



Universität Regensburg

Lattice study of continuity and finite-temperature transition in $SU(N) \times SU(N)$ Principal Chiral Model

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arXiv:1706.08954

Hadron Structure and Hadronic Matter, and Lattice QCD
Dubna 2017

Motivation

QCD is **strongly coupled theory** on large distances

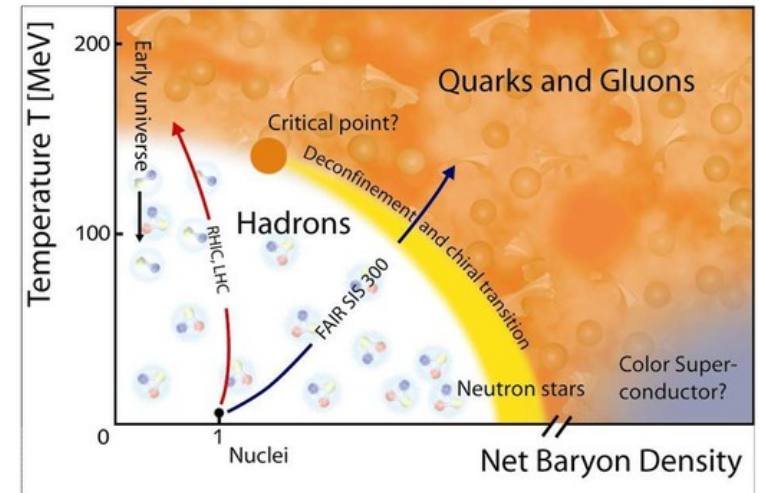
How to address ground state properties (mass gap, etc) ?

Idea: $\mathbb{R}^3 \times S^1$  L $L \rightarrow 0$ $\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}$

Now it is weakly coupled but..

$$T = 1/L$$

Beware of deconfinement phase transition / crossover



A generic phenomena:

- SU(N) Yang-Mills

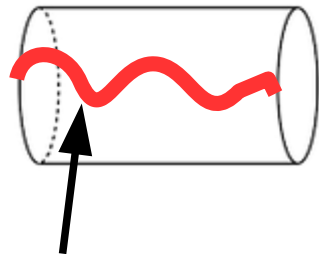
Breaking of Z(N) => failure of Eguchi-Kawai reduction

- CP(N), O(N) sigma models

Choice of boundary conditions

“What are boundary conditions of the universe?”

- A. Lehmann



L $L \rightarrow 0$

Different choices are possible

$$\Omega \quad \mathcal{Z} = \text{Tr} \left[e^{-LH} \right] \quad \longrightarrow \quad \mathcal{Z} = \text{Tr} \left[e^{-LH} \Omega \right]$$

Periodic BC

Twisted BC

Controlled interference of excited states

We would like to introduce **continuous** deformation:

- **Preserve center symmetry** $Z(N)$ and avoid phase transitions
- Match ground state in the limit $L \rightarrow \infty$

- QCD with periodic BC for fermions and with fermions in different representations (M. Unsal, A. Cherman)
- Twisted Eguchi-Kawai reduction in lattice QCD (A. Gonzales-Arroyo)

2d Principal Chiral Model

$$S = \frac{1}{g^2} \int d^2x \text{Tr} [\partial_\mu U(x) \partial^\mu U^\dagger(x)] \quad U(x) \in SU(N)$$

A **toy model** for **Yang-Mills** theory:

No gauge symmetry
No fermions

- Asymptotically free theory
- Integrable model
- Dynamically generated mass gap
- Matrix-like large N limit
- IR renormalon ambiguities
(Fateev, Kazakov, Wiegmann)

$$M_r = M \frac{\sin(r\pi/N)}{\sin(\pi/N)}$$

$$\Lambda^{\beta_0} = \mu^{\beta_0} e^{-\frac{4\pi}{g^2(\mu)}} \quad \beta_0 = N$$

$$\underbrace{e^{-\frac{8\pi}{g^2 N(Q)}}}_{1^{\text{st}} \text{ IR-renormalon}} \gg \underbrace{e^{-\frac{16\pi}{g^2 N(Q)}}}_{2^{\text{nd}} \text{ IR-renormalon}} \gg \dots$$

“Drawback”: no topologically protected non-perturbative saddle points $\pi_2[SU(N)] = 0$

Thermodynamics of PCM

Classical theory: N^2 non-linear waves

In quantum theory “deconfinement” transition:

$$\mathcal{F} \sim (N^2 - 1)T^2 \quad \xrightarrow{T \rightarrow 0} \quad \mathcal{F} \rightarrow NT^2 \left(\frac{m}{2\pi T} \right)^{1/2} e^{-m/T}$$

Evidence from lattice: “hadrons” at low temperature

E. Vicari, P. Rossi

Order and order parameter are not known

Twisted BC for PCM

$$\Omega = \text{diag} \left\{ 1, e^{i\frac{2\pi}{N}}, \dots, e^{i\frac{2\pi(N-1)}{N}} \right\}$$

$$\text{Tr}\Omega^n = \begin{cases} N, & n \equiv 0 \pmod{N} \\ 0, & \text{otherwise} \end{cases}$$

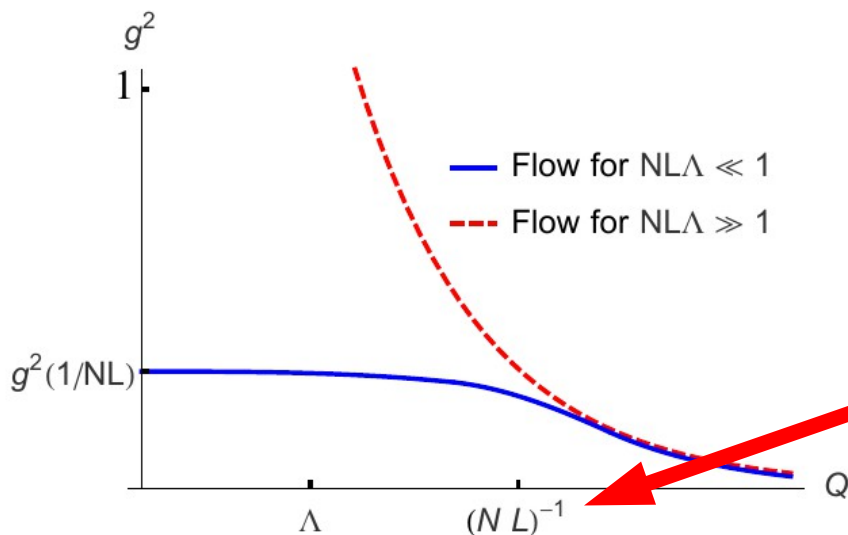
$$U(x_0 + L, x_1) = \Omega U(x_0, x_1) \Omega^\dagger$$

“Maximal” destructive interference =>
many excited states eliminated

$$e^{-LE_\sigma} \text{Tr}_\sigma \Omega$$

Explicit demonstration: exactly solvable $\mathbb{C}\mathbb{P}^{N-1}$ model

T. Sulejmanpasic, Phys. Rev. Lett. 118, 011601 (2017)



- Stabilize center group

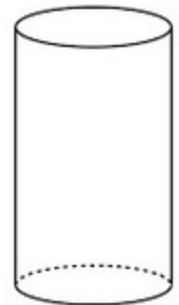
- Volume effectively increased N times
(~ Twisted EK)

- Sliding scale ΛN

M.Unsal, Phys. Rev. Lett. 102, 182002

- Does magic to saddle points

NL



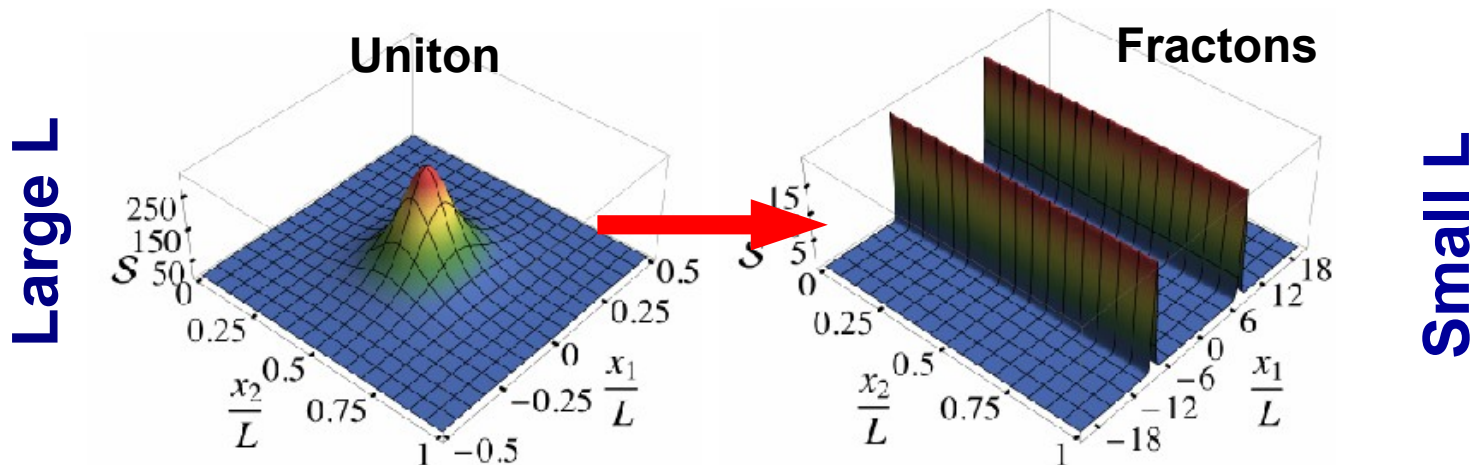
Non-perturbative saddle points

PCM on \mathbb{R}^2 : unstable **uniton** saddle points

Harmonic maps $S^2 \rightarrow SU(N)$

$$S_u = 8\pi/g^2$$

Non-trivial effect of the **twist** in the small L limit:



A. Cherman, D. Dorigoni, M. Unsal, arXiv:1403.1277

Emergent topology \Rightarrow N stable **fracton** constituents at small L

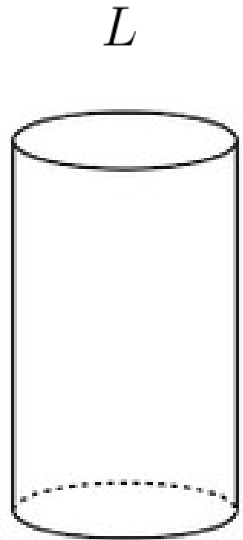
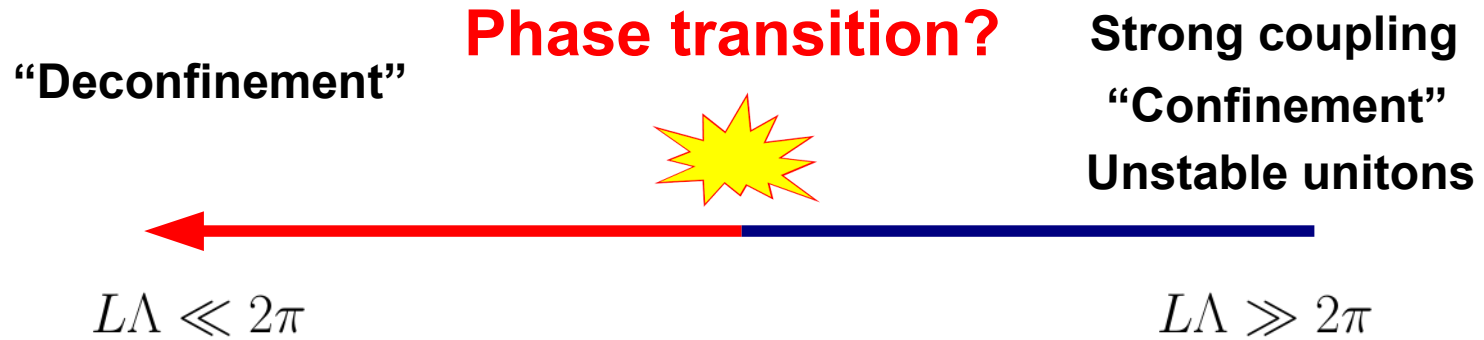
$SU(N) \rightarrow U(1)^{N-1}$ at energies smaller than $1/(NL)$

$$S_f = S_u/N$$

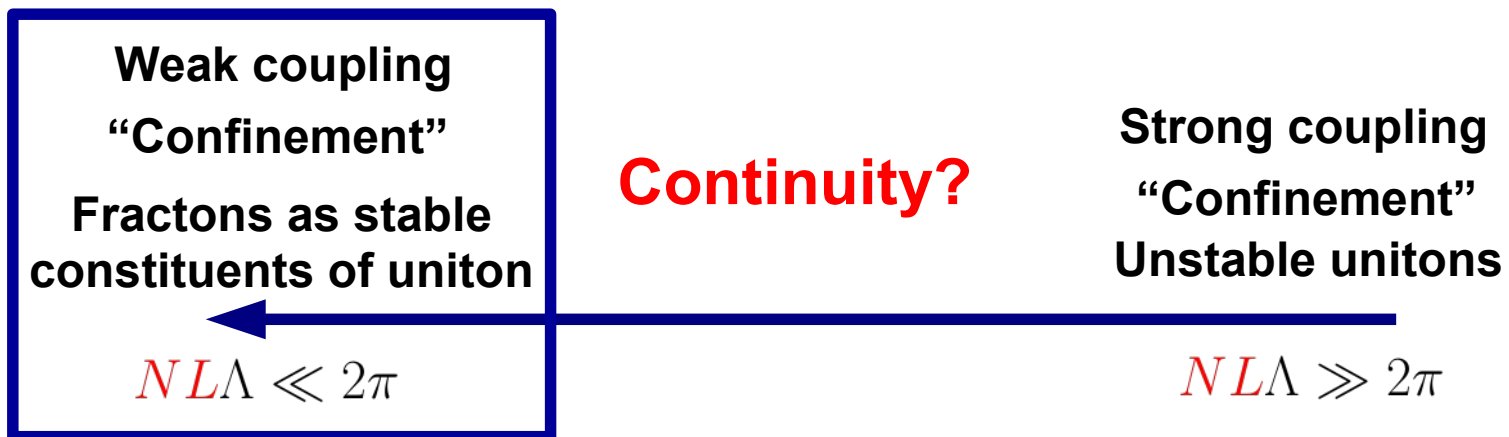
Fractons are responsible for mass gap generation and IR renormalon ambiguity regularization via resurgence theory

Continuity conjecture

Periodic BC

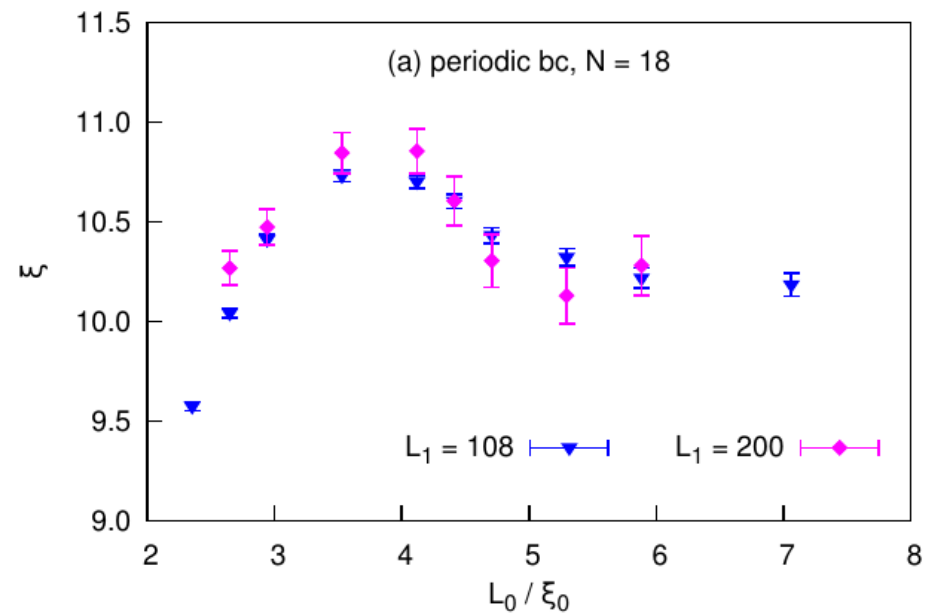
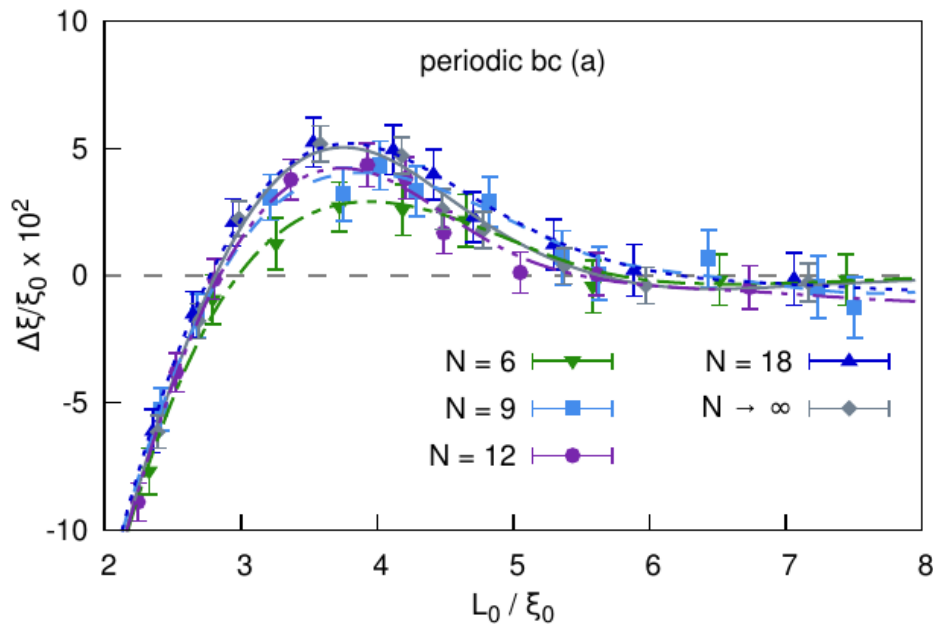


Twisted BC



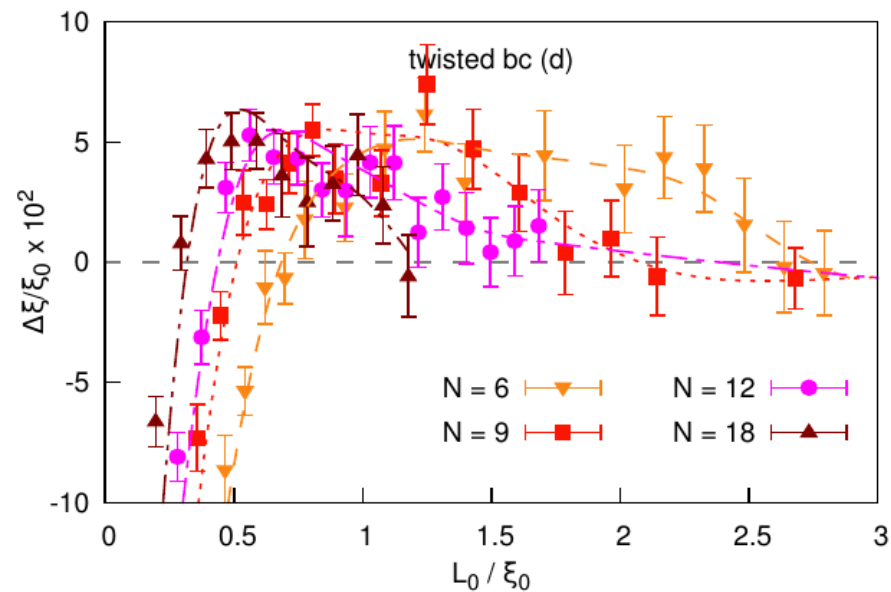
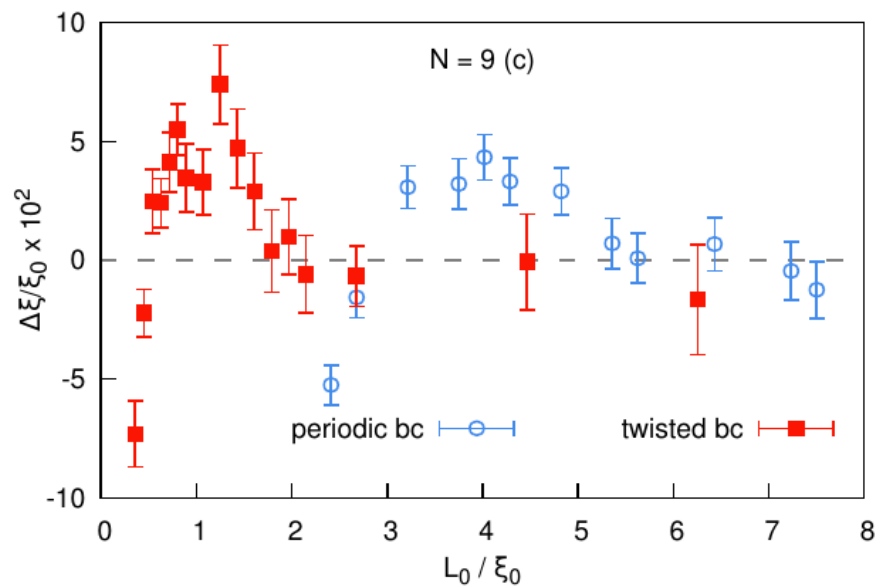
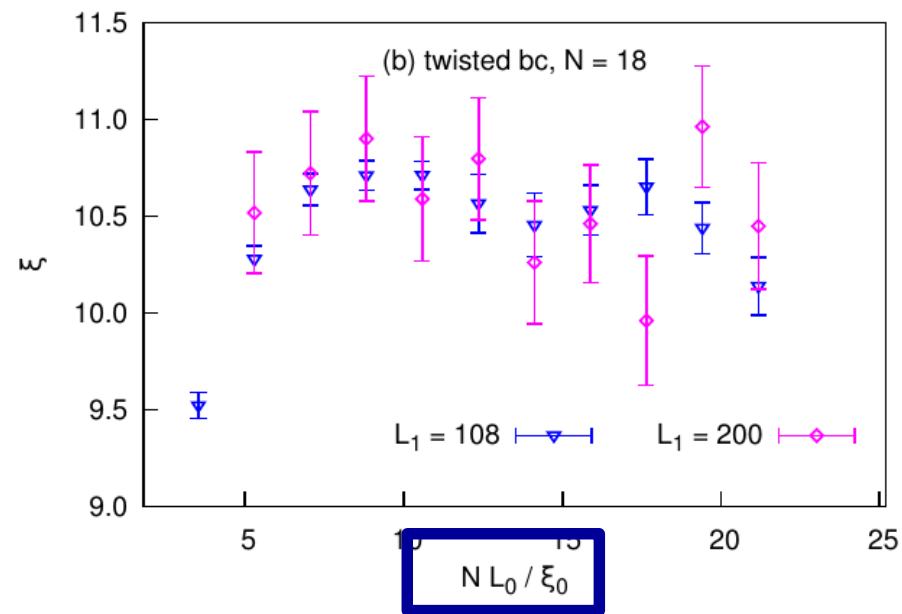
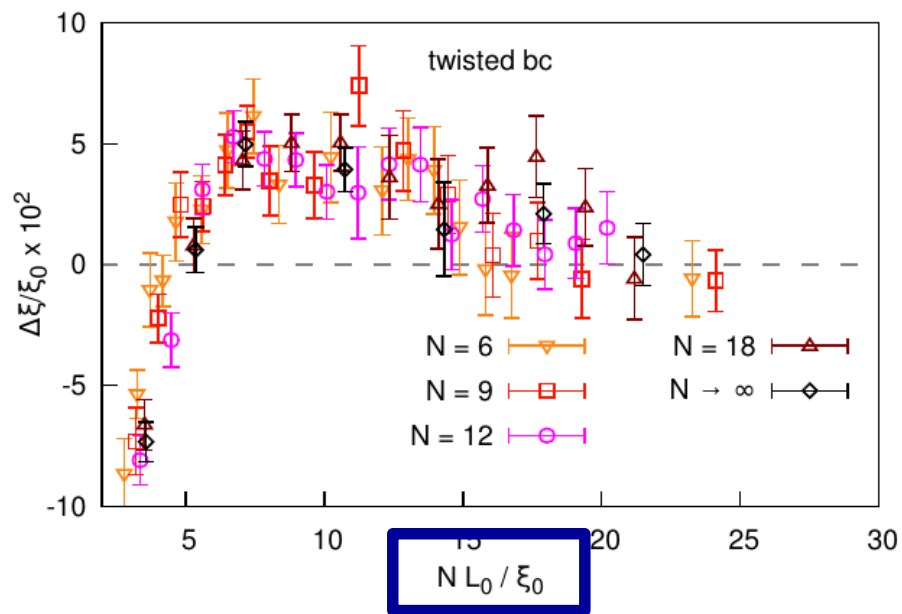
Unsal-Dunne regime

Periodic boundary conditions



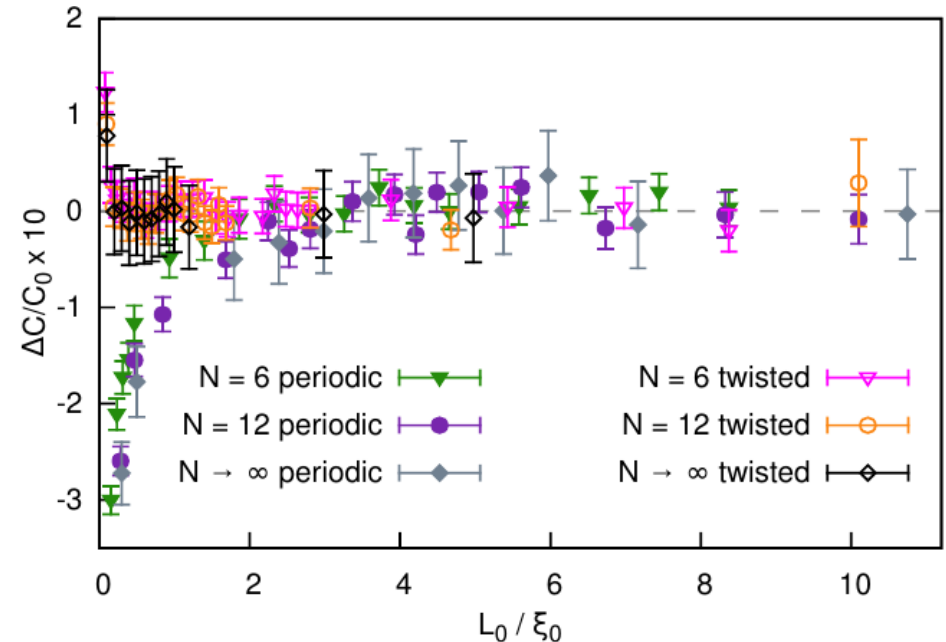
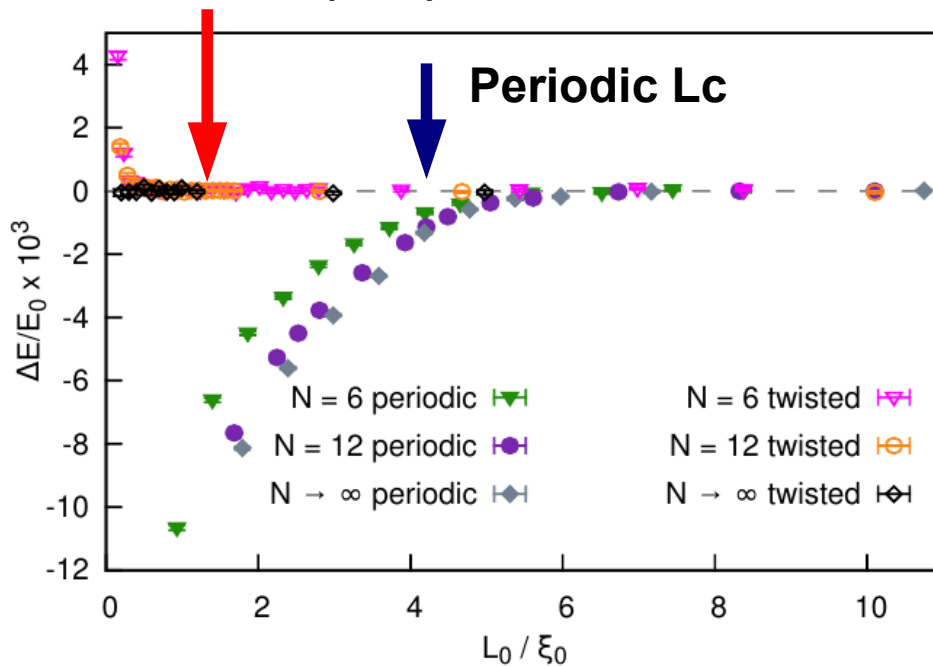
- **Weak enhancement** in the region 3...5 ($\sim 5\%$)
- Slightly **higher** and **narrower** when N increases
- **Infinite N** extrapolation suggests that **correlation length is finite**
Compatible with DiagMC arXiv:1705.03368
- Very mild volume dependence
- Large N **volume independence** in large volumes

Twisted boundary conditions



Mean energy and specific heat

Twisted Lc(N=6)



- **Volume independence** in large volumes
- **Different behavior** in small L limit
- Transition points agree with those for correlation length
- No signatures for phase co-existence

Gradient flow: non-perturbative objects

$$\frac{\partial U(\mathbf{x}, \tau)}{\partial \tau} = -\frac{i}{\beta N} \nabla_{\mathbf{x}}^a S[U(\mathbf{x}, \tau)] T_a U(\mathbf{x}, \tau)$$

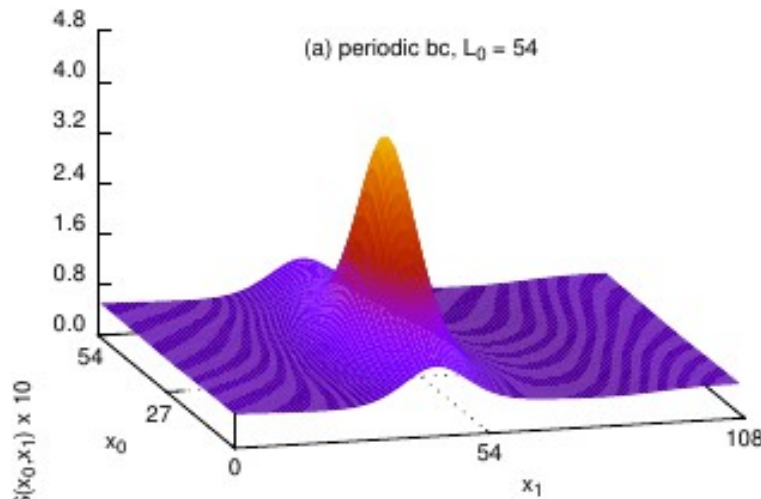
$$U(\mathbf{x}, \tau = 0) \equiv U(\mathbf{x})$$

$$\tau = 0 \dots 1.5 \times 10^3$$

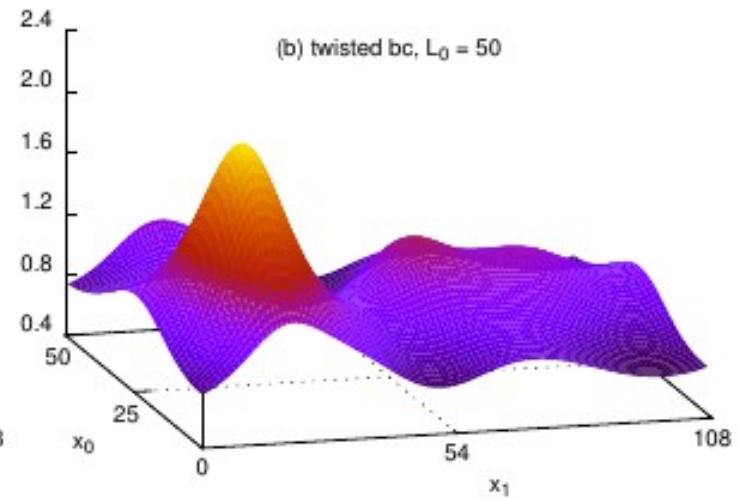
Periodic BC

Twisted BC

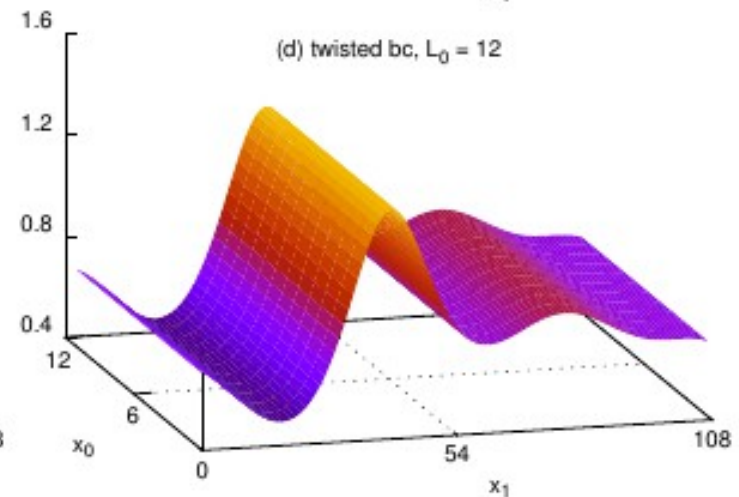
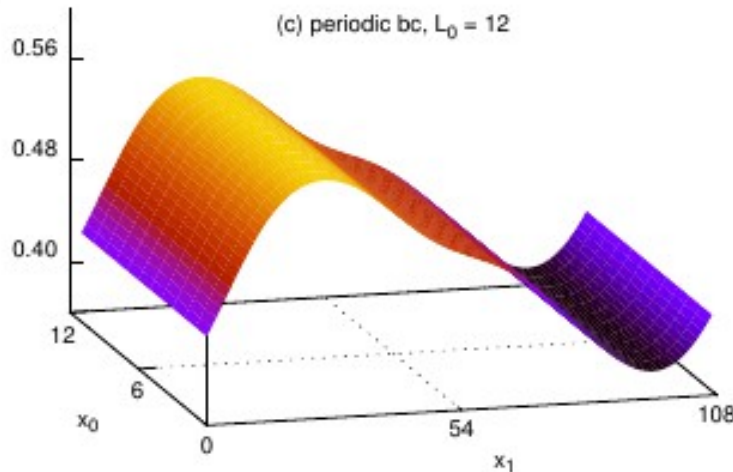
Large L



$N = 9$



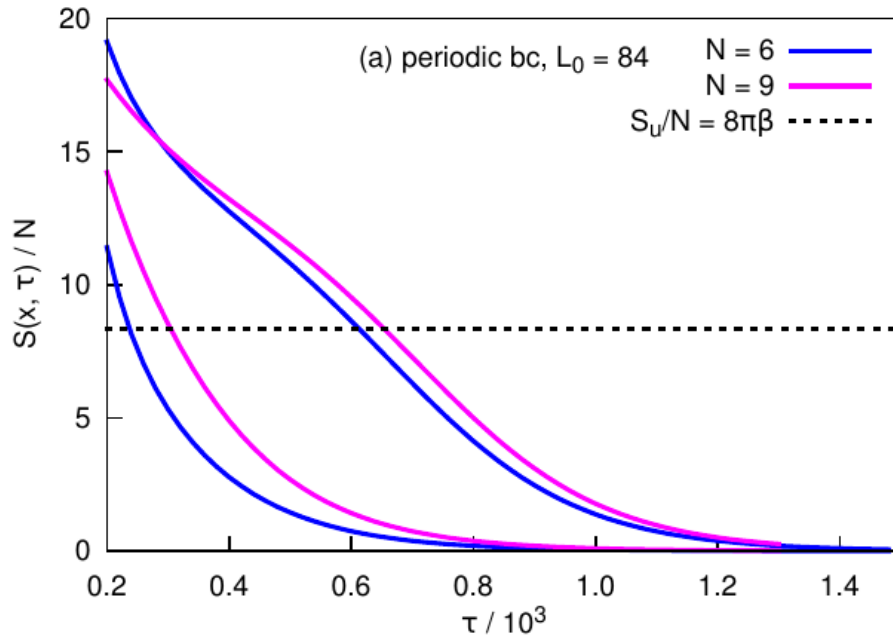
Small L



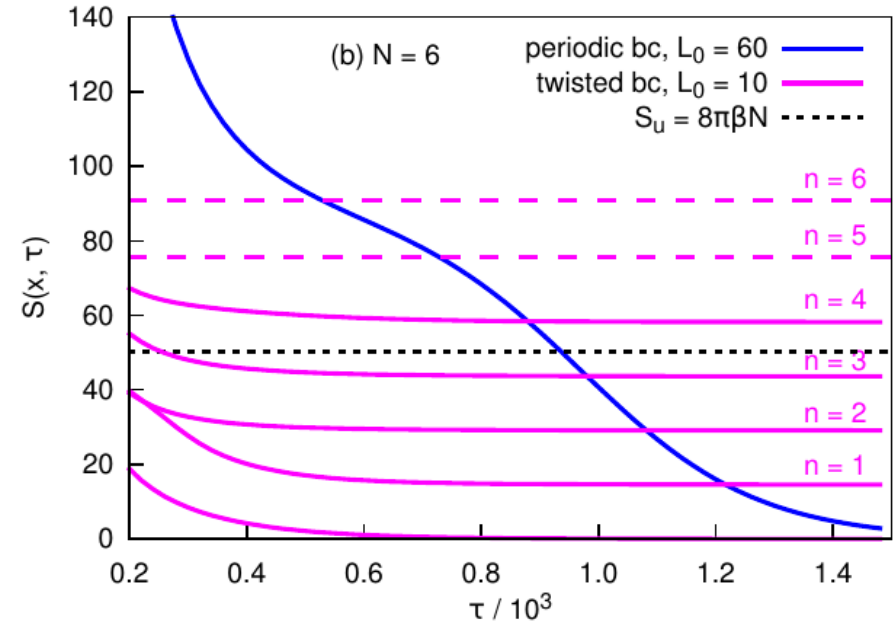
Gradient flow: the action

Uniton: $S_u = 8\pi\beta N$

Fracton: $S_f = 8\pi\beta$



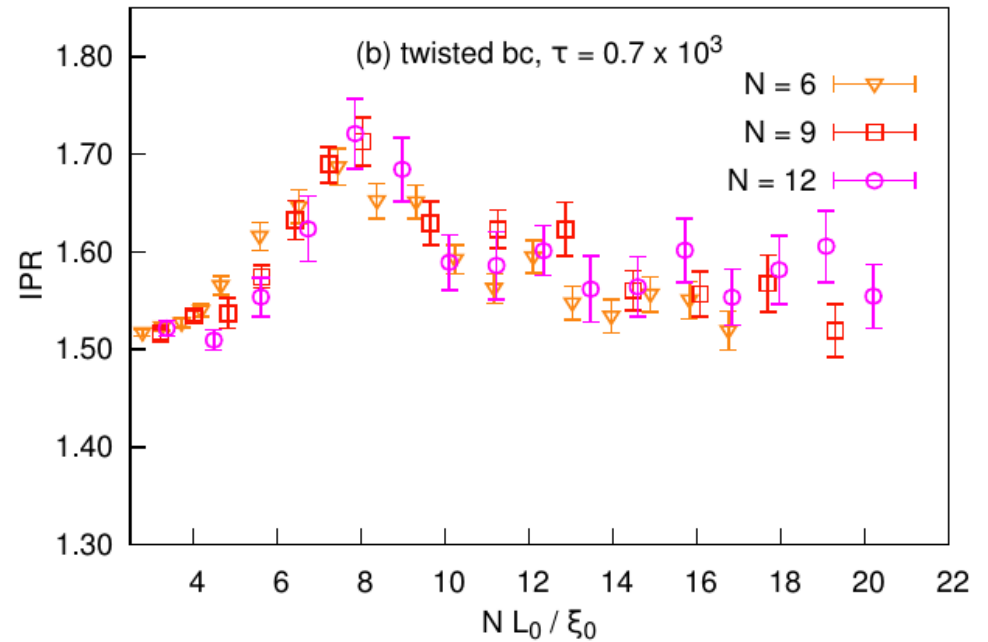
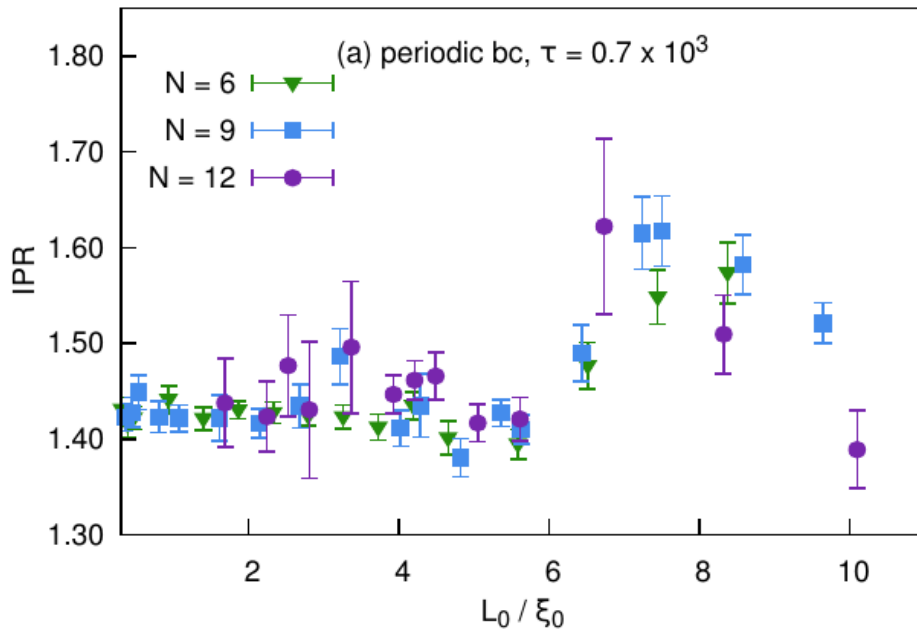
Unitons for $N=6,9$



Stable fractons compared to uniton with $NL^{\text{TBC}} = L^{\text{PBC}}$

- Very **stable saddle points** with **twist** in Unsal-Dunne limit, evidence for **emergent topology**
- Presumably, the plateaus can be associated with **unitons** and **fractons**

Inverse Participation Ratio (IPR)



$$\text{IPR}(\tau) = V \left\langle \frac{\sum_{\mathbf{x}} \tilde{S}^2(\mathbf{x}, \tau)}{\left(\sum_{\mathbf{x}} \tilde{S}(\mathbf{x}, \tau) \right)^2} \right\rangle$$

$$\tilde{S}(\mathbf{x}, \tau) = S(\mathbf{x}, \tau) - \min_{\mathbf{x}} S(\mathbf{x}, \tau)$$

We use IPR as a measure of action density localization

- Interesting **peak** in twisted case which **coincide with twisted NLc**

Conclusions

- We find **evidences** compatible with a **weak crossover or phase transitions** for both types of boundary conditions
- For **periodic** BC, correlation length **enhancement** become larger and narrower as N increases
- For **twisted** BC, correlation length **enhancement** is N independent if considered as a function **NL**
- **Volume scaling** seems to be very **mild** in both cases.
- Using **Gradient flow** equations, we find an evidence for **emergent topology** in Unsal-Dunne limit with **twisted BC**.
- **More work is needed**: combined study of volume and N scaling, continuum limit.
- Might be a challenge for resurgence theory if phase transition (possibly of infinite order) is confirmed