Real-Time Simulation of Heavy-Quarkonia

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Motivation

- **Question:**
  Do bound states of heavy quarks form in the early time evolution of a colour-glass condensate?
- If they do: What is their formation time?
- Investigation of the real-time evolution of the quarkonium-correlators necessary

State of the Work

- Crank-Nicholson scheme complete and tested against the free case (see plot below)

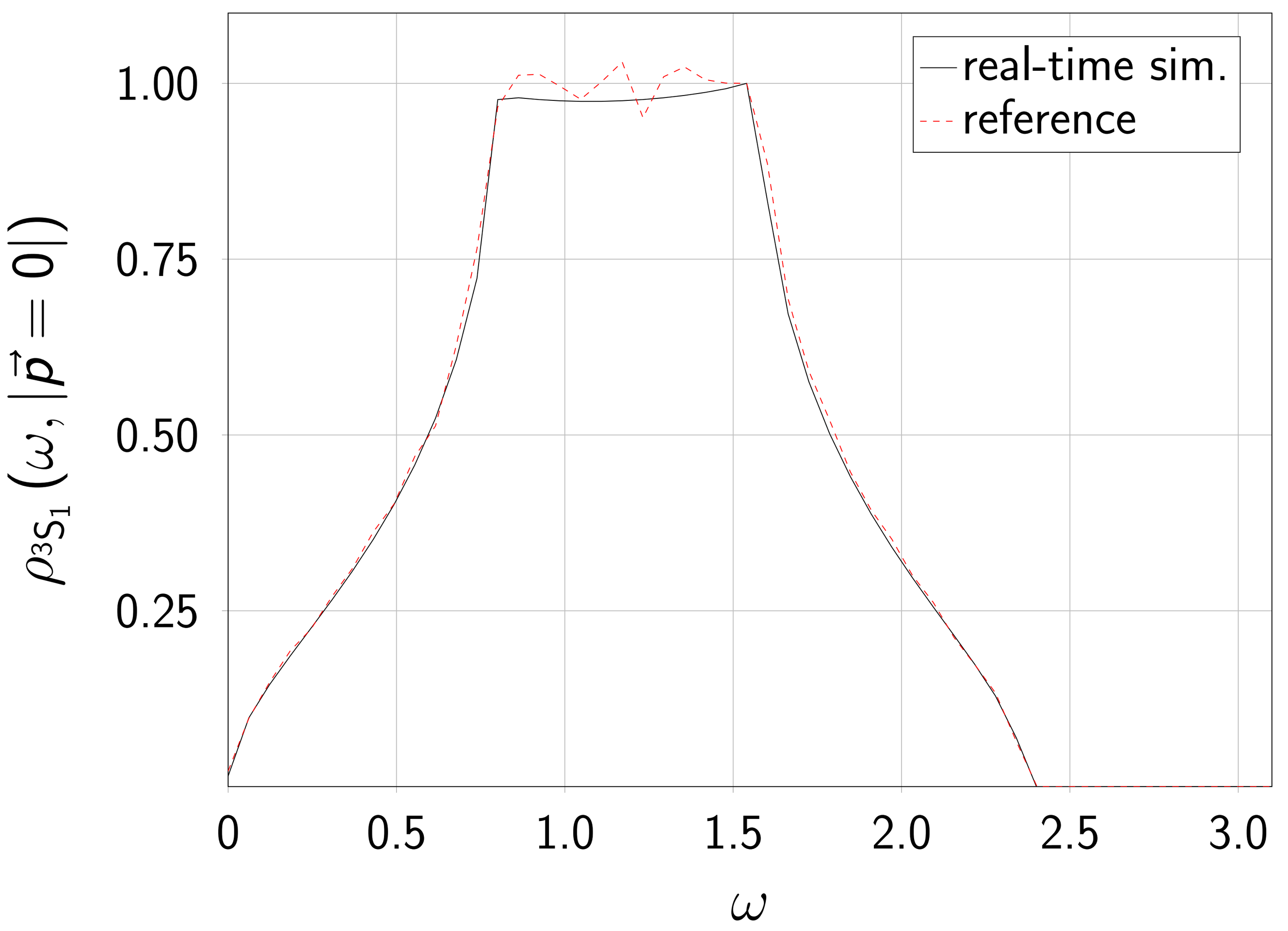

Figure 1: Free spectral density of $^3S_1$-channel of bottomonium. The real-time-simulation is compared to the reference, obtained from [1].
Setup: 64^3-lattice, $N_t = 100$, $a_t/a_s = 1$, $a_s m_b = 5$.

- Currently working on the gauge-field-simulation
- Initial setup for the gauge-field-occupancy corresponding to a colour-glass-condensate (review in [2]):

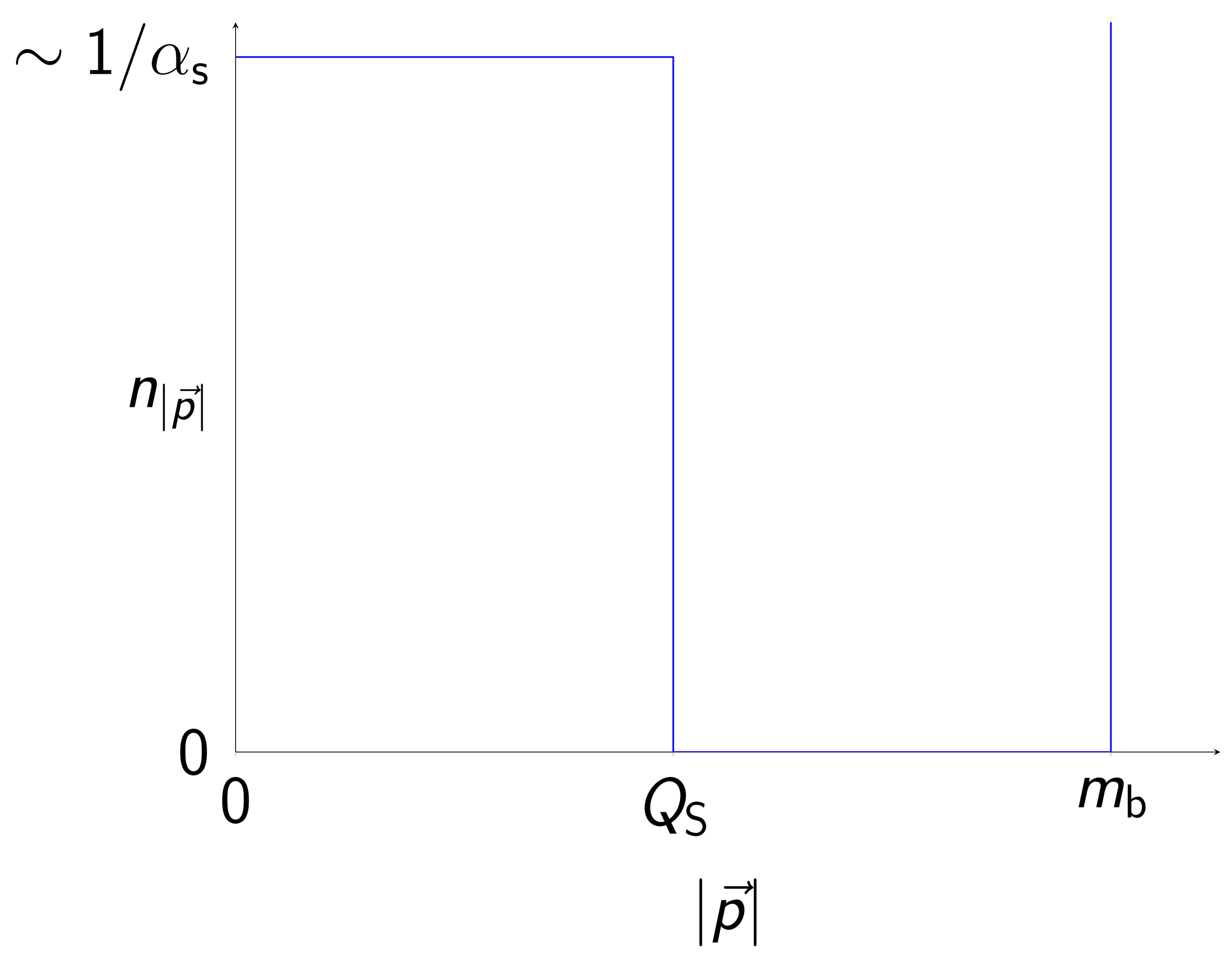

Figure 2: Distribution function w.o. quantum fluctuation. The heavy quark mass scale is given by $m_b \approx 4.6\text{GeV}$. The momentum cutoff is $Q_s \approx 1\text{GeV}$.

Methods

- Time-evolution of the gauge-fields via classical treatment:
  - Hamitonian mechanics (spatial links U_k and chromo-electric field E_k)

$$\partial_t E_k^b = -\frac{1}{\sqrt{N_C a_s^2}} \mathrm{Im}\mathrm{Tr}\left[T^b U_k \sum_{j\neq k}\left(S_j^{[} + S_j^{]}\right)\right], \quad \partial_t U_k = \left(\mathrm{i}2\sqrt{N_C} E_k^a T^a\right) U_k$$

$S_j^{[}(x), S_j^{]}(x)$ ... staple=plaquette in math. pos. or neg. dir. w.o. $U_j(x)$.
  - Justified due to very high occupancy for early times
  - Expectation values via ensemble instead of computer time averages
- Time-evolution of the quarks via an effective field theory:
  - Non-relativistic QCD (NRQCD) for heavy quarks (expansion in $1/m_b$)
  - Schrödinger-like equation for the quark and antiquark propagator
  - Numerical integration scheme: Crank-Nicholson (for matrix-valued field)

Some Formulas for the Discussion

- NRQCD-Lagrangian for heavy quarks [3]

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger\Big(\mathrm{i}D_0 + \underbrace{\frac{\vec{D}^2}{2m_b}}_{-\hat{H}}\Big)\psi + \chi^\dagger\Big(\mathrm{i}D_0 - \frac{\vec{D}^2}{2m_b}\Big)\chi + \mathcal{O}\left(m_b^{-1}\right)$$

- NRQCD meson correlator of S-wave in Minkowski space:

$$D_\Upsilon(x^F, x^I) = \langle D_\Upsilon^{[U]}(x^F, x^I)\rangle_{\mathrm{ensemble}},$$

$$D_\Upsilon^{[U]}(x^F, x^I) = \frac{1}{2N_C}\mathrm{Tr}\left\{\sum_{i=1}^{3}\sigma_i G_\psi\left(x^F, x^I\right)\cdot\sigma_i^*\left(-G_\chi^\dagger\left(x^F, x^I\right)\right)^*\right\}$$

- Crank-Nicholson scheme for the quark and antiquark propagator:

$$\begin{aligned} G_\psi(t + a_t) &= \left(1 - \mathrm{i}\frac{a_t}{2}\hat{H}\right)^{-1}\cdot\left(1 + \mathrm{i}\frac{a_t}{2}\hat{H}\right)\cdot G_\psi(t) + \mathcal{O}(a_t^2) \\ &= \prod_{k=x,y,z}\left(1 + \mathrm{i}\frac{D_k^2}{2m_b}\cdot\frac{a_t}{2}\right)^{-1}\left(1 - \mathrm{i}\frac{D_k^2}{2m_b}\cdot\frac{a_t}{2}\right) G_\psi(t) \end{aligned}$$

- Free spectral function of the s-wave channel:

$$\rho_S(\omega) = \frac{4\pi N_C}{N_S^3}\sum_{\vec{p}_{\mathrm{lat}}}\delta\left(\omega - 2E_p\right), E_p = \frac{\mathrm{i}}{a_t}\log\left(\frac{1 - \mathrm{i}\frac{a_t}{2}\frac{\vec{p}_{\mathrm{lat}}^2}{2m_b}}{1 + \mathrm{i}\frac{a_t}{2}\frac{\vec{p}_{\mathrm{lat}}^2}{2m_b}}\right),$$

$$\vec{p}_{\mathrm{lat}}^2 = 4\sum_{i=1}^{3}\sin^2\left(\frac{p_i}{2}\right), p_i = \frac{2\pi n_i}{N_s}, n_i = -\frac{N_s}{2} + 1, ..., \frac{N_s}{2}$$

References

[1] S. Kim, P. Petreczky, and A. Rothkopf, "Lattice nrqcd study of s- and p-wave bottomonium states in a thermal medium with $N_f = 2 + 1$ light flavors," *Phys. Rev. D*, vol. 91, p. 054511, Mar 2015.

[2] K. Fukushima, "Evolution to the quarkgluon plasma," *Rept. Prog. Phys.*, vol. 80, no. 2, p. 022301, 2017.

[3] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, "Effective field theories for heavy quarkonium," *Rev. Mod. Phys.*, vol. 77, p. 1423, 2005.

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