Chiral heat-vortical wave in cold Fermi liquid and the deformation of zero sound

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The plan

- Anomalous transport effects
- Chiral waves
- Chiral kinetic theory formalism for Fermi liquid
- Results

Motivation

- blah-blah

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- Namely, chiral waves should cause generation of quadrupole moment in quark-gluon plasma

Anomalous transport effects

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- Chiral vortical effect

$$\mathbf{j}_A = \left[\frac{(\mu_V^2 + \mu_A^2)}{2\pi^2} + \frac{T^2}{6}\right] \boldsymbol{\Omega}$$

$$\mathbf{j}_V = \frac{\mu_V \mu_A}{\pi^2} \boldsymbol{\Omega}$$

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$$\omega\chi\delta\mu_{V/A} + \frac{(\mathbf{k}\cdot\mathbf{B})\delta\mu_{A/V}}{2\pi^2} = 0$$

Dispersion relation

$$v_{CMW} = \frac{B}{2\pi^2\chi}$$

Chiral vortical wave

- Chiral vortical effect is quadratic in chemical potentials $\Rightarrow$
- We need finite vector charge density in the background

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Dispersion relation

$$v_{CVW} = \frac{\mu_V \Omega}{2\pi^2 \chi}$$

Chiral heat wave

- Anomalous energy transport due to rotation:

$$\mathbf{j}_E = \frac{\mu_A}{3}\left[\frac{3\mu_V^2 + \mu_A^2}{\pi^2} + T^2\right]\boldsymbol{\Omega}$$

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- For small fluctuations of vector charge and energy density above equilibrium

$$\delta\rho_V = \chi\delta\mu_V + \alpha\delta T$$

$$\delta\epsilon = C\delta T + \gamma\delta\mu_V$$

Chiral heat wave

Dispersion relation

$$v_{CHW} = \sqrt{\frac{T^3}{C\chi - \alpha\gamma}\frac{\Omega}{3}}$$

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Dispersion relation

$$v_{CHVW} = \sqrt{\frac{\Omega^2 |9\mu_V^2 (C - \alpha\mu_V) - 3\mu_V T\pi^2(\alpha T + \gamma - \chi\mu_V) + \pi^2\chi T^3|}{9\pi^4\chi|C\chi - \alpha\gamma|}}$$

Berry Fermi liquid

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- System of strongly interacting chiral particles
- Not far from equilibrium is described in terms of quasiparticles
- The anomalous effects are captured by Berry connection in momentum space

Kinetic equation

$$\frac{\partial n_{R/L}}{\partial t} + \dot{\mathbf{x}}_{R/L} \cdot \frac{\partial n_{R/L}}{\partial \mathbf{x}} + \dot{\mathbf{p}}_{R/L} \cdot \frac{\partial n_{R/L}}{\partial \mathbf{p}} = C_{R/L}[n_R, n_L]$$

- $n_{R/L}(t, \mathbf{x}, \mathbf{p})$ are right and left quasiparticles distribution functions
- $C_{R/L}$ are respective collision integrals

Equations of motion

$$\sqrt{G_{R/L}}\dot{\mathbf{x}}_{R/L} = \mathbf{v}_{R/L} + 2\epsilon_{R/L}\boldsymbol{\Omega}(\mathbf{v}_{R/L}\cdot\mathbf{b}_{R/L}) + \boldsymbol{\mathcal{E}}_{R/L}\times\mathbf{b}_{R/L}$$

$$\sqrt{G_{R/L}}\dot{\mathbf{p}}_{R/L} = \boldsymbol{\mathcal{E}}_{R/L} + 2\epsilon_{R/L}\mathbf{v}_{R/L}\times\boldsymbol{\Omega} + (\boldsymbol{\mathcal{E}}_{R/L}\cdot\boldsymbol{\Omega})2\epsilon_{R/L}\mathbf{b}_{R/L}$$

- Here $\epsilon_{R/L}$ are quasiparticles energy functionals,
- $\mathbf{v}_{R/L} = \dfrac{\partial\epsilon_{R/L}}{\partial\mathbf{p}}$, $\boldsymbol{\mathcal{E}}_{R/L} = -\dfrac{\partial\epsilon_{R/L}}{\partial\mathbf{x}}$
- $\mathbf{b}_{R/L} = \pm\dfrac{\hat{\mathbf{p}}}{2p^3}$ are Berry connections curvature in momentum space $(p = |\mathbf{p}|)$
- $\sqrt{G_{R/L}} = 1 + 2\epsilon_{R/L}(\mathbf{b}_{R/L}\cdot\boldsymbol{\Omega})$ modify phase space volume

Collective excitations in kinetic theory formalism

- We are considering the small fluctuations above the equilibrium distribution functions:

$$n_{R/L} = n^0_{R/L} + \delta n_{R/L} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$$

where

$$n^0_{R/L} = \frac{1}{e^{\beta(\epsilon^0 - \mu)} + 1}$$

- Here $\beta = T^{-1}$ and we assume $\mu \gg T$

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- Two regimes: $\omega \ll \tau^{-1}$ (hydrodynamic regime) $\omega \gg \tau^{-1}$ (collisionless regime). Here τ is characteristic relaxation time of collision integral

Hydrodynamic regime: chiral heat-vortical wave

Dispersion relation

$$\omega = \pm \frac{\mu(\mathbf{k}\cdot\mathbf{\Omega})}{2\pi^2\chi}\sqrt{F_1 F_2}\left[1 + \frac{T^2\pi^2}{6\mu^2 F_1}\left(1 - 4A + 4A^2\right)\right]$$

- $A = \dfrac{\mu}{v_F p_F}$
- F_1 and F_2 are taken from linearised energy functionals

Collisionless regime: zero sound

Non-modified implicit zero sound dispersion relation

$$\operatorname{arcotanh} s_0 = \frac{1}{s_0}\left(\frac{1}{2(F_S \pm F_A)} + 1\right)$$

- Here $s_0 = \dfrac{\omega}{v_f k}$, F_S and F_A are taken from linearised energy functionals

Modified zero sound

Modification due to the chiral heat-vortical wave

$$\delta s = s - s_0 \approx \mp \frac{\omega^2 \mu^2}{p_F^4} \left(1 - \frac{2\pi^2 T^2}{3 v_F^2 p_F^2} \right) \cdot$$

$$\frac{F_S L_2(s_0) + [L_1(s_0)^2 - 2 L_0(s_0) L_2(s_0)](F_S^2 - F_A^2)]}{F_A[\operatorname{arcotanh} s_0 - \frac{s_0}{2((s_0)^2 - 1)}]}$$

$$L_0(s_0) = s_0 \operatorname{arcotanh} s_0 - 1$$

$$L_1(s_0) = 3 s_0 (s_0 \operatorname{arcotanh} s_0 - 1)$$

$$L_2(s_0) = 2 s_0 [-3 s_0 + (3 s_0^2 - 1) \operatorname{arcotanh} s_0]$$

Conclusion

- The result for velocity of chiral heat-vortical wave coincides with the one known from hydrodynamics

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- There turn out to be two branches of modified zero sound with the correction to velocity being quadratic in angular velocity

THANK YOU FOR YOUR ATTENTION