Neutrino masses and mixing: status and prospects

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Comments on the phenomenology of neutrino oscillations

The Standard Model and neutrino

The simplest mechanism of the generation of neutrino masses

COMMENTS ON THE PHENOMENOLOGY OF NEUTRINO OSCILLATIONS

The award of the Nobel Prize to T. Kajita and A. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass" was a result of more than fifty years efforts of many experimentalists and theoreticians

First ideas of neutrino oscillations were pioneered in 1957-58 by B. Pontecorvo

First model independent evidence in favor of disappearance of atmospheric ν_μ's was obtained in 1998 by the Super-Kamiokande collaboration

First model independent evidence of the disappearance of solar ν_e's was obtained by the SNO collaboration in 2001

First model independent evidence of the disappearance of reactor $\bar{\nu}_e$'s was obtained by the KamLAND collaboration in 2002

The discovery of neutrino oscillations was confirmed by many experiments(K2K, MINOS, T2K, DayaBay, RENO, Double Chooz, IceCube, Nova)

The basic relation

$$\nu_{lL}(x) = \sum_i U_{li}\, \nu_{iL}(x), \quad j_\alpha^{CC} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha l_L$$

is the relation between fields

What are the states of flavor neutrinos ν_e, ν_μ, ν_τ which are produced in $\pi^+ \to \mu^+ + \nu_\mu$ and other processes?

$\frac{\Delta m^2_{ki}}{2E}$ $(\Delta m^2_{ki} = m^2_i - m^2_k)$ are very small; it follows from Heisenberg uncertainty relation that the states of flavor neutrino are coherent superposition of states of neutrinos with different masses

$$|\nu_l\rangle = \sum_i U^*_{li}\ |\nu_i\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i, momentum $\vec{p}$ and energy $E_i \simeq E + \frac{m_i^2}{2E}$

Transition probability $\nu_l \to \nu_{l'}$ during the time t

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i}\; e^{-iE_i t} U^*_{li}|^2$$

A common phase of the amplitude is arbitrary

$$P(\nu_l \to \nu_{l'}) = |\delta_{l'l} + \sum_{i \neq p} U_{l'i}\; (e^{-2i\Delta_{pi}} - 1)\; U^*_{li}|^2$$

p is a fixed index, $\Delta_{ki} = \frac{\Delta m^2_{ki} L}{4E}$.

For three neutrinos with masses m_1, m_2, m_3 two neutrino mass spectra are possible

Neutrino masses are labeled in such a way that $m_2 > m_1$ and $\Delta m^2_{12} = \Delta m^2_S > 0$

Possible neutrino mass spectra are determined by the mass m_3

1. Normal ordering (NO) $m_3 > m_2 > m_1$
2. Inverted ordering (IO) $m_2 > m_1 > m_3$

Transition probability

$$P^{NO(IO)}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4|U_{l3}|^2(\delta_{l'l} - |U_{l'3}|^2)\sin^2\Delta_A$$
$$-4|U_{l1(2)}|^2(\delta_{l'l} - |U_{l'1(2)}|^2)\sin^2\Delta_S$$
$$-8\ [B^{31(32)}_{l'l}\cos(\Delta_A + \Delta_S) \pm (\mp) A^{31(32)}_{l'l}\sin(\Delta_A + \Delta_S)]\sin\Delta_A\sin\Delta_S$$

$$B^{ik}_{l'l} = Re\ U_{l'i}U^*_{li}U^*_{l'k}U_{lk},\ A^{ik}_{l'l} = Im\ U_{l'i}U^*_{li}U^*_{l'k}U_{lk}$$

$$\Delta m^2_A = \Delta m^2_{23}\ \ (\mathrm{NO})\ \mathrm{and}\ \Delta m^2_A = |\Delta m^2_{13}|\ \ (\mathrm{IO})$$

Probability is the sum of atmospheric, solar and interference terms

Dominant neutrino oscillations

Two small parameters: $\frac{\Delta m_S^2}{\Delta m_A^2} \simeq 10^{-2}$, $\sin^2\theta_{13} \simeq 2.5 \cdot 10^{-2}$

If we neglect a few % contribution to the transition probabilities

- In the atmospheric range $\Delta_A \simeq 1, \quad \Delta_S \ll 1$ (atmospheric, accelerator experiments)

  $$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)\sin^2\Delta_A = 1 - \sin^2 2\theta_{23}\sin^2\Delta_A$$

  $\nu_\mu \to \nu_\mu$ oscillations

- In the reactor KamLAND experiment $\Delta_S \simeq 1, \quad \Delta_A \gg 1$

  $$P(\bar\nu_e \to \bar\nu_e) \simeq 1 - 4|U_{e2}|^2(1-|U_{e2}|^2)\sin^2\Delta_S = 1 - \sin^2 2\theta_{12}\sin^2\Delta_S$$

  $\bar\nu_e \to \bar\nu_{\mu,\tau}$ oscillations

No dependence on neutrino mass ordering

No difference between neutrino and antineutrino (If CP is violated in the lepton sector $P(\nu_l \to \nu_{l'}) \neq P(\bar\nu_l \to \bar\nu_{l'})$ $l \neq l'$)

Future major problems of neutrino oscillation experiments T2K, NOvA, JUNO, RENO-50, DUNE

I. Determine neutrino mass ordering

II. Determine CP phase δ

In future experiments a few % effects must be studied

Challenging problem (usual background problem, problem of neutrino cross sections, problem of statistics etc)

Results of a global analysis of the data

Parameter	Normal Spectrum	Inverted Spectrum
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.579^{+0.025}_{-0.037}$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0219^{+0.0011}_{-0.0010}$
δ (in $^\circ$)	(306^{+39}_{-70})	(254^{+63}_{-62})
Δm^2_S	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$
Δm^2_A	$(2.457^{+0.047}_{-0.047})\cdot 10^{-3}\ \mathrm{eV}^2$	$(2.449^{+0.048}_{-0.047})\cdot 10^{-3}\ \mathrm{eV}^2$

Neutrino and the Standard Model

Impressive agreement of the Standard Model with experiment. The most remarkable prediction of the SM, the existence of the scalar Higgs boson, was recently confirmed by LHC experiments

What general conclusions can we make from this success? Can we make any conclusions about neutrino?

The Standard Model is based on the following principles

- Local gauge $SU_L(2) \times U_Y(1)$ invariance of massless fields.
- Brout-Englert-Higgs mechanism of mass generation.
- Unification of the weak and electromagnetic interactions.

From the success of SM we can conclude that in the framework of these general principles Nature choose the simplest, most economical possibilities

Massless four-component fermion field satisfies the Dirac equation

$$i\gamma^{\alpha}\partial_{\alpha}\psi(x) = 0.$$

Two-component $\psi_L(x) = \frac{1}{2}(1-\gamma_5)\psi$ and $\psi_R(x) = \frac{1}{2}(1+\gamma_5)\psi$ satisfy Weil equations

$$i\gamma^{\alpha}\partial_{\alpha}\psi_L(x) = 0, \quad i\gamma^{\alpha}\partial_{\alpha}\psi_R(x) = 0$$

$\psi(x)$ possesses four degrees of freedom, $\psi_L(x)$ ($\psi_R(x)$) possesses two degrees of freedom If we look for economy we can try to choose for spin 1/2 massless field $\psi_L(x)$ (or $\psi_R(x)$)
This can not be done for charged particles because in the electromagnetic current four-component field enter

$$j^{EM}_{\alpha} = e\bar{\psi}\gamma_{\alpha}\psi = e(\bar{\psi}_L\gamma_{\alpha}\psi_L + \bar{\psi}_R\gamma_{\alpha}\psi_R)$$

Neutrinos, however, have no electric charge and direct electromagnetic interaction.

The most economical possibility for neutrino is ν_L (or ν_R). This is the famous two-component neutrino theory by Landau, Lee and Yang and Salam.

The confirmation of this theory by the Goldhaber et al experiment on the measurement of the neutrino helicity means that Nature has chosen this economical possibility (ν_L)

Neutrino ν_e, ν_μ, ν_τ participate in the CC weak interaction together with corresponding charged lepton e, μ, τ

Symmetry is a manifestation of simplicity. To ensure $e-\mu-\tau$ universality with only one interaction constant we need to assume local nonabelian symmetry group

The simplest nonabelian symmetry group is $SU_L(2)$ with doublets

$$\psi_{eL} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix},\ \psi_{\mu L} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix},\ \psi_{\tau L} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}$$

e'_L, μ'_L, τ'_L are left-handed massless Weil fields (like neutrino fields)

To provide local gauge invariance we need to change in the free Lagrangian the derivative $\partial_\alpha \psi_{lL}$ by the covariant derivative

$$\partial_\alpha \psi_{lL} \to (\partial_\alpha + ig\frac{1}{2}\vec{\tau}\vec{A}_\alpha \psi_{lL}) \quad (l = e, \mu, \tau).$$

$\vec{A}_\alpha$ is isovector gauge field

Interaction of leptons and vector gauge bosons

Minimal interaction is the sum of CC, NC and EM terms

$$\mathcal{L}_I = \left(-\frac{g}{2\sqrt{2}} j^{CC}_{\alpha} W^{\alpha} + \mathrm{h.c}\right) - \frac{g}{2\, cos\theta_W} j^{NC}_{\alpha} Z^{\alpha} - e\, j^{EM}_{\alpha} A^{\alpha}$$

Perfect agreement with data

The SM mechanism of the mass generation is the Brout-Englert-Higgs mechanism of the spontaneous symmetry breaking. Based on the assumption of the existence of the scalar Higgs field. To generate masses of $W^{\pm}$ and Z^0 we need three Goldstone degrees of freedom . Minimal possibility is a doublet of complex Higgs fields (four degrees of freedom)

$$H(x) = \begin{pmatrix} H_+(x) \\ H_0(x) \end{pmatrix}$$

Existence of the scalar Higgs particle is predicted Correspond to LHC finding

Lepton masses are generated by B-E-H mechanism via Yukawa interaction

$$\mathcal{L}_Y = -\sqrt{2}\sum_{l,l'} \bar{\psi}_{lL}\ Y_{ll'}\ l'_R\ H + \mathrm{h.c.} = \sum_{\mathrm{l}} \mathrm{m_l}\ \bar{\mathrm{l}}\ \mathrm{l}\ (1+\frac{\mathrm{h}}{\mathrm{v}})$$

$m_l = y_l\ v \quad l = e, \mu, \tau$

Masses of leptons (and quarks) are proportional to $v = \sqrt{2}G_F \simeq 246$ GeV. No constraints on Yukawa constants y_i from symmetry.

Neutrinos are neutral particles. No need to introduce ν_{lR}. ν_{lL} is the most economical possibility. Neutrinos after SSB remain two-component massless Weyl particles

Assume that not only ν_{lL} but also ν_{lR} are in the Lagrangian

Neutrino masses can be generated by B-E-H mechanism: $m_i = y_i\ v$

For the particles of third generation

$y_t \simeq 0.7, y_b \simeq 1.7 \cdot 10^{-2}, y_\tau \simeq 7 \cdot 10^{-3}, y_3 < 0.8 \cdot 10^{-12}$. It is very unlikely that neutrino masses are generated by B-E-H mechanism

If in the SM neutrinos are massless, neutrino masses are generated by a beyond the SM mechanism

The method of the effective Lagrangian is a general method which allows to describe effects of a beyond the SM physics.

Effective Lagrangian is nonrenormalizable, dimension five or more Lagrangian invariant under $SU_L(2) \times U_Y(1)$ transformations and built from SM fields

Consider $(\bar{\psi}_{lL}\tilde{H})$ ($\tilde{H} = i\tau_2 H^*$ conjugated Higgs doublet)

This term is $SU_L(2) \times U_Y(1)$ invariant and has dimension $M^{5/2}$

After SSB $(\bar{\psi}_{lL}\tilde{H}) = \frac{v}{\sqrt{2}}\bar{\nu}_{lL}$

The only possible effective Lagrangian which generates neutrino mass term (Weinberg)

$$\mathcal{L}_I^{\mathrm{eff}} = -\frac{1}{\Lambda} \sum_{l,l'}(\bar{\psi}_{lL}\tilde{H})\ Y_{ll'}\ (\tilde{H}^T(\psi_{l'L})^c) + \mathrm{h.c.}$$

Dimension five operator, violate L.

Λ characterizes a scale of a beyond the SM L-violating physics

$Y_{ll'}$ are dimensionless constants, no constraints from the symmetry

After spontaneous symmetry breaking we come to the Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\,\frac{v^2}{\Lambda}\sum_{l,l'}\bar{\nu}_{lL}\,Y_{ll'}(\nu_{l'L})^c + \mathrm{h.c.} = -\frac{1}{2}\sum_{i=1}^{3} m_i\bar{\nu}_i\,\nu_i$$

$$\nu_i = \nu_i^c \quad (\mathrm{Majorana\ \ field}), \qquad m_i = \frac{v^2}{\Lambda}y_i = \frac{v}{\Lambda}(y_i v),$$

y_i is a Yukawa coupling, $(y_i v)$ is a “typical fermion mass in the SM”.

Neutrino masses, generated by the Weinberg effective Lagrangian, are suppressed with respect to the masses of leptons and quarks by the factor

$$\frac{v}{\Lambda} = \frac{SM\ \ scale}{scale\ \ of\ \ a\ \ new\ \ physics}$$

The Weinberg effective Lagrangian can be generated by the exchange of heavy virtual Majorana leptons N_i with mass M_i through the interaction

$$\mathcal{L}_I = -\sqrt{2} \sum_{l,i} \bar{\psi}_{lL} \tilde{H} y_{li} N_{iR} + \mathrm{h.c.}$$

(Like low-energy Fermi effective Lagrangian of the β-decay is generated by $W^{\pm}$-exchange)

In this case

$$\frac{1}{\Lambda} Y_{ll'} = \sum_{i} \frac{y_{li} y_{l'i}}{M_i}$$

Λ is determined by M_i (seesaw)

What is the scale of Λ ? (scale of masses of heavy Majorana leptons) Different possibilities are considering (from TeV to $(10^{14} - 10^{15})$ GeV

$\Lambda = y_i \; \frac{v^2}{m_i}$. We do not know y_i and m_i

Consider the third family. If the lightest mass is much smaller than other masses $m_3 \simeq \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}\ \mathrm{eV}$

$\frac{v^2}{\sqrt{\Delta m_A^2}} \simeq 1.2 \cdot 10^{15}$ GeV (huge number)

if $\Lambda \simeq \mathrm{TeV} \quad y_3 \simeq 10^{-12}$ (too small, fine tuning)

If $y_3 \simeq 1$, $\Lambda \simeq 10^{15}$ GeV (GUT scale)

$\Lambda \gg v$ looks as a plausible possibility

Majorana leptons with masses much larger than v can not be observed directly. However, their CP-violating decays in the early Universe can create the lepton asymmetry which could explain the Barion asymmetry of the Universe. This mechanism (leptogenesis) is considered as a most viable mechanism of the Barion asymmetry.

Successful leptogenesis is an indirect indication in favor of existence of heavy Majorana leptons

Major indirect consequences of the mechanism of the neutrino mass generation we considered

I. ν_i are Majorana particles

The most sensitive to small Majorana m_i process is $0\nu\beta\beta$-decay of even-even nuclei $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

$$\frac{1}{T^{0\nu}_{1/2}} = |m_{\beta\beta}|^2 \; |M^{0\nu}|^2 \; G^{0\nu}(Q,Z)$$

$m_{\beta\beta} = \sum_i U^2_{ei} m_i$ effective Majorana mass, $|M^{0\nu}|$ is NME

(challenging nuclear problem, 5 models, 2-3 times differences)

Some latest data

GERDA, Heidelberg-Moscow

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 5.2 \cdot 10^{25}\ \mathrm{y}$ $\;|m_{\beta\beta}| < (1.6 - 2.6) \cdot 10^{-1}\ \mathrm{eV}$

EXO-200

$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 1.1 \cdot 10^{25}\ \mathrm{y}$ $\;|m_{\beta\beta}| < (1.9 - 4.5) \cdot 10^{-1}\ \mathrm{eV}$

KamLAND-Zen

$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 11 \cdot 10^{25}\ \mathrm{y}$ $\;|m_{\beta\beta}| < (0.6 - 1.6) \cdot 10^{-1}\ \mathrm{eV}$

Next generation of the experiments on the search for $0\nu\beta\beta$-decay will be sensitive to $|m_{\beta\beta}| \simeq \mathrm{a\ few} \cdot 10^{-2}\ \mathrm{eV}$

II. The number of ν_i is equal to the number of the lepton flavors (three) No transitions into sterile states

Sterile neutrinos have no standard weak interaction and can not be detected directly. There are two ways to reveal existence of the sterile neutrinos

- Detect flavor neutrinos and prove that transition (survival) probability depends on additional large mass-squared difference(s)
- Detect neutrinos via NC processes. If there are no transitions into sterile neutrinos no oscillations will be observed in NC processes

Indications in favor of transition into sterile states were obtained in short baseline LSND ($\bar{\nu}_\mu \to \bar{\nu}_e$), MiniBooNE ($\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$)), reactor ($\bar{\nu}_e \to \bar{\nu}_e$) and source ($\nu_e \to \nu_e$) experiments.

Best fit of the data $\Delta m^2_{14} \simeq 1 \text{ eV}^2$.

From data of recent experiments no indications in favor of transitions into sterile states were found and strong tension with old data were obtained

From analysis of the data of MINOS ($\nu_\mu \to \nu_\mu$) and DayaBay ($\bar{\nu}_e \to \bar{\nu}_e$) experiments large region of the LSND allowed region ($\Delta m^2_{14} < 0.8 \text{ eV}^2$) was excluded

From LSND and reactor experiments an allowed region for $\nu_\mu \to \nu_\mu$ transition can be found. This prediction was not confirmed by the IceCube experiment.

However, not all allowed regions were excluded. More than 20 new accelerator, reactor and source experiments on the search for sterile neutrinos are in preparation

For example, in SBN experiment (Fermilab) three detectors (600 m, 470 m, 110 m) will be used. Light sterile neutrino problem will be definitely solved

CONCLUSIONS

The Standard Model teaches us that the simplest possibilities are more likely to be correct. Massless two-component left-handed Weyl neutrinos is the simplest, most elegant and most economical possibility

Majorana neutrino mass term generated by the effective Lagrangian is the simplest possibility for neutrinos to be massive, naturally light and mixed

Small neutrino masses discovered by neutrino oscillation experiments signify that exist a new physics at a large scale with (very) heavy Majorana leptons which decay into leptons and Higgs boson. Decay of heavy Majorana leptons in the early Universe can explain the barion asymmetry of the Universe

Plausible scenarios, not proved

Observation of $0\nu\beta\beta$-decay would be important confirmation of such a picture

Nonobservation of sterile neutrinos would be a confirmation of an idea of simplicity applied to a beyond the SM physics