A fresh view of cosmological models describing very early Universe: general solution of dynamical equations with application to inflation.

Alexandre T. Filippov, JINR, Dubna (Helmholtz -- DIAS-TH School, 2016)

The dynamics of any spherical cosmology with a scalar field **(`scalaron')** coupling to gravity is described by the nonlinear second-order differential equations for two metric functions and the scalaron depending on the `time' parameter ***t***. The equations depend on the scalaron potential and on arbitrary gauge function that describes reparametrizations ***t --> f(t)***.

This dynamical system can be integrated for flat, isotropic models with very special potentials. But, somewhat unexpectedly, replacing the independent variable ***t*** by one of the **metric functions** allows us to **completely integrate the general spherical theory in any gauge and with arbitrary `potential'.** In this approach, inflationary solutions are easily identified, explicitly derived, and compared to the standard approximate expressions.

This approach can also be applied to intrinsically anisotropic models with a massive vector field (`vecton') as well as to some non-inflationary models.

More recent work on inflation and quantum cosmology is also very briefly reviewed.

Note: If we consider the origin of the universe `from nothing', we must be ready to treat it with some sort of a generalized quantum mechanics. The simplest idea – to consider small perturbations on a classical background (like in the theory of solitons) or quasiclassics. But, deeper approaches try to include a `back reaction' – to quantize the `background' (Q-gravity). Note that the standard quantization of the spherically symmetric gravity is still useful.

* Beginning of **Inflationary** models and of ideas on **Quantum Creation of Universe**

A. A. Starobinsky, "A New Type of Isotropic Cosmological Models Without Singularity", Phys.Lett. **B 91** (1980) 99.

V. F. Mukhanov and G. V. Chibisov, "Quantum fluctuations in nonsingular Universe", JETP Lett. **33** (1981) 532.

A. H. Guth, "The inflationary Universe: a possible solution to the horizon and flatness problem", Phys. Rev. **D 23** (1981) 347.

A. D. Linde, "Chaotic inflation", Phys. Lett. **B 129** (1983) 177.

A. D. Linde, *Particle physics and inflationary cosmology*, Harwood, Chur, Switzerland (1990); arXiv:hep-th/0503203 (2005).

V. Mukhanov, *Physical foundations of cosmology*, Cambridge Univ. Press, NY, 2005.

D. Gorbunov and V. Rubakov, *Introduction to the theory of the early universe: cosmological perturbations and inflationary theory*, World Sci. Publ. Co., Singapore, 2010.

A. Linde, "Inflationary cosmology after Planck 2013", arXiv:1402.0525 (2014).

J. Martin, "The observational status of cosmic inflation after Planck", arXiv:1502.05733

V. Mukhanov, Quantum cosmological perturbations arXiv:1303.3925 (2013).

Andrei Linde, **A brief history of the multiverse,** arXiv:1512.01203

Beginning of quantum cosmology

J.Halliwell, Introductory lectures on quantum cosmology, 1990 (arXiv:0909.2566)
(a thorough review of approaches to quantum cosmology, including work of J.Wheeler, B.De Witt, S.Hawking, J.Hartle, A.Vilenkin, V.Rubakov, etc.).
[see also *A.Vilenkin*, `Many worlds in one'(2006). @Rus.tr.]

Note the later papers of *J.Hartle* on quantum mechanics of the spacetime in cosmology:
(see, e.g., gr-qc/0602013, 1608.0414 and many other papers).
We only give a hint of these ideas.

Ideas on relation between entanglement in quantum theory and gravity

Raphael Bousso and Leonard Susskind, The multiverse interpretation of quantum mechanics, 1105.3796 (discussion of decoherence of q-states in modern approach)

Mark van Raamsdonk, Comments on quantum gravity and entanglement, 0907.2939
Building up spacetime with quantum entanglement, 1005.3035 (short essay)

Dmitri Fursaev, Proof of holographic formula for entanglement entropy, hep-th/0606184

Juan Maldacena and *L. Susskind,* Cool horizons for entangled black hole, 1306.0533

L.Susskind, Copenhagen vs **Everett**, **teleportation**, and **ER=EPR**, 1604.02589,

Two quotations

J. Hartle:

The founders of quantum theory thought that the indeterminacy of quantum theory "reflected the unavoidable interference in measurement dictated by the magnitude of the quantum of the action" (Bohr). But what then is the origin of quantum indeterminacy in a closed quantum universe which is never measured? Why enforce the principle of superposition in a framework for prediction of the universe which has but a single quantum state? In short, the endpoint of this journey of generalization forces us to ask John Wheeler's famous question, "How come the quantum?" [60].

Could quantum theory itself be an emergent effective theory? Many have thought so (Section 2). Extending quantum mechanics until it breaks could be one route to finding out. 'Traveler, there are no paths, paths are made by walking.'

J.Maldacena and L.Susskind: Cool horizons for entangled black holes, 1306.0533

General relativity contains solutions in which two distant black holes are connected through the interior via a wormhole, or Einstein-Rosen bridge. These solutions can be interpreted as maximally entangled states of two black holes that form a complex EPR pair. We suggest that similar bridges might be present for more general entangled states.

Generalizing Quantum Mechanics for Quantum Spacetime[1]

James B. Hartle

Three features of quantum theory are striking from the present perspective: its success, its rejection by some of our deepest thinkers, and the absence of compelling alternatives.

A Short History of Spacetime and Quantum Theory

Newtonian Physics	Fixed 3-d space and a single universal time t.	**Non-relativistic Quantum Theory:** The Schrödinger equation $$i\hbar(\partial\Psi/\partial t) = H\Psi$$ holds between measurements in the Newtonian time t.
Special Relativity	Fixed flat, 4-d spacetime with many diifferent timelike directions.	**Relativistic Quantum Field Theory:** Choose a Lorentz frame with time t. Then (between measurements) $$i\hbar(\partial\Psi/\partial t) = H\Psi\,.$$ The results are unitarily equivalent to those from any other choice of Lorentz frame

General Relativity	Fixed, but curved spacetime geometry	**Quantum Field Theory in Curved Spacetime:** Choose a foliating famliy of spacelike surfaces labeled by t. Then (between measurements) $$i\hbar(\partial\Psi/\partial t) = H\Psi\,.$$ But the results are *not* generally unitarily equivalent to other choices.
Quantum Gravity	Geometry is *not* fixed, but rather a quantum variable	**The Problem of Time:** What replaces the Schrödinger equation when there is no fixed notion of time(s)?
M-theory, Loop quantum gravity, Posets, etc.	Spacetime is not even a fundamental variable	?

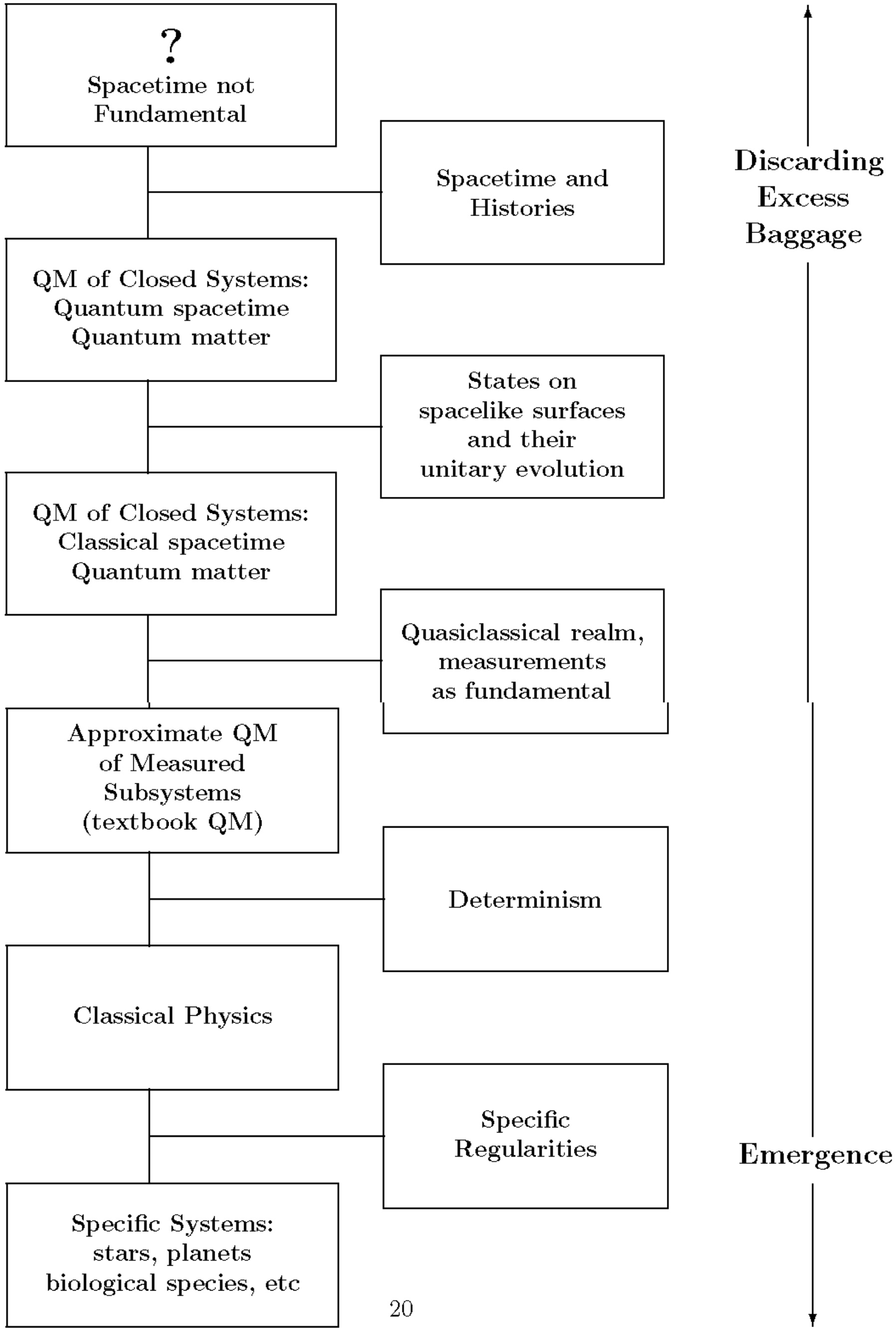
Here, we must clarify the meaning of what is the `fixed geometry'. Below we discuss the desciption of the spherical cosmology in terms of classical dynamics for the `matter' and `gravity' variables on equal footing and then `naively' quantize these variable using the standard quantum mechanical rules. In this example, the geometry is not comletely fixed (I propose to call it **`partially fixed').** We may also note that the Schroedinger equation is not the unique tool for quantizing and, probably, not the best in tis context.

SUPERSTRING vacua ?

NB: partly fixed spacetime (symm.): **metric&matter** are **quantum variables**

are miniuniverses in the MULTIVERSE closed ?

Are **BH** and **COSMOLOGIES** CLASSICAL **?** Possibly, only `partially' classical? Also, what about macroscopic quantum effects?
On **entanglement**?



Solving dynamical equation for black holes, cosmologies, and waves,
described by relativistic gravity coupled to scalarons (treated as gauge `finite DOF' systems).

**Quantizations of static states and Cosmoljgies (discrete gauge theories) [intr. by DG, API, ATF]*
(Constrained Dynamics). ATF, Vitttorio de Alfaro and M.Cavaglia 1994 - 1997
esp.: gr-qc/9508062 (BH), gr-qc/9502062 (cosm)

Gravity + scalaron* ***integrable models* *unifying BH+Cosm+`S-gravity' waves. Dynamical `portraits'. Search for 2-dim and 1-dim general exact solutions. 1996 - 2006*
The first papers on integrability : hep-th/9605008, gr-qc/9612068 (ATF). S-C -W: h-t/060527, 0612258 (ATF+VdA)

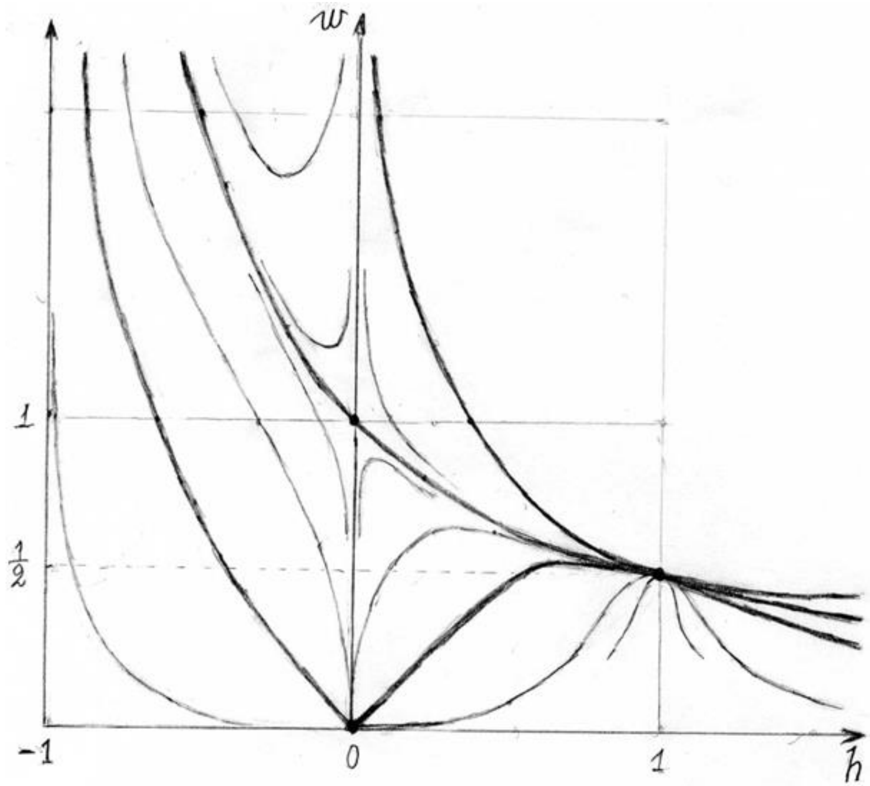
Multiexponential models (especially, integrable Toda-Liouville models describing (ATF+VdA)* ***Black Holes+Cosmology+Waves**) hep-th/09024445, 1302.6372 ,… 2006 - 2013*
(On Toda models see A.Leznov and M.Saveliev; in QG see P.Fre and A.Sorin)

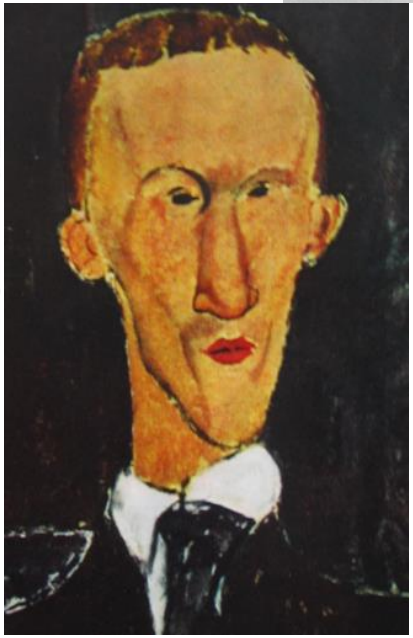
**Weyl-Eddington-Einstein (WEE) - inspired affine extension of gravity: GR+vecton.*
Intrinsically non-isotropic cosmology. Reduction to scalaron. 2008 - 2014 (ATF)
0812.2616v3; 1003.0782; 1302.6372; 1403.6815; …

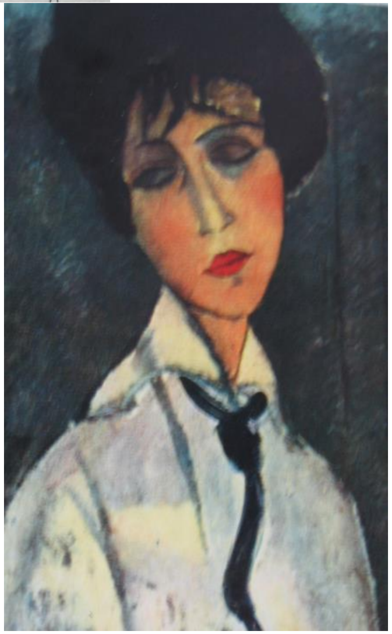
****Fresh view*** *of cosmology and of* ***inflation*** *by* ***solving the standard cosmological dynamics***
in an unusual formulation. Solving scalaron equations in case of curved space and anisotropy.
150601664v3; 1605,03948v2 2014 - 2016 (ATF)

****Lessons****: use of different gauges, drawing `portraits', clever choice of independent variables*
****Plans****: Inflation in WEE – inspired models. `S-gravity' waves. Study of bouncing cosmologies?*

The idea of the **topological portrait**
of dilaton gravity model coupled to scalaron







DG coupled to a massless scalaron.
h is the LC metric, w – the dilaton
h>0 - cosmologies, h<0 – static states,
[h = 0, w = 1] corresponds to horizon.

This portrait includes **both static and cosmological** solutions, and the most important information is on the structure of horizons and of other `fixed points'

$$\bar{\mathcal{L}}^{(1)} = \frac{1}{\bar{l}(\tau)}\left(-\dot{\phi}\,\dot{F} + \sum_{n=3}^{N} \dot{\psi}_n^2\right) + \bar{l}(\tau)\,\varepsilon\, e^F\, \bar{V}(\phi,\psi)$$

N-Liouville -- StaticCosmWave

Multiexp. with orthog. restrict.

$\bar{l}(\tau)$ =1

$$\bar{V}(\phi,\psi) = \sum_{n=1}^{N} g_n \exp \bar{q}_n^{(0)}\,, \quad \bar{q}_n^{(0)} = \bar{a}_n\,\phi + \sum_{m=3}^{N} \psi_m\, a_{mn}\,,$$

$$\sum_{n=1}^{N} \gamma_n = 0\,. \quad \sum_{l=3}^{N} a_{lm}\, a_{ln} - 2\,(\bar{a}_m + \bar{a}_n) = \bar{\gamma}_n^{-1}\,\delta_{mn}\,.$$

$$\psi_n = \sum_{m=1}^{N} \epsilon_n a_{nm} \gamma_m\, q_m \quad, \epsilon_1 = -1,\ \epsilon_{n>1} = 1 \qquad X_n(u,v) \equiv \exp[\,-q_n(u,v)/2\,]\,.$$

$$X_n(u,v) = \frac{1}{2\sqrt{\mu_n\nu_n}}\Big[C_{11}^{(n)}\, e^{-\mu_n u - \nu_n v} + C_{22}^{(n)}\, e^{\mu_n u + \nu_n v} + $$
$$+ C_{12}^{(n)} e^{-\mu_n u + \nu_n v} + C_{21}^{(n)} e^{\mu_n u - \nu_n v}\Big],$$

$$C_{11}^{(n)}\, C_{22}^{(n)} - C_{12}^{(n)}\, C_{21}^{(n)} = -\frac{1}{2}\tilde{g}_n$$

$$\sum_{n=1}^{N} \gamma_n\, \mu_n^2(u) = 0 = \sum \gamma_k \nu_k^2$$

$$X_n = \frac{1}{\sqrt{\mu_n\nu_n}}\left\{C_n^+ \cosh\left(\lambda_n\, r + \bar{\lambda}_n\, t + \delta_n^+\right) + C_n^- \cosh\left(\lambda_n\, t + \bar{\lambda}_n\, r + \delta_n^-\right)\right\}$$

This solution describes static states, cosmologies and waves, including soliton-like configurations. Similar results in the Toda-Liouville case.

$$\mu_n = \lambda_n + \bar{\lambda}_n,\ \nu_n = \lambda_n - \bar{\lambda}_n$$

Begin with the paper: ATF, arXiv:1506.01664 v.3

our aim is to spell out the mathematical structure

first using *differentiable maps* $\psi(\alpha)$ *and* $\alpha(\psi)$

which we call the **PORTRAITS of COSMOLOGY**

having in mind that the portraits may be more fundamental than solutions

Fundamental functions are: $\chi(\alpha) \equiv d\psi/d\alpha$ and $\bar{\chi}(\psi) \equiv d\alpha/d\psi$

Gauge invariant *equation for* $\chi^2(\alpha)$ *can be explicitly solved*

if we formally replace $v(\psi)$ *by* $\bar{v}(\alpha) \equiv v[\psi(\alpha)]$

In this paper, the main goal is to derive the most general solutions of general homogeneos isotropic models of the scalaron cosmologies.

The next paper 1605.03948 v.2 solves this problem for non-isotropic universes:
weak anisotropy (scalaron cosmology), **essential** anisotropy (vecton cosmology).

Spherical = perfect!?

SEPARATE and further REDUCE !

4D spherically reduced metric: $e^{2\alpha}dr^2 + e^{2\beta}d\Omega^2(\theta,\phi) - e^{2\gamma}dt^2 + 2e^{2\delta}drdt$

Lagrangian in 2D (duality of ***r*** and ***t***):

$$e^{\alpha+2\beta-\gamma}(\dot{\psi}^2 - 2\dot{\beta}^2 - 4\dot{\beta}\dot{\alpha}) - e^{-\alpha+2\beta+\gamma}(\psi'^2 - 2\beta'^2 - 4\beta'\gamma')$$
$$- e^{\alpha+2\beta+\gamma}V(\psi) + 2\bar{k}\, e^{\alpha+\gamma}$$

The momentum constraint:

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' = \frac{1}{2}\dot{\psi}\psi'.$$

(to account for the *drdt* terms in the metric)

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r), \quad \gamma = \gamma_0(t) + \gamma_1(r).$$

separation of ***r*** and ***t***

If r.h.s. of Momentum Constraint **= 0** we find simple separation conditions.

1. FLRW cosmology:

>> @ isotropic reduction   isotropy condition   **3**-dimensional curvature

$$\dot{\alpha} = \dot{\beta}\,, \quad \gamma' = 0\,, \quad \beta_1'' + \bar{k}\,e^{-2\beta_1} = 0\,, \quad 2\beta_1'' + 3\beta_1'^2 - \bar{k}\,e^{-2\beta_1} = 3k$$

$$\beta_1'^2 - \bar{k}\,e^{-2\beta_1} = k$$

<< homogeneity & isotropy condition

$$\mathcal{L}^{(1)} = 6\bar{k}e^{\alpha+\gamma} - e^{2\beta}[e^{\alpha+\gamma}(V + 2\Lambda) - e^{\alpha-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha} - \dot{\psi}^2)]$$

(Here must be *k*, without `bar')

effective cosmological Lagrangian

$$\mathcal{L}^{(2)} = e^{3\alpha-\gamma}(\dot{\psi}^2 - 6\dot{\alpha}^2) - e^{3\alpha+\gamma}v(\psi) - 6ke^{\alpha+\gamma}$$

$\gamma + c\,\alpha = 0.$ $\quad \mathcal{L}_c = e^{(3+c)\alpha}(\dot{\psi}^2 - 6\dot{\alpha}^2) - e^{(3-c)\alpha}v(\psi) - 6ke^{(1-c)\alpha}$

$$\mathcal{H}_c \equiv \eta^2 - 6\,\xi^2 + e^{-2c\alpha}\,v(\psi) + 6k\,e^{-2(1+c)\alpha} = 0\,;$$

momentum - like variables $\eta\,,\xi$ $\qquad \dot{\psi} = \eta\,, \quad \dot{\alpha} = \xi$

$$2\,\dot{\eta} + 2(3+c)\,\eta\,\xi + e^{-2c\alpha}\,v'(\psi) = 0$$

$$6\,\dot{\xi} + (3+c)\,\eta^2 + c\,e^{-2c\alpha}\,v(\psi) + (1+c)\,6k\,e^{-2(1+c)\alpha} = 0$$

Equation for the **Hubble parameter** $H(\alpha) \equiv \bar{\xi}(\alpha)$

$$2\,\dot{\xi} + \eta^2 + 2c\,\xi^2 + 2ke^{-2(1+c)\,\alpha} = 0$$

Independence of *z* on potential

$$\frac{dz}{d\alpha} + \bar{\eta}^2 = 0\,; \qquad z \equiv \bar{\xi}^2 - ke^{-2\alpha}$$

***t*-reparametrizing**

$$d\tau \equiv e^{-c\alpha}\,dt\,, \qquad d/dt \equiv e^{-c\alpha}\,d/d\tau$$

Corresponding gauge transformations, invariance

$$\eta \equiv e^{-c\alpha}\,\bar{\eta}\,, \qquad \xi \equiv e^{-c\alpha}\,\bar{\xi}$$

$$d\psi/d\tau = \bar{\eta}\,, \qquad 2\,d\bar{\eta}/d\tau + 6\,\bar{\eta}\,\bar{\xi} + v'(\psi) = 0\,,$$

$$d\alpha/d\tau = \bar{\xi}\,, \qquad 2\,d\bar{\xi}/d\tau + \bar{\eta}^2 + 2k\,e^{-2\alpha} = 0\,.$$

$$\bar{\mathcal{H}} \equiv e^{2c\alpha}\mathcal{H}_c = \bar{\eta}^2 - 6\,\bar{\xi}^2 + v(\psi) + 6k\,e^{-2\alpha} = 0\,.$$

Simple examples from upside down standpoint

$$v = v_0 \equiv 2\Lambda \qquad \eta = \eta_0 \exp[-(3+c)\alpha] \qquad t_c = \int d\alpha/\dot{\alpha}(\alpha)$$

$$6\dot{\alpha}^2 e^{2c\alpha} = v_0 + 6k\, e^{-2\alpha} + \eta_0^2\, e^{-6\alpha}$$

$$\text{when } c = -3 \qquad \psi = \eta_0\,(t - t_0)$$

$$\eta_0^2\, e^{-6\alpha} = v_0 \sinh^2[\sqrt{3/2}\,\eta_0\,(t-t_0)]$$

Portrait: $$= v_0 \sinh^2(\sqrt{3/2}\,\psi)$$

K=0 **exponential case,** ***c=-3:*** $$v = v_0 e^{g\psi} \qquad \psi + g\alpha = C_0(t - t_0)$$

$$2\,\ddot{\psi} + e^{6\alpha}\, v'(\psi) = 0\,, \qquad 2\,\ddot{\alpha} - e^{6\alpha}\, v(\psi) = 0$$

$$e^{-(g\psi + 6\,\alpha)} = 2\bar{g}C_1^{-2} \cosh^2[\psi + g\alpha + C_0(t_0 - t_1)/2]$$

Integrable bi--Liouville

$$v \;=\; v_1\, e^{g_1 \psi} \;+\; v_2\, e^{g_2 \psi}$$

$$6\alpha + g_i \psi = \mu_i\, \psi_i \qquad\qquad g_2 = 6/g_1$$

$$\mathcal{L}_c = -\dot{\psi}_1^2 + \dot{\psi}_2^2 + v_1\, e^{\mu_1 \psi_1} + v_2\, e^{\mu_2 \psi_2}$$

How to derive the potential in simple cases?

$$2\,\dot{\xi} + \eta^2 + 2k e^{-2\alpha} = 0\,, \quad 2\,\dot{\eta} + 6\,\eta\,\xi + v'(\psi) = 0\,,$$

$$v(\psi) = 6\,\xi^2 - \eta^2 - 6\,k\,e^{-2\alpha}\,.$$

The first eqn. can be solved if $\quad 1.\ \dot{\xi} = C_0$ or $2.\ \dot{\eta} = C_0.$

When $C_0 = 0$ $$\eta = k_0\, e^{-\alpha(\tau)}\,, \qquad (k_0^2 \equiv -2k)$$

Hubble function $$\dot{\alpha}(\tau) \equiv \xi = \xi_0\,, \qquad \alpha(\tau) = \xi_0\,(\tau - \tau_0)\,,$$

$$\chi(\alpha) = \frac{d\psi}{d\alpha} = \frac{\eta}{\xi} = \frac{k_0}{\xi_0}\, e^{-\alpha}\,; \qquad \chi:\ \alpha \mapsto \psi,$$

portrait $$\tilde{\psi} \equiv (\psi - \psi_0) = \int \chi(\alpha) = -\frac{k_0}{\xi_0}\, e^{-\alpha}$$

$$\bar{v}(\alpha) \;=\; 6\,\xi_0^2 \;+\; 2\,k_0^2\, e^{-2\alpha} \;=\; \bar{v}\,[\alpha(\psi)]$$

potential $$=\; 6\,\xi_0^2 \;+\; 2\,\xi_0^2\,(\psi - \psi_0)^2 = v(\psi)\,.$$

Gauge-invariant equations for differentials of the map

Definiitions

$$\chi(\alpha) \equiv d\psi/d\alpha \equiv \dot\psi/\dot\alpha \equiv \eta/\xi\,, \qquad \bar\chi(\psi) \equiv d\alpha/d\psi \equiv \dot\alpha/\dot\psi \equiv \xi/\eta$$

$$\frac{d}{dt} = \dot\alpha\frac{d}{d\alpha} = \xi\frac{d}{d\alpha} = \dot\psi\frac{d}{d\psi} = \eta\frac{d}{d\psi} \qquad\qquad \frac{d}{d\psi} = \frac{d\alpha}{d\psi}\frac{d}{d\alpha} = \bar\chi(\psi)\frac{d}{d\alpha}$$

$$2\frac{d\chi}{d\alpha} \equiv \xi^{-3}(\xi\dot\eta - \dot\xi\eta) = (\chi^2-6)\left(\chi + [\ln v(\psi)]'\right) +$$

$$+\,2k\,\xi^{-2}e^{-2(1+c)\alpha}\left(\chi + 3\,[\ln v(\psi)]'\right)$$

equation for $\chi(\alpha)$ $\qquad \bar v(\alpha) \equiv v[\psi(\alpha)] \qquad \bar l'(\alpha) \equiv [\ln \bar v(\alpha)]'$

$$\frac{d\chi^2}{d\alpha} = (\chi^2-6)\left(\chi^2 + \bar l'(\alpha)\right) + \frac{2k}{\xi^2(\alpha)}\,e^{-2(1+c)\alpha}\left(\chi^2 + 3\,\bar l'(\alpha)\right)$$

The Solution for $k=0$

$$\chi^2(\alpha) = 6 - e^{6\alpha}\,\bar{v}(\alpha)\Big[\,C_0 + \int e^{6\alpha}\bar{v}(\alpha)\,\Big]^{-1}$$

$$\bar{v}(\alpha) = v_0 \exp(g\alpha):$$

$$\chi^2\,(\alpha) = 6 - (g+6)\,[\,1 + C_1 e^{-(6+g)\alpha}\,]^{-1}$$

Now it is possible to derive $\psi(\alpha)$ by integrating $\psi'(\alpha) \equiv \chi(\alpha)$

along the 'physical' paths

complex α-plane and thus to find the 'portrait'

Once we know this expression we can reconstruct the full cosmology iusing the expression for the Hubble function H(t).

$$d\alpha \equiv H d\tau$$

$$H(\alpha) \equiv \bar{\xi}(\alpha) \qquad \bar{\xi}^2(\alpha) \;=\; e^{-\int \chi^2(\alpha)}\Big[C_0 - 2k \int e^{-2\alpha + \int \chi^2(\alpha)}\,\Big]$$

Equation for $\bar{\chi}(\psi)$

Generally not integrable

$$2\,\frac{d\bar{\chi}}{d\psi} = (6\,\bar{\chi}^2 - 1)\,(1 + \bar{\chi}\,l'(\psi))\,;$$

$$x = \sqrt{2/3}\,\psi,\ \ z(x) = \sqrt{6}\,\bar{\chi}.$$

$$dz/dx = (z^2 - 1)(z\,u(x) + 1)$$

$$u(x) \equiv d\ln\sqrt{v}/dx$$

Asymptotic at large *x*

$$\bar{\chi}(\psi)/\sqrt{6} \equiv z(x) = -[\,1 - e^{-2\,(x+c_0)}\,v(x)\,]\,[\,1 + e^{-2\,(x+c_0)}\,v(x)\,]^{-1}$$ **+**

can be derived by the important Ansatz

$$z(x) = -\frac{1 - \varepsilon e^{-2y}}{1 + \varepsilon e^{-2y}} = -\tanh^{\varepsilon} y(x)$$

$$z(x) = -\frac{1 - \varepsilon e^{-2y}}{1 + \varepsilon e^{-2y}} = -\tanh^{\varepsilon} y(x)$$

Then

$$dy/dx = 1 - \, u(x) \, \tanh^{\varepsilon}(y)$$

that is the main equation in *psi* version with which we can find asymptotic and power series expansions for z(x)

Depends on the`Logarithmic' potential

$$u(x) = \, \tilde{v}'(x)/2\, \tilde{v}(x)$$

It is important that for positive, `inflationary' potentials Z(x) can be large for large x.

$$\varepsilon = \, \mathrm{sign}(1 - z^2) = \, \mathrm{sign}(-v)$$

Taking the popular 'cosmological' potential $v(\psi) = v_0 \, \psi^2$ we find

the example of the asymptotic portrait

$$\sqrt{6}\, \alpha(\psi) = \psi_0 - \psi + \bar{C}_0 \, [1 - (3\, \psi^2 + \sqrt{6}\, \psi + 1) \exp(-\sqrt{6}\, \psi) + \ldots]$$

$$\frac{d\eta^2}{d\alpha} + 2(3+c)\,\eta^2 + e^{-2c\alpha}\,\bar{v}'(\alpha) = 0 \qquad y(\alpha) = e^{2c\alpha}\,\eta^2$$

define

$$\frac{d\xi^2}{d\alpha} + 2c\,\xi^2 + \eta^2 + 2k\,e^{-2(1+c)\alpha} = 0 \qquad x(\alpha) = e^{2c\alpha}\,\xi^2.$$

In **alpha** picture we have two equations and one constraint

$$y'(\alpha) + 6y(\alpha) + \bar{v}'(\alpha) = 0$$

$$x'(\alpha) + y(\alpha) + 2k\,e^{-2\alpha} = 0$$

α-version.

$$y(\alpha) - 6\,x(\alpha) + \bar{v}(\alpha) + 6k\,e^{-2\alpha} = 0$$

Exact solution

$$y(\alpha) = 6\,e^{-6\alpha} \int e^{6\alpha}\,\bar{v}(\alpha) - \bar{v}(\alpha)$$

For this version the solution of the dynamics is trivial in isotropic case with arbitrary ***k.*** Anisotropic case is more difficult, but for ***k***=0 we find explicit general solution. In general – perturb. in anisotropy.

$x(\alpha)$ is immediately derived from the constraint

If we know ***X*** – we trivially find the potential

$$I(\alpha) \equiv [\, C_0 + \int e^{6\alpha} \bar{v}(\alpha)\,]$$

$$x(\alpha) = e^{-6\alpha} I(\alpha) \, + \, k\, e^{-2\alpha} \, .$$

$$\dot{\alpha} = e^{-c\alpha} \sqrt{x(\alpha)} =$$

Exact result!

$$= e^{-(1+c)\alpha} \, [\, e^{-4\alpha} I(\alpha) \, + \, k\,]^{1/2}$$

EXACT FORMULA for
any potential and curvature

$$\chi^2(\alpha) \equiv y(\alpha)/x(\alpha) =$$

$$[\, 6\, e^{-6\alpha} I(\alpha) \, - \, \bar{v}(\alpha)\,]\, [\, e^{-6\alpha} I(\alpha) \, + \, k\, e^{-2\alpha}\,]^{-1}$$

A fresh look at inflation

We begin with the standard conditions for inflation and the parameters accessible to measurements. In our language the obvious necessary condition for inflation is $\bar{\chi}(\psi) \gg 1$ on a small interval of ψ, or, equivalently, $\chi^2(\alpha) \ll 1$ on the corresponding large interval $(\alpha_i < \alpha < \alpha_f)$, where $\alpha_f - \alpha_i = N \sim 50$ is the so-called number of e-foldings (see [3], [15]). Using equations (11) and (63) with $k = 0$ [16] we derive the exact relations

$$\underline{-2\,\dot{\xi}/\xi^2 \equiv -x'(\alpha)/x(\alpha)} = y(\alpha)/x(\alpha) \equiv \chi^2(\alpha) \equiv 2\,\hat{\epsilon}\,.$$

$$\hat{r} \equiv \dot{\psi}^2/v(\psi) \equiv y(\alpha)/\bar{v}(\alpha) = \frac{\hat{\epsilon}}{3}\,(1 - \hat{\epsilon}/3)^{-1} \simeq \frac{\hat{\epsilon}}{3}$$

In the standard approach to the inflationary cosmology one introduces one more `parameter':

$$-2\,\dot{\eta}/\eta\xi \equiv -y'(\alpha)/y(\alpha) \equiv 2(\hat{\eta} - \hat{\epsilon})\,, \qquad \hat{\eta} = \chi^2(\alpha) - \chi'(\alpha)/\chi(\alpha)$$

$$6x(\alpha) \;=\; 6e^{-6\alpha} I(\alpha) = C_0 e^{-6\alpha} + \sum_0^N (-6)^{-n} \bar{v}^{(n)}(\alpha) + (-6)^{-(N+1)} e^{-6\alpha} \int_{-\infty}^{\alpha} e^{6\alpha} \bar{v}^{(n+1)}(\alpha)$$

This is the ***crucial formula*** for our **INFLATIONARY PERTURBATION THEORY**

$$\hat{r}(\alpha) \equiv \frac{y(\alpha)}{\bar{v}(\alpha)} = 6\,\frac{x(\alpha)}{\bar{v}(\alpha)} - 1 =$$

for the **inflationary** solution with

$$C_0 = 0$$

we find the **perturbative expansion** (by iterations)

$$= \frac{\chi^2}{6}\left(1 - \frac{\chi^2}{6}\right)^{-1} = \sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\alpha)}{6^n\,\bar{v}(\alpha)}$$

$$\chi = -l'(\psi)\left(1 - \frac{\chi^2}{6}\right)\left[1 + \sum_2^\infty \frac{D_n * v(\psi)}{(-6)^{n-1} v\, l'}\right] \qquad d_\alpha^n = \chi D_n$$

$$d_\alpha \equiv \chi d_\psi$$

$$d_\alpha^2 = \chi[\,\chi d_\psi^2 + (d_\psi \chi) d_\psi]$$

<<definitioins>>

$$\frac{D_2 * v}{v\, l'(\psi)} = \frac{\chi}{l'}[\,l'' + (l')^2] + (d_\psi \chi)$$

The **corrections** to the standard inflationary expression

$$\chi_2 = -l'(\psi)\left[1 - \frac{1}{6}(l')^2\right]\left\{1 + \frac{1}{6}[2l'' + (l')^2]\right\} = -l'(\psi)\left[1 + \frac{1}{3}l'' + O(\chi_1^4)\right]$$

$$\hat{\eta}_2 = \chi_2^2 - d_\psi\,\chi_2 = \hat{\eta}_1 + (1/3)\,[\,2\,l^{(2)}\,\hat{\eta}_1 + l^{(1)} l^{(3)} - (l^{(2)})^2]$$

Our corrections to the number of e-foldings

$$N_e = \int_{\psi_i}^{\psi_f} d\psi\, \bar{\chi}(\psi) = \int_{\psi_f}^{\psi_i} \frac{d\psi}{l'(\psi)} \left[1 - \frac{1}{3} l'' + O(\chi_1^4)\right] = N_e^{(0)}(\psi) + \frac{1}{3} \ln[\, l'(\psi)]_i^f$$

$$N_e = [\, \psi^2/4N + \ln \psi/3\,]_f^i \text{ when } v = v_0 \psi^{2N}$$

Possible relation to the ψ **version:** $z(x) = -\tanh^{\varepsilon} y(x)$

$$dy/dx = 1 - u(x)\, \coth(y) \qquad u(x) = \tilde{v}'(x)/2\, \tilde{v}(x) \qquad z = \sqrt{6}\, \bar{\chi}\,, \quad x = \sqrt{3/2}\, \psi$$

asymptotic expansion of $y(x)$ in powers of $1/x \sim u(x) \sim l'$ for $v = v_0 x^{2N}$:

$$y(x) = \sum_0^{\infty} y_{2n+1}\, x^{-(2n+1)} \qquad y_1 = N\,, \quad y_3 = \frac{1}{3} N^3 - N^2, \ldots,$$

if $u(x) = \sum_1^{\infty} u_n\, e^{-ngx}$ $\qquad \bar{y} = \sum_1^{\infty} y_n\, e^{-ngx}$

In the recent paper **1605.03948v2** we derive the general gauge independent solution for non-isotropic scalar cosmologies and applied our approacg to the vector one, which is an approximation to the remake of the WEE affine gravity

The dynamics of any spherical cosmology with a scalar field coupling to gravity is described by the nonlinear second-order differential equations for two metric functions and the scalaron depending on the `time' parameter. The equations depend on the scalaron potential and on the arbitrary gauge function that describes time parameterizations. This dynamical system can be integrated for flat isotropic models only with very special potentials. But, somewhat unexpectedly, replacing the time variable by one of the metric functions allows us to completely integrate the general spherical theory in any gauge and with apparently arbitrary potentials. The main restrictions on the potential arise from positivity of the derived analytic expressions for the solutions, which are essentially the squared canonical momenta. An interesting consequence is emerging of classically forbidden regions for these analytic solutions. It is also shown that in this rather general model the in inflationary solutions can be identifed, explicitly derived, and compared to the standard approximate expressions. This approach can be applied to intrinsically anisotropic models with a massive vector field (`vecton')as well as to some non-inflationary models.

The approximate cosmological Lagrangian can be written in the form ($A \equiv A_z(t)$):

$$\mathcal{L}_c = e^{2\beta}\left[e^{-\alpha-\gamma}\dot{A}^2 - e^{-\alpha+\gamma}m^2A^2 - e^{\alpha+\gamma}(V+2\Lambda) - e^{\alpha-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha} - \dot{\psi}^2)\right] + 6k\,e^{\alpha+\gamma}\,.$$

To write the equations of motion in a more compact form, we introduce the notation

$$3\,\rho \equiv (\alpha+2\beta)\,, \quad 3\,\sigma \equiv (\beta-\alpha)\,, \quad 3\,A_{\pm} = e^{-2\rho+4\sigma}(\dot{A}^2 \pm m^2 e^{2\gamma} A^2)\,, \quad v(\psi) \equiv V(\psi)+2\Lambda\,.$$

Then the exact Lagrangian for vecton-scalar cosmology is:

$$\mathcal{L}_c = e^{3\rho-\gamma}(\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2)\,e^{3\rho+\gamma} - v(\psi) + 6k\,e^{\alpha+\gamma} + e^{3\rho-\gamma}\,3\,A_-\,.$$

Here e^{γ} is the Lagrangian multiplier, variations of which yield the energy constraint:

$$\mathcal{H}_c \equiv \dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2 + e^{2\gamma}\,V - 6k\,e^{-2(\rho+\sigma-\gamma)} + 3\,A_+ = 0\,.$$

This and the following equations denoted by (S, V; k) generate 4 reduced systems: (S, 0; 0), (S, 0; k), (0, V; 0), (0, V; k). At the moment, the first 3 are well understood (but generally not integrable).

matter $$\ddot{\psi} + (3\dot{\rho} - \dot{\gamma})\dot{\psi} + \frac{1}{2}e^{2\gamma}\,v'(\psi) = 0; \qquad \ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\,m^2 A = 0\,,$$

metric $$4\ddot{\rho} + 6\dot{\rho}^2 - 4\dot{\rho}\dot{\gamma} - 6\dot{\sigma}^2 + \dot{\psi}^2 - e^{2\gamma}\,v(\psi) = -2k\,e^{-2(\rho+\sigma-\gamma)} + A_-\,,$$

aniso tropy $$\ddot{\sigma} + 3\,\dot{\sigma}\dot{\rho} - \dot{\sigma}\dot{\gamma} = -k\,e^{-2(\rho+\sigma-\gamma)} + A_-\,,$$

$$\ddot{\sigma} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\sigma} = k\,e^{2\gamma-2(\rho+\sigma)} + A_- \,;$$

$$\ddot{\psi} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\psi} + e^{2\gamma}\,v'(\psi)/2 = 0;$$

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\,m^2 A = 0\,,$$

$$\ddot{\rho} + 3\,\dot{\sigma}^2 - \dot{\rho}\dot{\gamma} + \dot{\psi}^2/2 = -k\,e^{2\gamma-2(\rho+\sigma)} - (3A_+ + A_-)/4$$

Introduce momenta and write the Hamiltonian, which defines the general WdW equation

$$(p_\rho,\, p_\psi,\, p_\sigma) = 2\,e^{3\rho-\gamma}(-6\dot{\rho},\, \dot{\psi},\, 6\,\dot{\sigma})\,, \quad p_A = 2\,e^{\rho+4\sigma-\gamma}\,\dot{A}$$

$$\frac{1}{24}(6p_\psi^2 + p_\sigma^2 - p_\rho^2 + 6p_A^2\,e^{2\rho-4\sigma})\,e^{\gamma-3\rho} + v(\psi)\,e^{\gamma+3\rho}$$

$$+ 6k\,e^{\gamma+\rho-2\sigma} + m^2 A^2\,e^{\gamma+\rho+4\sigma} = 0$$

In A=0 case introduce the definitions similar to the isotropic case

$$(\dot\rho,\ \dot\psi,\ \dot\sigma)\ \equiv [\,\xi(\rho)\,,\eta(\rho)\,,\zeta(\rho)\,] =$$

$$= [\,\xi(\rho),\ \xi\,\psi'(\rho),\ \xi\,\sigma'(\rho)\,]\ \equiv\ \xi(\rho)[\,1,\ \chi(\rho),\ \omega(\rho)]$$

$$v'(\psi) = \frac{dv}{d\psi} = \frac{dv}{d\rho}\frac{d\rho}{d\psi} = \bar v'(\rho)\frac{\xi}{\eta} = \bar v'(\rho)/\chi(\rho)$$

Define the gauge independent (invariant) functions

$$\mathbf{S}(\rho) \equiv [\,x(\rho),\ y(\rho),\ z(\rho)\,] \equiv\ \exp\,(6\rho - 2\gamma)\,[\,\xi^2(\rho)\,,\eta^2(\rho),\ \zeta^2(\rho)\,]$$

Most important! $\mathbf{S}(\rho) > 0$

The gauge invariant equations in the general sclaron theory

$$y'(\rho) + V'(\rho) - 6V(\rho) = 0\,, \qquad V \equiv e^{6\rho}\,\bar{v}(\rho)$$

$$x'(\rho) - V(\rho) = 4k\,e^{4\rho-2\sigma}$$

$$z'(\rho) = 2k\,\rho^{\rho-2\sigma}\,\sigma'(\rho)$$

$$6\,x(\rho) \;=\; y(\rho) + V(\rho) \;+\; 6z(\rho) \;+\; 6k\,e^{4\rho-2\sigma}$$

For large ρ we can first derive approximation for x and y and then find σ

Note that x, y, z must be positive that is not garanteed for the general solution

The exact general solution for *y* and the general expression for *x* are

$$y(\rho) = 6\left(C_y + \int V(\rho)\right) - V(\rho)\,, \qquad x(\rho) = \left(C_x + \int V(\rho)\right) + 4k\int e^{4\rho - 2\sigma(\rho)}$$

The exact expression for *z* is obtained from the constraint

$$6\,x(\rho) \;=\; y(\rho) + V(\rho) \;+\; 6z(\rho) \;+\; 6k\,e^{4\rho - 2\sigma}$$

$$z \equiv x(\rho)\,\sigma'^{2}(\rho) = C_z + k\left[4\int e^{4\rho-2\sigma(\rho)} - e^{4\rho-2\sigma(\rho)}\right]$$

$$\equiv\; C_x - C_y + 2k\int \sigma'(\rho)\,e^{4\rho-2\sigma(\rho)}\,.$$

The conditions for positivity are

$$6I(\rho) - V(\rho) > 0\,, \qquad I(\rho) + k\,e^{4\rho} > 0 \qquad\qquad I(\rho) \equiv \int_{-\infty}^{\rho} V(\rho)$$

1) $k = 0,\ \sigma' = 0,\ C_z = 0$ – isotropic FLRW cosmology

2) $k = 0,\ \sigma' \neq 0,\ C_z \neq 0$ anisotropic solution

becomes isotropic at large ρ, i.e. $\sigma' \to 0$ for $\rho \to +\infty$

3) $k \neq 0,\ \sigma' = 0,\ C_z = 0$ – isotropic cosmology, not of FLRW type

4) $k \neq 0,\ \sigma' \neq 0$ – general anisotropic solution - can vanish at $\rho \to +\infty$.

Most general definition of **INFLATION**:

$\omega^2 \ll 1$ (small anisotropy) and $\chi^2 \leq 1$

$$\chi^2 = -\bar{l}'(\rho) + \mathrm{o}(\bar{l}') = -\chi\, v'(\psi)/v(\psi) + \ldots$$

$$\chi = -v'(\psi)/v(\psi) + \ldots \equiv -l'(\psi) + \mathrm{o}(l')$$

THE

END

SHORT SUMMARY

1. The *cosmological dynamical equations* are formulated *in different gauges and versions*. We illustrate relations between them on simple solutions and by integrable models.
2. The *general properties of gauge independent* χ*-equations* (26)-(27), describing the main (α, ψ) portraits of isotropic cosmologies, are established in α and ψ versions.
3. Equations (29)-(30) allow us to derive the complete solution if $\chi(\alpha)$ or $\bar{\chi}(\psi)$ are known. Taking into account equations (31) we can in addition derive $\bar{v}(\alpha)$ or $v(\psi)$.
4. We discussed *different ways to determine cosmologies not using potentials*. A most natural one seems to first derive $\chi^2(\alpha)$ using (32), with the Hubble function $\xi^2(\alpha)$ as an input.
5. Although the χ-equations depend only on $v'(\psi)/v(\psi)$ and are thus insensitive to the sign of $v(\psi) \equiv \bar{v}(\alpha)$, *this sign is critically important for global properties of the solutions*. From (28), (44), (75) it follows that the solutions in the intervals with $v(\psi) > 0$ are isolated from those in the intervals with $v(\psi) < 0$ and must be studied separately.
6. We mostly considered potentials not changing the sign and studied in detail models with *positive potentials for which inflationary scenarios are natural.* We also can use and actually used our solutions and their expansions near the points ψ_0 where $v(\psi_0) = 0$ and thus $v'(\psi)/v(\psi)$ behaves as $(\psi - \psi_0)^{-1} \to \pm\infty$. This is a problem in the ψ-version because $(\chi^2 - 6)$ may change the sign with the potential, as follows from (44), (75). But in the α-version it is no problem at all, as can be seen from from expression (62) for $\chi^2(\alpha)$.

New results: anysotropic case completely solved for k=0. In general, it is analytically solved in case of small anisotropy. The result is completely gauge independent.

7. Probably, the *most important results* are presented in Section 4, where we have found the *exact solution of all equations for arbitrary* $\bar{v}(\alpha)$ *and* k. The necessary condition for inflation is $\chi^2(\alpha) < 6$ ($6\bar{\chi}^2(\psi > 1)$. To derive from $\chi(\alpha)$ standard inflationary scenarios we first suppose that the spatial curvature vanishes, $k = 0$. Then, by fixing the arbitrary integration constant, $C_0 = 0$, we preserve the v-scale invariance of inflationary solution χ and derive its expansion from Eq.(82) as a sum, the n-th term of which for $n \geq 1$ has the form:

$$-l'(\psi) \sum_k c_n(k_1, ..., k_{2n}) \prod_{i=1}^{2n} [\, l^{(i)}(\psi)\,]^{k_i}\,, \quad \text{where } \sum_i ik_i = 2n\,, \;\; k_i \geq 0\,.$$

This *inflationary perturbation expansion* can be obtained by the well-defined recursive algebraic iterations and gives higher-order corrections to the inflationary parameters $\hat{\epsilon}$, $\hat{\eta}\,, N_e$.
8. When $v(\psi) < 0$ and thus $\chi^2(\alpha) > 6\,,\; 6\bar{\chi}^2(\psi) < 1$, it is also convenient to use expansions of $\bar{\chi}(\psi)$ when it is small or close to $1/6$. In the last case we have derived *asymptotic approximation* (54) for $\psi \to \infty$ valid *for a broad class of potentials* $v(\psi)$. We should mention an interesting one-parameter class of 'bouncing' solutions (see (59) and (60)), which exist when $v'(\psi)/v(\psi) \sim 1/\psi$, and a special solution (56) that probably is a separatrix. The *global picture of solutions* with such properties are of great interest for ekpyrotic-bouncing scenarios and *must be studied in future.*

I believe that *visualization of these structures*, drawing the (α, ψ) portraits, and using perturbative expansions for concrete inflationary, ekpyrotic, bouncing and other, more strange isotropic cosmologies may stimulate their better theoretical understanding.

ATF: arXiv:1506.01664 v.3

The general gauge invarinace and g.f. were introduced and discussed in the work on `discrete strings' (`86-`96)...
gauging (super)canon. symm

2 Dynamical equations

2.1 Gauges and gauge independence . . . .

2.1.1 Gauge-invariant equations and general remarks .

2.2 Simple examples from 'upside-down' standpoint . . . . .

2.2.1 Solutions of equations with exponential potentials

2.2.2 Note on independence from potentials . . . .

2.3 Equations for $\chi(\alpha)$, $\bar{\chi}(\psi)$ and their main properties . . .

2.3.1 On what is the solution . . . .

Generalization of Emden-Fowler eqn.: $\ddot{\psi} + 3\,\dot{\alpha}\dot{\psi} + v'(\psi)/2 = 0 \qquad v = a\psi^2 - b\psi^p$,

3 Dynamics in ψ-version

3.1 Main cosmological equations . . . .

3.2 Exact and asymptotic solutions of $\bar{\chi}(\psi)$ - equation . . . .

3.2.1 Solution with $v(\psi) = v_0\, e^{2\,g\psi}$ . . . .

3.2.2 Important transformation of $\bar{\chi}(\psi)$ and properties of $v'(\psi)/v(\psi)$.

3.2.3 Large ψ behavior of $\bar{\chi}(\psi)$ . . . .

3.2.4 Small ψ behavior of $\bar{\chi}(\psi)$ . . . .

Exact and approxim. solutions `03-`10
N-Liouville and Toda systems (ATF, VdA)
BH-Cosm-Waves

4 Dynamics in α -version

4.1 Exact solution of $\chi^2(\alpha)$ -equation for $k = 0$ . . . .

4.2 Exact solutions $\eta^2(\alpha)$, $\xi^2(\alpha)$, $\chi^2(\alpha)$ for arbitrary $\bar{v}(\alpha)$ and k .

4.2.1 On replacing potential by kinetic energies . . . .

4.3 A fresh look at inflation and inflationary perturbation theory .

A few remarks about non-isotropic and curved universes:
weak anisotropy (scalaron), and essential anisotropy (vecton). To be published soon.

4-2000 BC. **First astronomical observations** in Egypt, Central America, England (Stonehenge)
260 BC. Greek Aristarchus of Samos (c. 315–230 BC) proposes a sun-centered universe.
c. 150 AD. Greek-Egyptian Ptolemy (2nd century AD) proposes an earth-centered universe.

1543. Copernicus publishes his sun-centered theory of the universe (solar system).

1576. English mathematician Thomas Digges (c. 1546–1595) proposes that
Universe infinite because **stars are at varying distances**
1576-1597. Tycho de Brahe's most complete observations of stars positions.
1584. Italian philosopher Giordano Bruno (1548–1600) states that the **Universe is infinite**.

1609-1610. Galileo's observations with his \`telescope'. Kepler's telescope with 2 lenses.
1632. Galileo champions Copernicus's sun-centered universe, but is forced Inquisition to recant.
1666-1671. Newton constructs the first telescope - reflector…
1687, 1713, 1726 Newton's \`Principia'. 1729 edition with added *"The Laws of the Moon's Motion, according to Gravity"* by John Machin
1705. Edmond Halley discovers the **proper movements of stars.**
1779-1784. F. W. Herschel (astronomer and composer) discovers binary and multiply stars

1845. William Parsons (Lord Ross) discovers **spiral structure** of some nebulae

Differential geometry: Gauss, Lobachevski; Riemann, Hemholtz; Beltrami, Levi-Civita, Ricci…

1912-1914. Vesto Slipher determines high velocities of 11 **spiral nebulae fast fleeing away**…

COSMOLOGICAL OBSERVATIONS - THEORIES - PREDICTIONS - DISCOVERIES

1915-1916. Einstein's **General Relativity** (GR) (the final equations; the detailed review 20.03.16)
1917. Einstein proposes a closed static universe theory (the **first relativistic cosmology**)
Further predictions: **Black Holes, Cosmological constant, Gravitational Waves**…(*the story …*)
1922. Alexander A. Friedmann : the **expanding Universe** solutions of Einstein's equations
1923: Einstein: Extension of GR to **affine geometry** with additional vector field (DE and DM)
1927. G. Lemaître proposes a detailed theory of the **expanding Universe** (using Slipher's and Hubble's data). *Earlier, he independently derived the Friedmann solutions (unpublished)*

Mathematical work: Weyl, Cartan, O.Klein

1929. Edwin Hubble demonstrates *(in fact)* the expansion of the Universe (also with Sl.'s dt.)
1937. Einstein and Nathan Rosen derived exact cylindrical **gravitational waves**
1946. George A. Gamov – the **Hot Big-Bang** theory, prediction of **Relic photons at 3 K (CMB).**
1965. Al. Penzias and Rob. Wilson – observation of bkg. radiation (*rem.* Dicke e.a.)
……….

Now certainly discovered

Bcgr. Rad., Homogeneity and Isotropy of the Universe (with checking corrections); Black Holes, Dark Energy, Dark Matter; the `Age' and `Radius of the Universe.

Well established theoretical models

Realistic FLRW cosmological models (homogeneous, isotropic, based on GR+scalaron e.a) …
Inflationary Models with possibility to confront them to observations (*small amplitude* GW).
Observed! Nonlinear Gravitational Waves! (visit tht seminar on March 22, A.F.Zakharov)
The problem of **baryons** in our Universe (abundance clarified, antibarions problem still-?)