

# Non-conformal anisotropic solution in 5-dimensional Maxwell-dilaton gravity

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# Motivation

## Holography needs:

- **Natural** theory with metric as solution of EOM.
- Non-zero chemical potential  $\mu \neq 0$ .

2016 *I.Ya. Aref'eva*

“Holography for Heavy Ions Collisions at LHC and NICA”  
(arXiv:1612.08928 [hep-th])

IA's talk on Monday

$$ds^2 = L^2 \frac{b(z)}{z^2} \left[ -g(z)dt^2 + dx^2 + z^{2-\frac{2}{\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)} \right]$$

# AGG solution

$$ds^2 = \frac{L^2}{z^2} \left[ -g(z)dt^2 + dx^2 + z^{2-\frac{2}{\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)} \right]$$

$$\mu = 0, \quad b(z) = 1$$

2015 *I.Ya. Aref'eva, A.A. Golubtsova*

JHEP **1504** 011 (arXiv:1410.4595 [hep-th])

2016 *I.Ya. Aref'eva, A.A. Golubtsova, E. Gourgoulhon*

JHEP **1609** 142 (arXiv:1601.06046 [hep-th])

Since phenomenally require  $\mu \neq 0$ ,  $b \neq 1$ ,  $\nu \neq 1$  *Aref'eva 1612.08928*

OUR GOAL



Generalizations:



$$\mu \neq 0$$



$$b(z) \neq 1$$

# Action in Einstein frame, metric ansatz

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \times$$
$$\times \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$
$$A_\mu^{(1)} = A_t(z) \delta_\mu^0 \quad F^{(2)} = q dy^1 \wedge dy^2 \quad \phi = \phi(z)$$

$$ds^2 = L^2 \frac{b(z)}{z^2} \left[ -g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)} \right]$$

*Boundary conditions:*

$$g(0) = 1, \quad g(z_h) = 0 \quad A_t(0) = \mu, \quad A_t(z_h) = 0 \quad \phi(z_h) = 0$$

$\nu = 1$ : 2017 M.-W. Li, Y. Yang, P.-H. Yuan (arXiv:1703.09184 [hep-th])

# Equations of motion

$$\phi'' + \phi' \left( \frac{g'}{g} + \frac{3b'}{2b} - \frac{\nu + 2}{\nu z} \right) + \frac{z^2 A_t'^2}{2bg} \frac{\partial f_1}{\partial \phi} - \frac{q^2 z^{-2 + \frac{4}{\nu}}}{2bg} \frac{\partial f_2}{\partial \phi} - \frac{b}{z^2 g} \frac{\partial V}{\partial \phi} = 0$$

$$A_t'' + A_t' \left( \frac{b'}{2b} + \frac{f_1'}{f_1} - \frac{2 - \nu}{\nu z} \right) = 0$$

$$\text{1-st: } g'' + g' \left( \frac{3b'}{2b} - \frac{1}{z} - \frac{2}{\nu z} \right) - \frac{z^2}{b} f_1 A_t'^2 = 0$$

$$\text{2-nd: } 2g' \left( 1 - \frac{1}{\nu} \right) + g \left( 1 - \frac{1}{\nu} \right) \left( \frac{3b'}{b} - \frac{4}{z} - \frac{4}{\nu z} \right) + \frac{q^2 z^{-1 + \frac{4}{\nu}}}{b} f_2 = 0$$

$$\text{3-rd: } b'' - \frac{3b'^2}{2b} + \frac{2b'}{z} - \frac{4b}{3\nu z^2} \left( 1 - \frac{1}{\nu} \right) + \frac{b}{3} \phi'^2 = 0$$

$$\begin{aligned} \text{4-th: } & -V - \frac{z^4}{2b^2} A_t'^2 f_1 - \frac{3z^2 b' g'}{2b^2} - \frac{3z^2 g b'^2}{b^3} + \frac{9z g b'}{2\nu b^2} + \frac{15z g b'}{2b^2} + \\ & + \frac{z g'}{\nu b} + \frac{2z g'}{b} + \frac{z^2 g \phi'^2}{2b} - \frac{8g}{\nu b} - \frac{4g}{b} = 0 \end{aligned}$$

# General anisotropic solution

$$b(z) = \exp P(z), \nu \neq 1$$

$$f_1 = z^{-2 + \frac{2}{\nu}}$$

$$A_t = \tilde{\mu} \int_z^{z_h} e^{-\frac{P(\xi)}{2}} \xi d\xi, \quad \tilde{\mu} = \frac{\mu}{\int_0^{z_h} e^{-\frac{P(\xi)}{2}} \xi d\xi}$$

$$g = 1 + \tilde{\mu}^2 \int_0^z e^{-\frac{3P(\xi)}{2}} \left( \int_0^\xi e^{-\frac{P(\chi)}{2}} \chi d\chi \right) \xi^{1 + \frac{2}{\nu}} d\xi -$$
$$- \frac{1 + \tilde{\mu}^2 \int_0^{z_h} e^{-\frac{3P(\xi)}{2}} \left( \int_0^\xi e^{-\frac{P(\chi)}{2}} \chi d\chi \right) \xi^{1 + \frac{2}{\nu}} d\xi}{\int_0^{z_h} e^{-\frac{3P(\xi)}{2}} \xi^{1 + \frac{2}{\nu}} d\xi} \int_0^z e^{-\frac{3P(\xi)}{2}} \xi^{1 + \frac{2}{\nu}} d\xi$$

$$\phi(z) = C_5 + \int_0^z \sqrt{-3P''(\xi) + \frac{3}{2} P'^2(\xi) - \frac{6}{\xi} P'(\xi) + 4 \frac{\nu - 1}{\xi^2 \nu^2}} d\xi$$

# General anisotropic solution

$$b(z) = \exp(cz^2/2), \nu \neq 1$$

$$f_1 = z^{-2+\frac{2}{\nu}} \quad A_t(z) = \mu \frac{e^{-\frac{cz^2}{4}} - e^{-\frac{cz_h^2}{4}}}{1 - e^{-\frac{cz_h^2}{4}}}$$

$$c = 0.9$$

$$g(z) = 1 - \frac{\Gamma(1 + \frac{1}{\nu}) - \Gamma(1 + \frac{1}{\nu}; \frac{3cz^2}{4})}{\Gamma(1 + \frac{1}{\nu}) - \Gamma(1 + \frac{1}{\nu}; \frac{3cz_h^2}{4})} -$$
$$- \mu^2 e^{\frac{cz_h^2}{2}} \frac{\Gamma(1 + \frac{1}{\nu}) - \Gamma(1 + \frac{1}{\nu}; cz^2)}{4 c^{1/\nu} \left(1 - e^{\frac{cz_h^2}{4}}\right)^2} +$$
$$+ \mu^2 e^{\frac{cz_h^2}{2}} \frac{\Gamma(1 + \frac{1}{\nu}) - \Gamma(1 + \frac{1}{\nu}; cz_h^2)}{4 c^{1/\nu} \left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \frac{\Gamma(1 + \frac{1}{\nu}) - \Gamma(1 + \frac{1}{\nu}; \frac{3cz^2}{4})}{\Gamma(1 + \frac{1}{\nu}) - \Gamma(1 + \frac{1}{\nu}; \frac{3cz_h^2}{4})}$$

# Scalar field $\phi(z)$ for general solution

$$b(z) = \exp(cz^2/2), \nu \neq 1$$

$$\begin{aligned} \phi(z) = & C_5 + c \sqrt{\frac{3}{8}} \left\{ \sqrt{(\mathcal{A}^2 - z^2)(\mathcal{B}^2 - z^2)} + \mathcal{A}\mathcal{B} \ln(z^2) - \right. \\ & - \frac{\mathcal{A}^2 + \mathcal{B}^2}{2} \ln \left( 2\sqrt{(\mathcal{A}^2 - z^2)(\mathcal{B}^2 - z^2)} - \mathcal{A}^2 - \mathcal{B}^2 + 2z^2 \right) - \\ & \left. - \mathcal{A}\mathcal{B} \ln \left[ - \left( 2\mathcal{A}^2\mathcal{B}^2 - \mathcal{A}^2z^2 - \mathcal{B}^2z^2 + 2\mathcal{A}\mathcal{B}\sqrt{(\mathcal{A}^2 - z^2)(\mathcal{B}^2 - z^2)} \right) \right] \right\} \\ & z \leq \mathcal{A} \end{aligned}$$

$$c = 0.9, \nu = 4.5$$

↓

$$\mathcal{A} = 0.29406, \mathcal{B} = 2.565$$

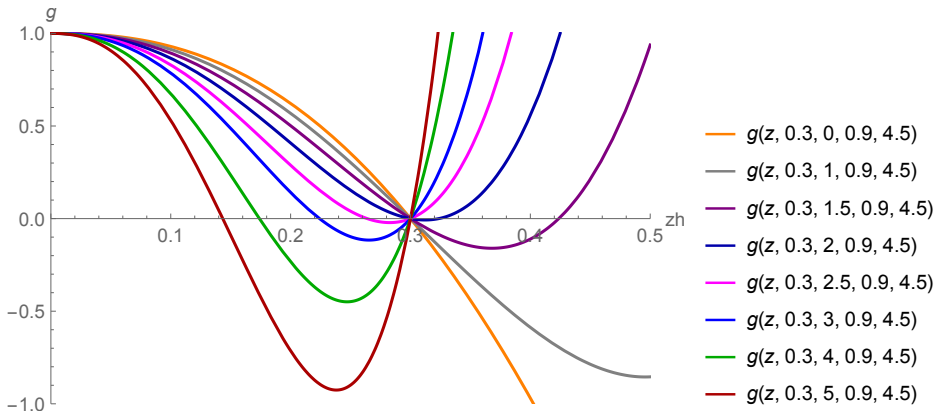
↓

$$z \leq 0.29406$$



# Blackening function $g(z)$ for various $\mu$

$$z_h = 0.29, c = 0.9, \nu = 4.5$$



# Solution for anisotropic metric ansatz

$$b(z) = 1, \nu \neq 1$$

$$f_1 = z^{-2+\frac{2}{\nu}}$$

$$A_t(z) = \mu \left( 1 - \frac{z^2}{z_h^2} \right)$$

$$g(z) = 1 - \left( \frac{z}{z_h} \right)^{2+\frac{2}{\nu}} \left[ 1 + \frac{\mu^2 \nu z_h^{\frac{2}{\nu}}}{1+2\nu} \left( 1 - \frac{z^2}{z_h^2} \right) \right]$$

$$\phi = 2 \frac{\sqrt{\nu-1}}{\nu} \log z + C_5$$

# Anisotropic solution

$$b(z) = 1, \nu \neq 1, A_t(z) = 0$$

$$g(z) = 1 - \left( \frac{z}{z_h} \right)^{2 + \frac{2}{\nu}}$$

$$f_2(z) = \frac{4z^{-4/\nu}}{q^2} \frac{(\nu - 1)(1 + 3\nu + 2\nu^2)}{\nu^2 (1 + 2\nu)}$$

$$\phi(z) = C_5 \pm 2 \frac{\sqrt{\nu - 1}}{\nu} \log(z)$$

$$V(z) = -2 \frac{(1 + \nu)(1 + 2\nu)}{\nu^2}$$

Corresponds to AGG solution

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JHEP **1609** 142 (arXiv:1601.06046 [hep-th])

# Thermodynamics for anisotropic solution

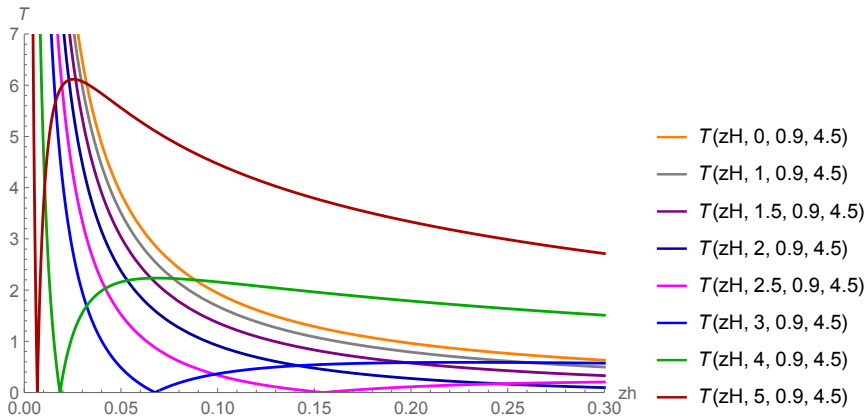
$$b(z) = \exp(cz^2/2), \nu \neq 1$$

$$s = \frac{e^{\frac{3cz_h^2}{4}}}{4z_h^{1+\frac{2}{\nu}}}$$

$$T = \frac{z_h^{1+\frac{2}{\nu}}}{8\pi} e^{-\frac{3cz_h^2}{4}} \left| \left( \frac{3c}{2} \right)^{1+\frac{1}{\nu}} \frac{2^{1-\frac{1}{\nu}}}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}; \frac{3cz_h^2}{4}\right)} + \frac{\mu^2 c e^{\frac{cz_h^2}{4}}}{\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \left[ 1 - \left( \frac{3}{4} \right)^{1+\frac{1}{\nu}} e^{\frac{cz_h^2}{4}} \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}; cz_h^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}; \frac{3cz_h^2}{4}\right)} \right] \right|$$

# Temperature $T(z_h)$ for various $\mu$

$c = 0.9, \nu = 4.5$



# Conclusions

- The *AdS* anisotropic black hole solution for 5-dimensional Maxwell-dilaton gravity is found.
- The AGG solution is generalized for the non-zero chemical potential case.
- Scalar field restricts the BH radius.

## To do:

- Shock-wave consideration  
(Aref'eva, Mamedov, KR, Arifulov) – in progress.
- Confinement  $\leftarrow$  axion?

Thank you  
for your attention