup of quantum algorithms and entanglement in pure states

$$|\alpha\rangle=\sum_{i_1,i_2,\ldots i_n} a_{i_1,i_2,\ldots i_n}|i_1\rangle|i_2\rangle\ldots.|i_n\rangle$$

One-qubit gate U:

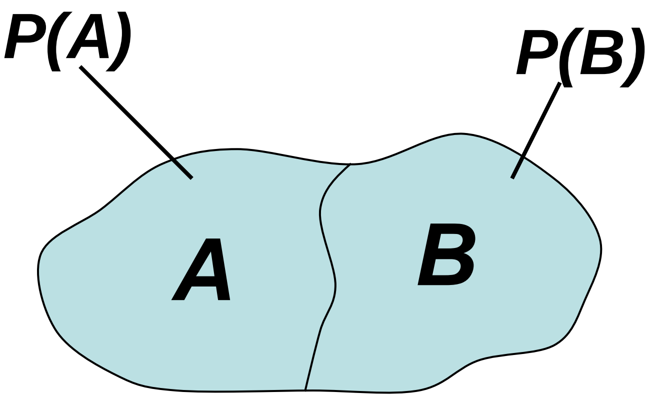
$$|\alpha\rangle\rightarrow|\alpha'\rangle=\sum_{i_1,i_2,\ldots i_n} a'_{i_1,i_2,\ldots i_n}|i_1\rangle|i_2\rangle\ldots.|i_n\rangle$$

$$a'_{i_1,i_2,\ldots i_n}=\sum_{j_1} U_{i_1 j_1} a_{j_1,i_2,\ldots i_n}$$

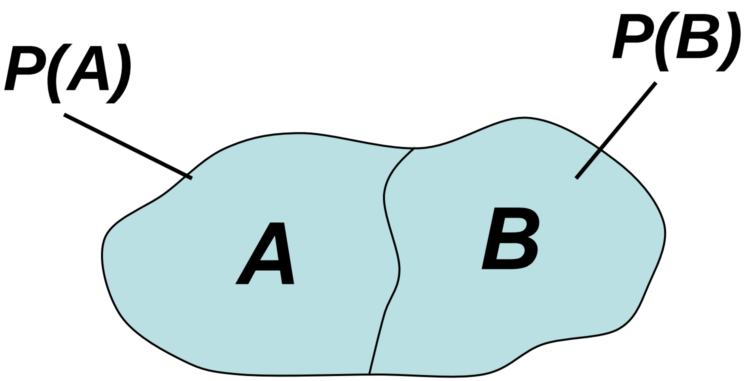
Unentangled (separable) states:

$$a_{j_1,i_2,\ldots i_n}=\alpha_{j_1}\alpha_{i_2}\ldots\alpha_{i_n}$$

Correlations in two-partite systems



$$P(A,B) = P(A)P(B)$$



$$P(A,B) \neq P(A)P(B)$$

Correlation coefficient:

$$\rho_{A,B} = \frac{E[AB]}{\sqrt{E[A^2]\cdot E[B^2]}} \quad (E[A]=E[B]=0)$$

If P(A,B)=P(A)P(B) then $\rho_{A,B} = 0.$ If $\boldsymbol{\rho_{A,B} = 0},$ then the condition P(A,B)=P(A)P(B) may not be satisfied.