Universität Regensburg

Lattice study of continuity and finite-temperature transition in SU(N) x SU(N) Principal Chiral Model

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arXiv:1706.08954

Hadron Structure and Hadronic Matter, and Lattice QCD
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Motivation

QCD is strongly coupled theory on large distances

How to address ground state properties (mass gap, etc) ?

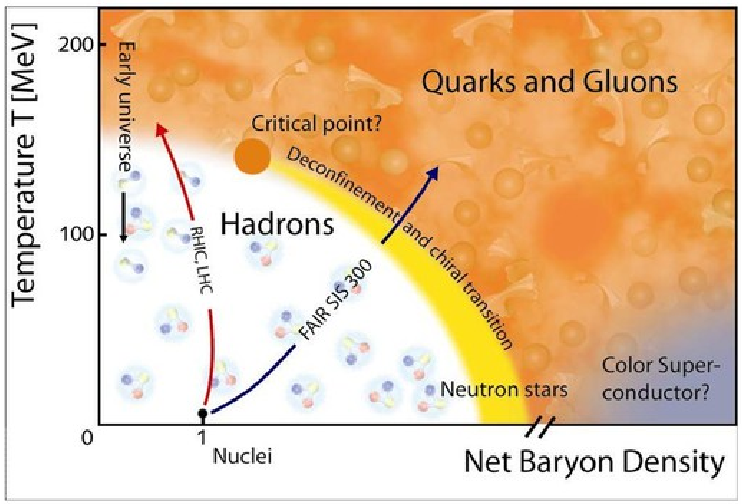
Idea: $\mathbb{R}^3 \times S^1$

L $\quad L \to 0 \quad \alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}$

Now it is weakly coupled but..

$$T = 1/L$$

Beware of deconfinement phase transition / crossover



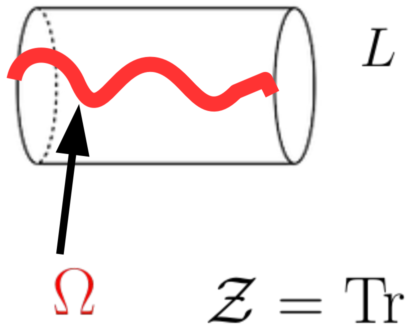
A generic phenomena:

- **SU(N) Yang-Mills**
  - **Breaking of Z(N) => failure of Eguchi-Kawai reduction**
- **CP(N), O(N) sigma models**

Choice of boundary conditions

"What are boundary conditions of the universe?"

- A. Lehmann



$L \to 0$

Different choices are possible

$$\mathcal{Z} = \mathrm{Tr}\left[e^{-LH}\right] \longrightarrow \mathcal{Z} = \mathrm{Tr}\left[e^{-LH}\Omega\right]$$

Periodic BC — **Twisted BC**

Controlled interference of excited states

We would like to introduce continuous deformation:

- **Preserve center symmetry Z(N) and avoid phase transitions**
- **Match ground state in the limit** $L \to \infty$

- **QCD with periodic BC for fermions and with fermions in different representations (M. Unsal, A. Cherman)**
- **Twisted Eguchi-Kawai reduction in lattice QCD (A. Gonzales-Arroyo)**

2d Principal Chiral Model

$$S = \frac{1}{g^2} \int d^2x \mathrm{Tr}\left[\partial_\mu U(x) \partial^\mu U^\dagger(x)\right] \quad U(x) \in SU(N)$$

A toy model for Yang-Mills theory:

No gauge symmetry
No fermions

- **Asymptotically free theory**
- **Integrable model**

$$M_r = M \frac{\sin(r\pi/N)}{\sin(\pi/N)}$$

- **Dynamically generated mass gap**

$$\Lambda^{\beta_0} = \mu^{\beta_0} e^{-\frac{4\pi}{g^2(\mu)}} \qquad \beta_0 = N$$

- **Matrix-like large N limit**
- **IR renormalon ambiguities**

  (Fateev, Kazakov, Wiegmann)

$$\underbrace{e^{-\frac{8\pi}{g^2 N(Q)}}}_{1^{\text{st}}\ \text{IR-renormalon}} \gg \underbrace{e^{-\frac{16\pi}{g^2 N(Q)}}}_{2^{\text{nd}}\ \text{IR-renormalon}} \gg \cdots$$

"Drawback": no topologically protected non-perturbative saddle points $\pi_2[SU(N)] = 0$

Thermodynamics of PCM

Classical theory: N^2 **non-linear waves**

In quantum theory "deconfinement" transition:

$$\mathcal{F} \sim (N^2 - 1)T^2 \quad \xrightarrow{T \to 0} \quad \mathcal{F} \to NT^2 \left(\frac{m}{2\pi T}\right)^{1/2} e^{-m/T}$$

Evidence from lattice: "hadrons" at low temperature

E. Vicari, P. Rossi

Order and order parameter are not known

Twisted BC for PCM

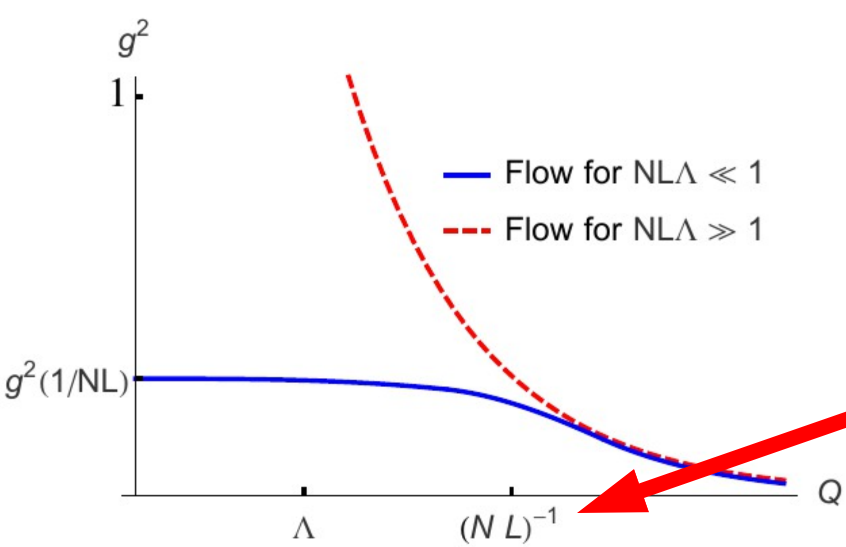
$$\Omega = \mathrm{diag}\left\{1, e^{i\frac{2\pi}{N}}, \ldots, e^{i\frac{2\pi(N-1)}{N}}\right\} \qquad \mathrm{Tr}\Omega^n = \begin{cases} N, & n \equiv 0 \bmod N \\ 0, & \text{otherwise} \end{cases}$$

$$U(x_0 + L, x_1) = \Omega U(x_0, x_1)\Omega^\dagger$$

“Maximal” destructive interference =>
many excited states eliminated $e^{-LE_\sigma}\mathrm{Tr}_\sigma\Omega$

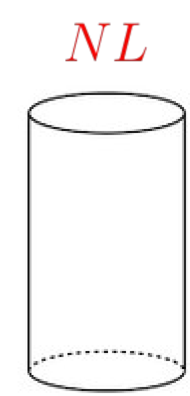
Explicit demonstration: exactly solvable $\mathbb{CP}^{N-1}$ model

T. Sulejmanpasic, Phys. Rev. Lett. 118, 011601 (2017)



A.Cherman, D.Dorigoni, M.Unsal, Phys. Rev. Lett. 112, 021601 (2014)

- **Stabilize center group**
- **Volume effectively increased N times**
  (~ Twisted EK)
- **Sliding scale** ΛN
  M.Unsal, Phys. Rev. Lett. 102, 182002
- **Does magic to saddle points**



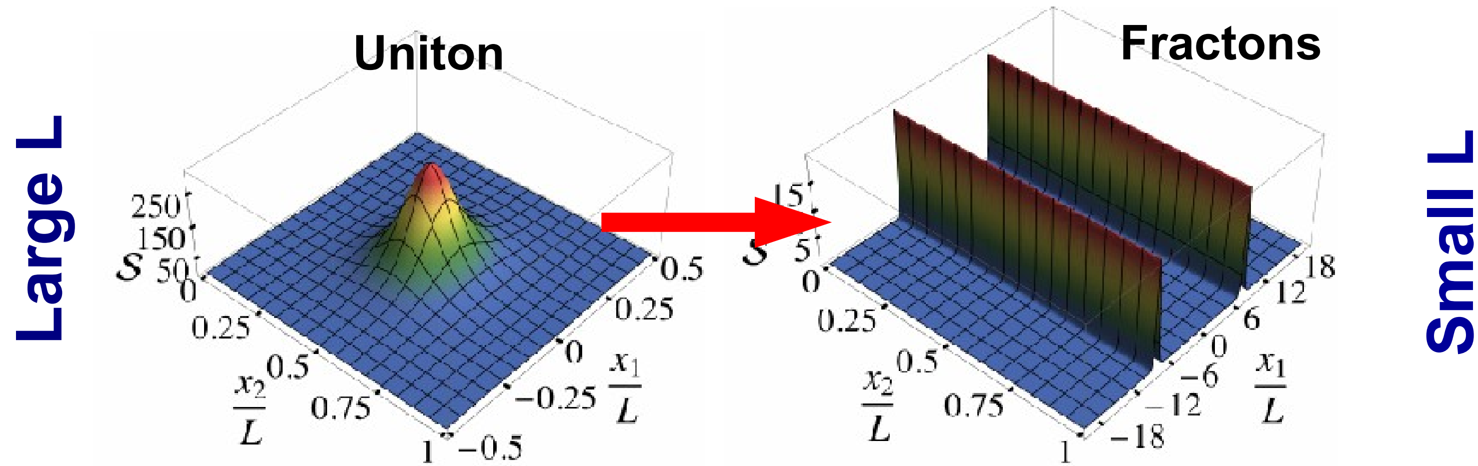
Non-perturbative saddle points

PCM on $\mathbb{R}^2$: unstable uniton saddle points

Harmonic maps $S^2 \to SU(N)$

$$S_u = 8\pi/g^2$$

Non-trivial effect of the twist in the small L limit:



A. Cherman, D. Dorigoni, M. Unsal, arXiv:1403.1277

Emergent topology => N stable fracton constituents at small L

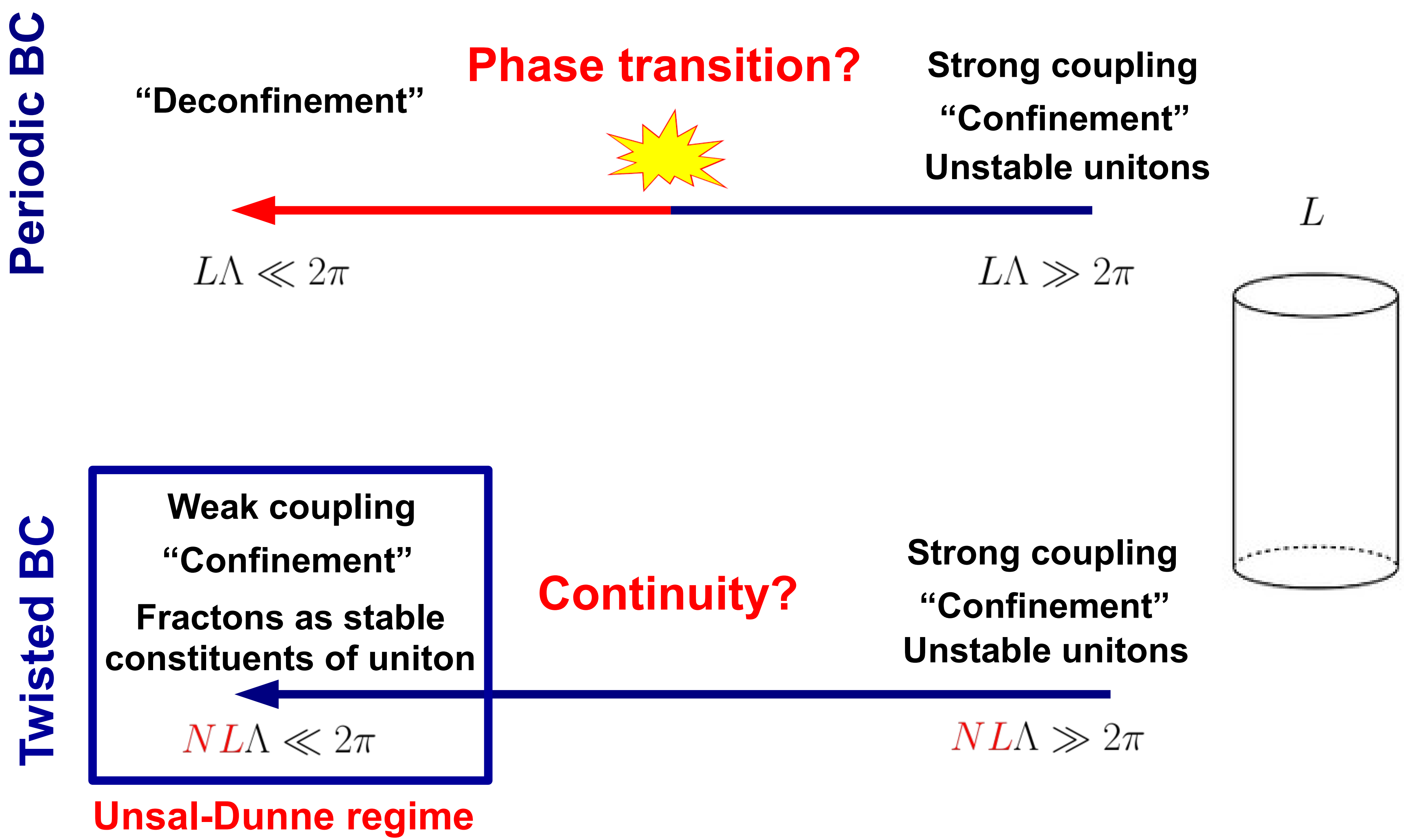
$SU(N) \to U(1)^{N-1}$ **at energies smaller than** $1/(NL)$

$$S_f = S_u/N$$

Fractons are responsible for mass gap generation and IR renormalon ambiguity regularization via resurgence theory

Phys. Rev. Lett. 112, 021601 (2014)   arXiv:1403.1277

Continuity conjecture



Lattice PCM

$$S = -2\beta N \mathrm{Tr} \sum_{x,\mu} \left[U(x) U^\dagger(x+e_\mu) \right] \qquad \beta = 1/\lambda = 1/(g^2 N)$$

Simulation setup

Cabbibo-Marinari algorithm

- $N = 6, 9, 12, 18$ **+ fits** $O(L_0, N) = \tilde{O}(L_0) + c_1/N^2$
- **Lattice size:** $L_1 = 108$, $L_1 = 200, N = 18$ — $1 \leq L_0 \leq L_1$ compact direction
- **Boundary conditions: periodic and twisted**
- **Coupling:** $\beta = 0.332$ $\beta > \beta_c = 0.305$ **(weak coupling)**

Observables

$\xi_0 \equiv \xi(L_0 = L_1, PBC)$
scale

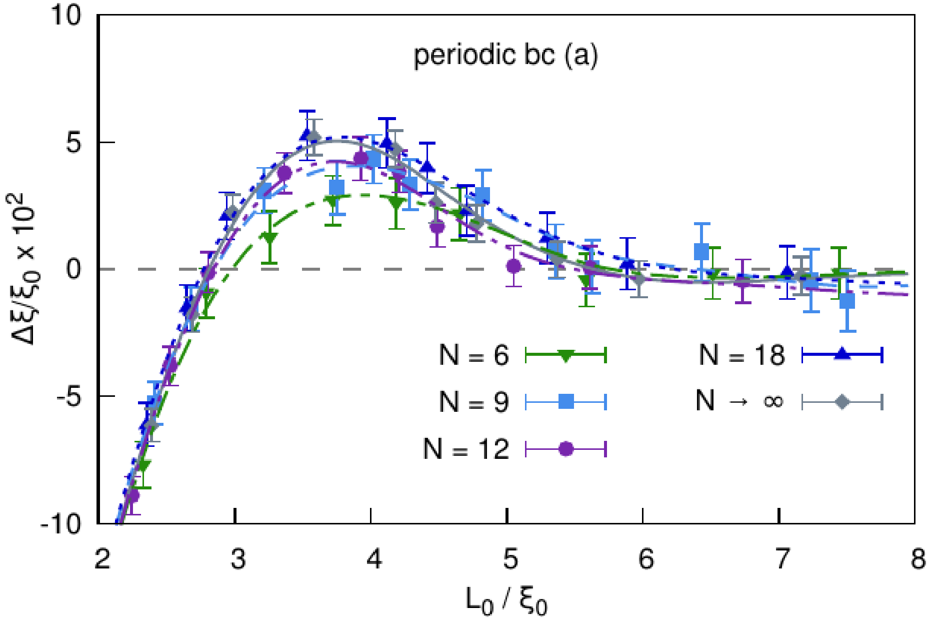
- **Static correlation length** $\xi(L_0)$
- **Mean energy** $E(L_0)$
- **Specific heat** $C(L_0)$

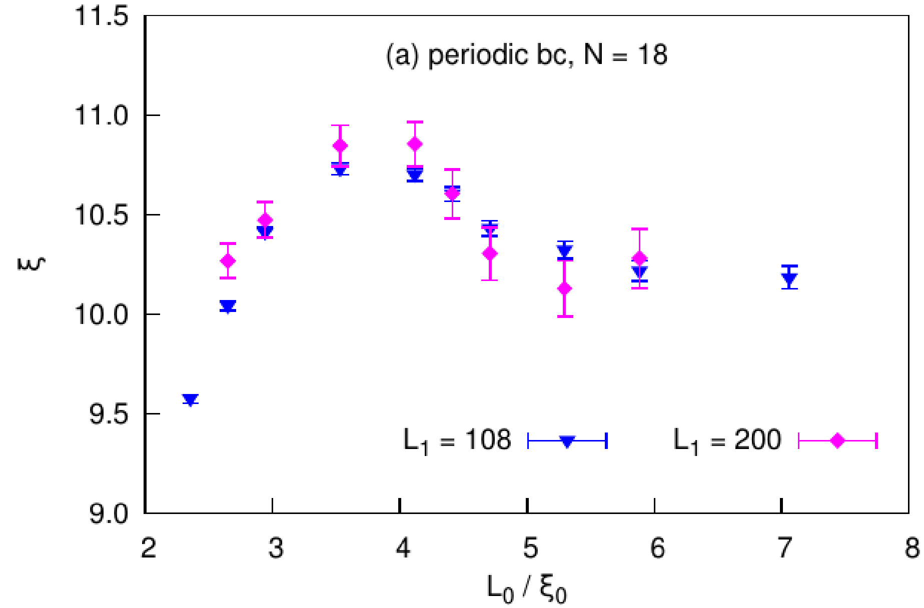
Notations:

$O_0 \equiv O(L_0 = L_1, PBC)$

$$\frac{\Delta O(L_0)}{O_0} \equiv \frac{O(L_0) - O_0}{O_0}$$

Periodic boundary conditions

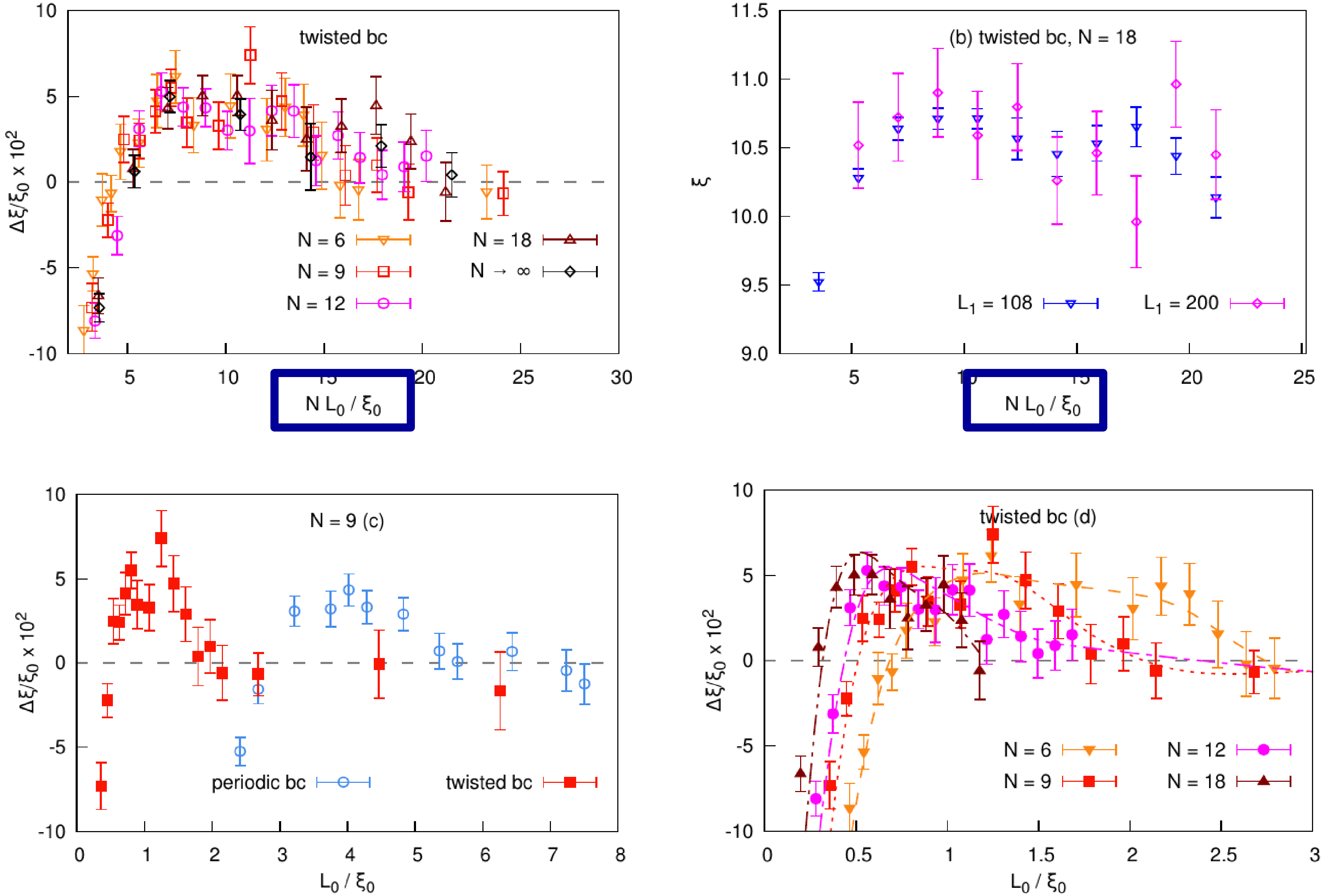




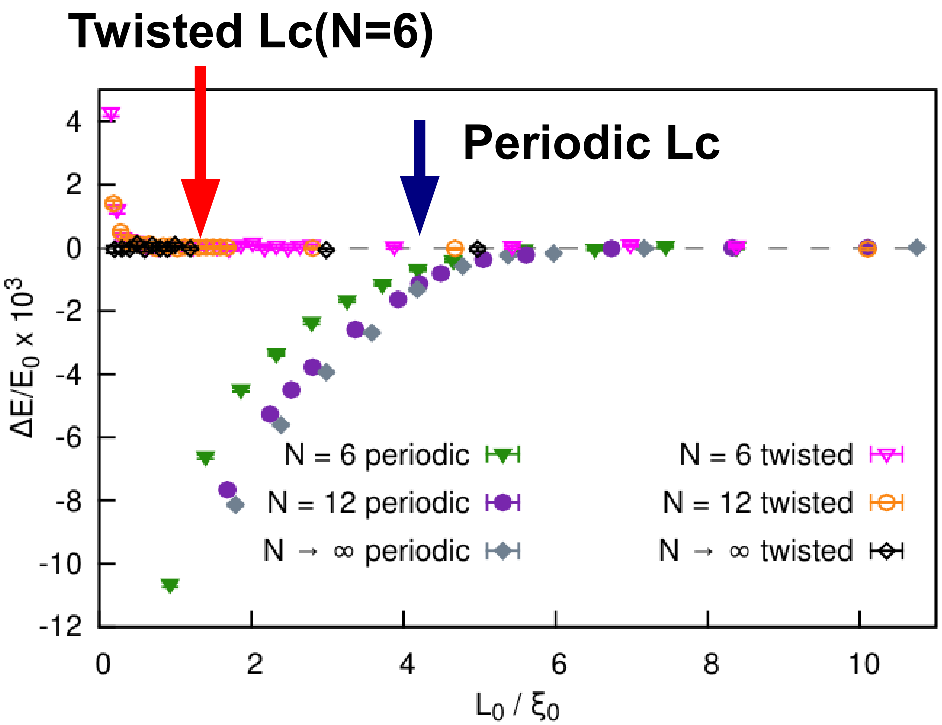
- **Weak enhancement in the region 3...5 (~ 5%)**
- **Slightly higher and narrower when N increases**
- **Infinite N extrapolation suggests that correlation length is finite**

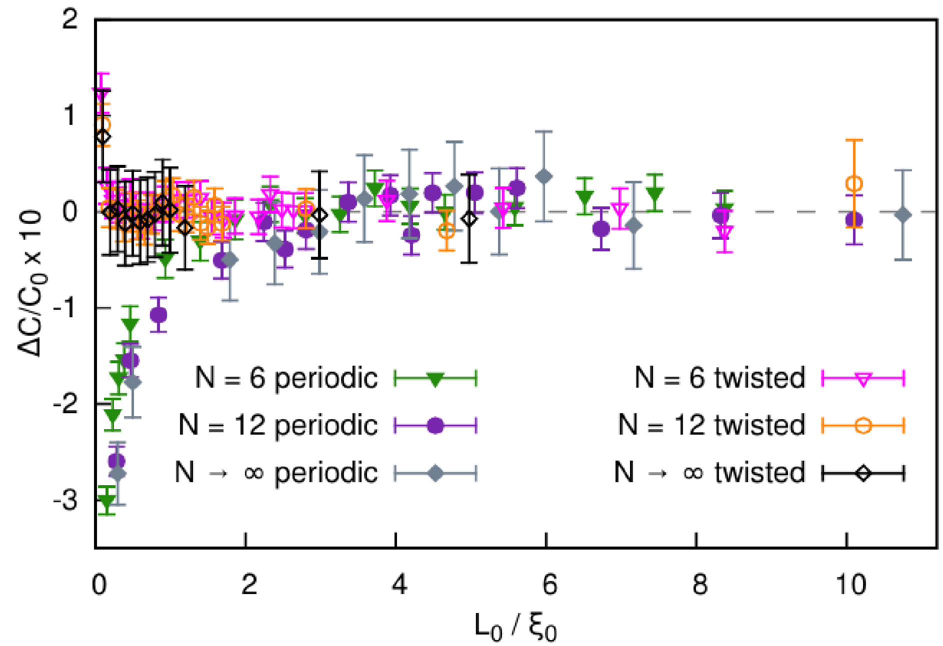
  Compatible with DiagMC arXiv:1705.03368
- **Very mild volume dependence**
- **Large N volume independence in large volumes**

Twisted boundary conditions



Mean energy and specific heat



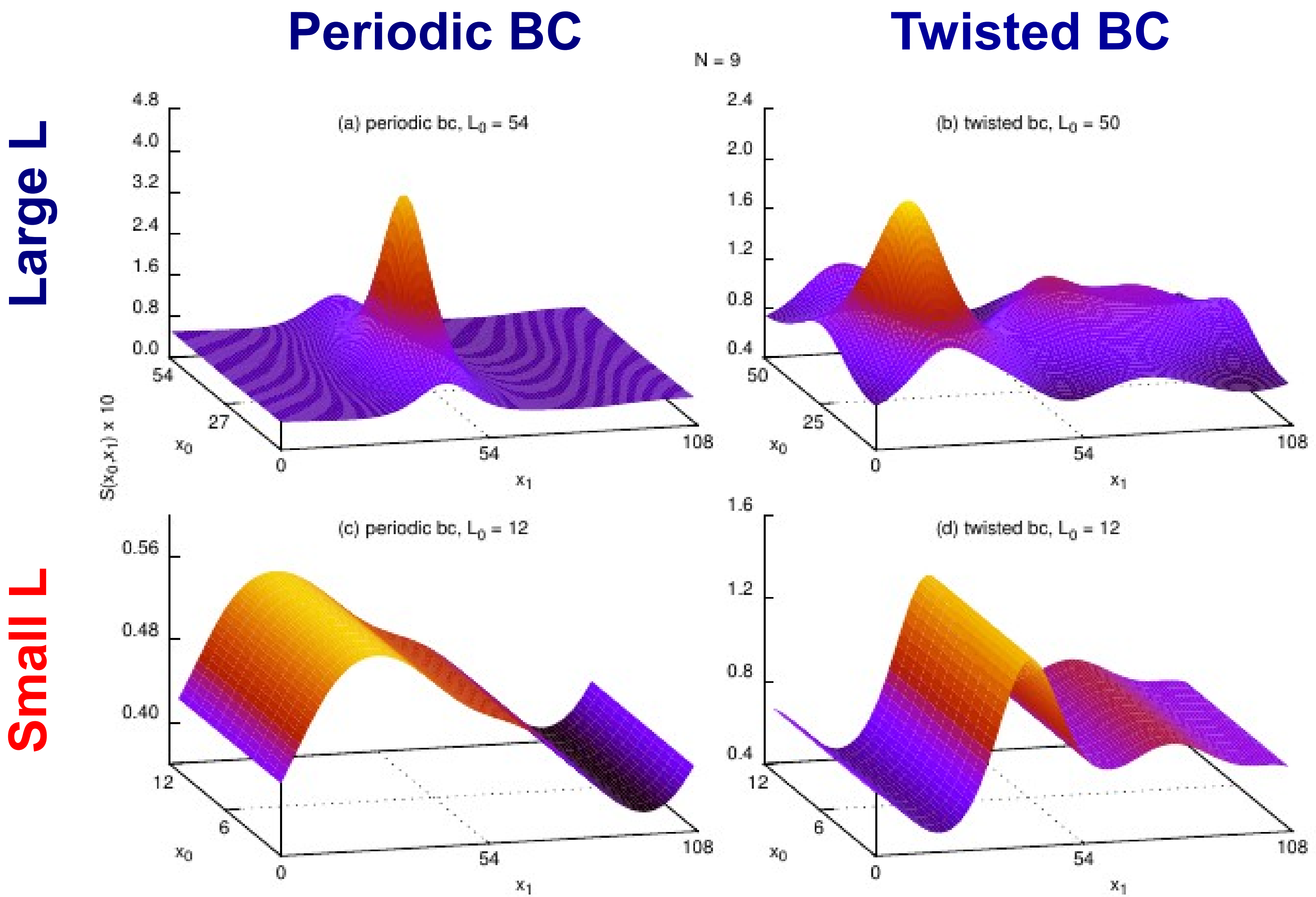


- **Volume independence** in large volumes
- **Different behavior** in small L limit
- **Transition points agree with those for correlation length**
- **No signatures for phase co-existence**

Gradient flow: non-perturbative objects

$$\frac{\partial U(\mathbf{x},\tau)}{\partial \tau} = -\frac{i}{\beta N} \nabla^a_{\mathbf{x}} S\left[U(\mathbf{x},\tau)\right] T_a U(\mathbf{x},\tau)$$
$$U(\mathbf{x},\tau=0) \equiv U(\mathbf{x})$$

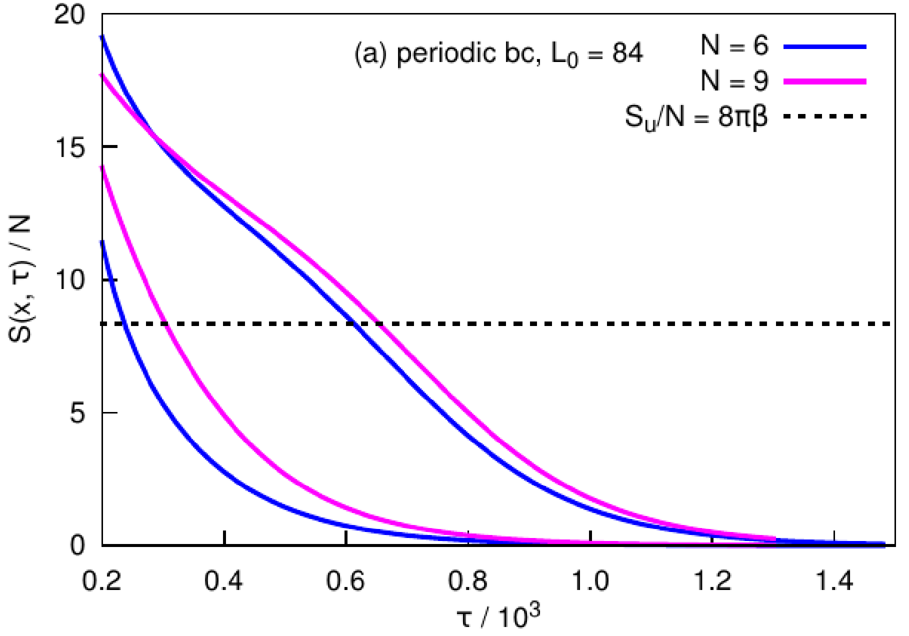
$$\tau = 0 \ldots 1.5 \times 10^3$$

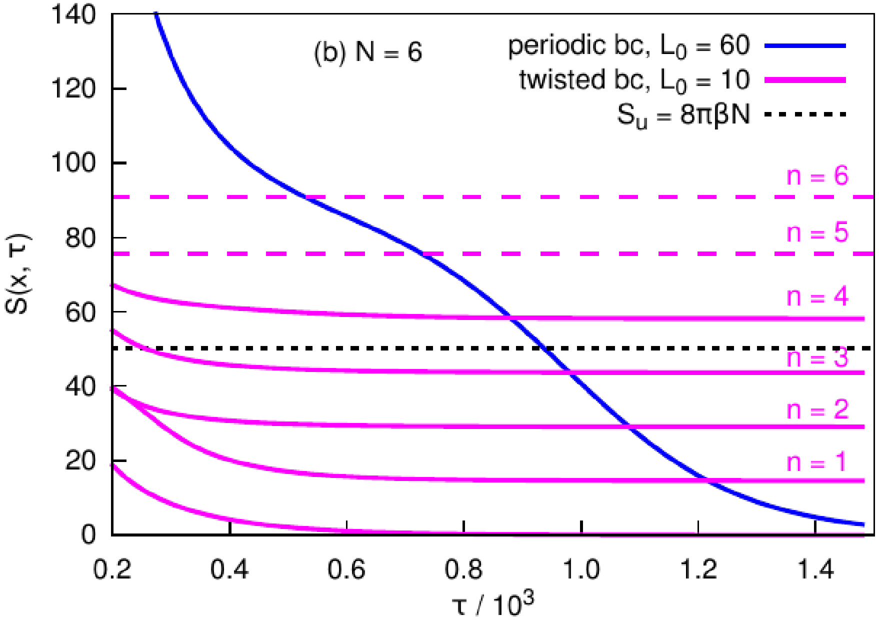


Gradient flow: the action

Uniton: $S_u = 8\pi\beta N$

Fracton: $S_f = 8\pi\beta$



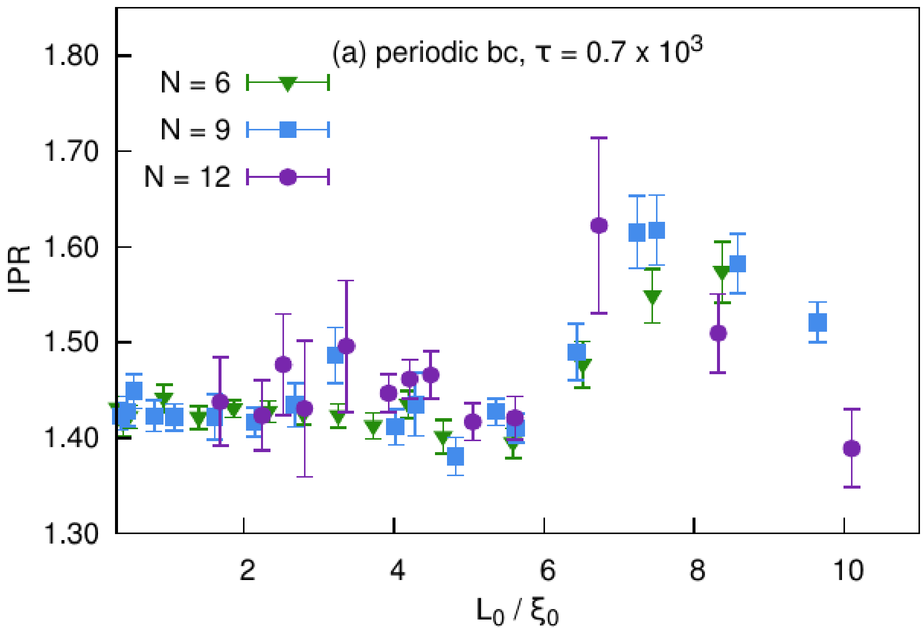


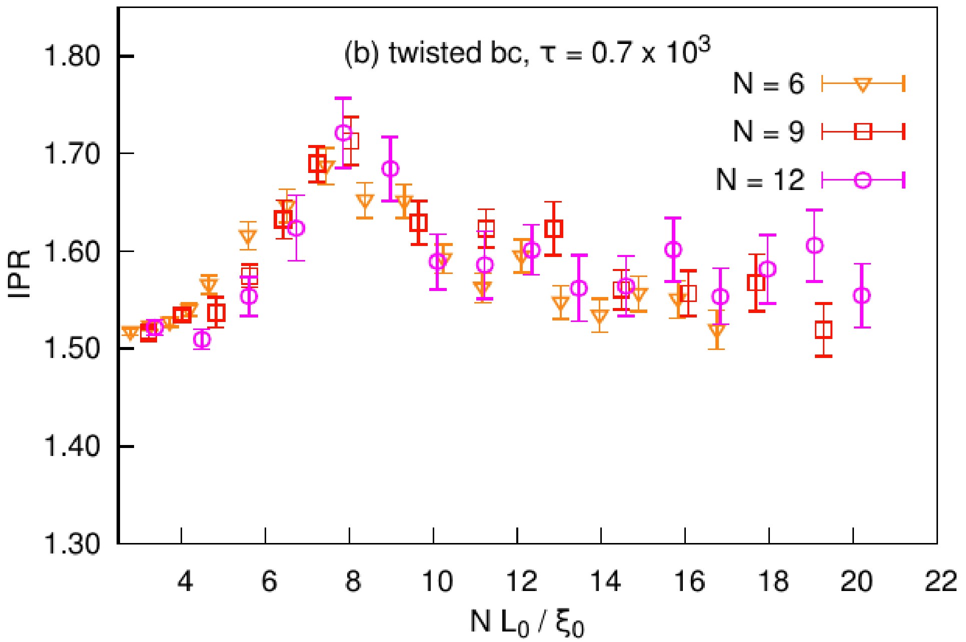
Unitons for N=6,9

Stable fractons compared to uniton with $N L^{\mathrm{TBC}} = L^{\mathrm{PBC}}$

- **Very stable saddle points with twist in Unsal-Dunne limit, evidence for emergent topology**
- **Presumably, the plateaus can be associated with unitons and fractons**

Inverse Participation Ratio (IPR)





$$\mathrm{IPR}\,(\tau) = V \left\langle \frac{\sum_{\mathbf{x}} \tilde{S}^2(\mathbf{x},\tau)}{\left(\sum_{\mathbf{x}} \tilde{S}(\mathbf{x},\tau)\right)^2} \right\rangle$$

$$\tilde{S}(\mathbf{x},\tau) = S(\mathbf{x},\tau) - \min_{\mathbf{x}} S(\mathbf{x},\tau)$$

We use IPR as a measure of action density localization

- **Interesting peak in twisted case which coincide with twisted NLc**

Conclusions

- We find evidences compatible with a weak crossover or phase transitions for both types of boundary conditions
- For periodic BC, correlation length enhancement become larger and narrower as N increases
- For twisted BC, correlation length enhancement is N independent if considered as a function NL
- Volume scaling seems to be very mild in both cases.
- Using Gradient flow equations, we find an evidence for emergent topology in Unsal-Dunne limit with twisted BC.
- More work is needed: combined study of volume and N scaling, continuum limit.
- Might be a challenge for resurgence theory if phase transition (possibly of infinite order) is confirmed