Heavy hadrons spectra on lattice using NRQCD

Protick Mohanta
National Institute of Science Education and Research

DIAS - Helmholtz International Summer School -2017, JINR, Dubna

1. Introduction
2. NRQCD Action
   - Foldy-Wouthuysen Transformation
3. Lattice NRQCD
4. Improvement upto $O(a^4)$
5. Green's Function
6. Relativistic fermions
   - Wilson Fermions
7. Tuning and mass calculation
   - Heavy-heavy correlator
   - Mass difference of Υ and η_b
   - Heavy-light correlator
8. B_c and B_s mesons
   - B_c meson
   - B_s meson
9. Baryons
   - Ω_{bbb}
   - Ω_{bbs}

Introduction

- Lattice methods are powerful techniques in analyzing the spectrum of hadrons. However for hadrons containing heavy quarks particullarly bottom quark are difficult to analyze.
- For spectrum calculation it is necessary that $aM << 1$. For light quarks it is true but for charm quark $aM_c > 0.7$ and for bottom qurak $aM_b > 2$ with lattice spacing $a = 0.12fm$.
- However in hadrons containing heavy quarks the velocities of heavy quarks are non-relativistic. One can use effective theories like NRQCD. $M_\Upsilon = 9390$ MeV where as $2 \times M_b = 8360$ MeV ($\overline{MS}$ Scheme) and $M_{J/\psi} = 3096$ MeV where as $2 \times M_c = 2580$ MeV.

Foldy-Wouthuysen Transformation

- The Dirac equation $H\psi = i\frac{\partial\psi}{\partial t}$ where

$$H = \overrightarrow{\alpha}.(\overrightarrow{P} - e\overrightarrow{A}) + e\phi + m\beta$$

- Non-relativistic limit is reached by making the following transformation $\psi' = e^{iS}\psi$ where $S = -\frac{i}{2m}\beta\overrightarrow{\alpha}.(\overrightarrow{P} - e\overrightarrow{A})$.
- We get $i\frac{\partial\psi'}{\partial t} = H'\psi'$ where

$$\begin{aligned} H' &= e^{iS}He^{-iS} - ie^{iS}\frac{\partial e^{-iS}}{\partial t} \\ &= H + i[S,H] - \frac{1}{2}[S,[S,H]] - \frac{i}{6}[S,[S,[S,H]]] + ... \\ &\quad -\dot{S} - \frac{i}{2}[S,\dot{S}] + \frac{1}{6}[S,[S,\dot{S}]] + ... \end{aligned}$$

- Defining $\theta = \overrightarrow{\alpha}.(\overrightarrow{P} - e\overrightarrow{A})$ we get (up to O(v^4/c^4))

$$
\begin{aligned}
H' &= \beta(m + \frac{\theta^2}{2m} - \frac{\theta^4}{8m^3}) + e\phi - \frac{e}{8m^2}[\theta, [\theta, \phi]] - \frac{i}{8m^2}[\theta, \dot{\theta}] \\
&+ \frac{e\beta}{2m}[\theta, \phi] + i\beta\frac{\dot{\theta}}{2m} - \frac{\theta^3}{3m^2}
\end{aligned}
$$

- writting

$$
\psi' = \begin{pmatrix} u \\ v \end{pmatrix}
$$

$$
\begin{aligned}
i\frac{\partial u}{\partial t} &= [m - \frac{1}{2m}\sum_j D_j^2 - \frac{e}{2m}\sigma.B - \frac{1}{8m^3}(\sum_j D_j^2)^2 \\
&+ e\phi - \frac{e}{8m^2}\nabla.E - \frac{ie}{8m^2}\sigma.(\nabla \times E - E \times \nabla)]u
\end{aligned}
$$

- Similarly like QED we write NRQCD Lagrangian upto $O[(v/c)^6]$
  $\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}_{v^4} + \delta\mathcal{L}_{v^6}$

$$\mathcal{L}_0 = \psi(x)^\dagger (iD_0 + \frac{\vec{D}^2}{2m})\psi(x)$$

$$\begin{aligned}\delta\mathcal{L}_{v^4} &= c_1\frac{1}{8m^3}\psi^\dagger D^4\psi + c_2\frac{g}{8m^2}\psi^\dagger(\vec{D}.\vec{E} - \vec{E}.\vec{D})\psi \\ &+c_3\frac{ie}{8m^2}\psi^\dagger\vec{\sigma}.(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})\psi + c_4\frac{g}{2m}\psi^\dagger\vec{\sigma}.\vec{B}\psi\end{aligned}$$

$$\begin{aligned}\delta\mathcal{L}_{v^6} &= c_5\frac{g}{m^3}\psi^\dagger\{\vec{D}^2, \vec{\sigma}.\vec{B}\}\psi + c_6\frac{ig^2}{m^3}\psi^\dagger(\vec{\sigma}.\vec{E}\times\vec{E})\psi \\ &+c_7\frac{ig}{m^4}\psi^\dagger\{\vec{D}^2, \vec{\sigma}.(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})\}\psi\end{aligned}$$

- $\mathcal{L}_0$ merely gives us Schrodinger equation.
- $c_1, c_2, c_3, c_4 = 1$(tree level).

- To calculate c_7 let us consider the term $T_E = \bar{\psi}(q)\gamma^0 g\phi(q-p)\psi(p)$ with the positive energy spinor

$$\psi(p) = \left(\frac{E_p + m}{2E_p}\right)^{\frac{1}{2}} \left[\begin{array}{c} u \\ \frac{\sigma.p}{E_p+m} u \end{array} \right]$$

$$\begin{aligned} T_E &= \sqrt{\frac{(E_p+m)(E_q+m)}{4E_pE_q}} \\ &\quad \times u^\dagger \left[1 + \frac{p.q + i\sigma.q \times p}{(E_q+m)(E_p+m)}\right] g\phi(q-p)u \end{aligned}$$

- Term containg σ

$$V(p,q) = \left[\frac{i}{4m^2} - \frac{3i}{32m^4}(p^2+q^2)\right] u^\dagger \sigma.(q \times p) g\phi(q-p)u$$

- $c_7 = \frac{3}{64}$.
- c_5 can be calculated from $T_B(p,q) = -\bar{\psi}(q) g\gamma.A(q-p)\psi(p)$ and so on
- $c_5 = \frac{1}{8}, c_6 = -\frac{1}{8}$

- Replace continuum derivatives by lattice derivaties.For quark fields

$$
\begin{aligned}
a\Delta^{+}_{\mu}\psi(x) &= U_{\mu}(x)\psi(x+a\hat{\mu})-\psi(x) \\
a\Delta^{-}_{\mu}\psi(x) &= \psi(x)-U^{\dagger}_{\mu}(x-a\hat{\mu})\psi(x-a\hat{\mu})
\end{aligned}
$$

For gauge fields

$$
\begin{aligned}
a\Delta^{+}_{\rho}F_{\mu\nu}(x) &= U_{\rho}(x)F_{\mu\nu}(x+a\hat{\rho})U^{\dagger}_{\rho}(x)-F_{\mu\nu}(x) \\
a\Delta^{+}_{\rho}F_{\mu\nu}(x) &= F_{\mu\nu}(x)-U^{\dagger}_{\rho}(x-a\hat{\rho})F_{\mu\nu}(x)U_{\rho}(x-a\hat{\rho})
\end{aligned}
$$

Here a is the lattice spacing and $U_{\mu}(x)$ is link variable.

- Symmetric derivatie

$$\Delta^{\pm}=\frac{1}{2}(\Delta^{+}+\Delta^{-})$$

- Laplacian

$$\Delta^{2}\equiv\sum_{i}\Delta^{+}_{i}\Delta^{-}_{i}=\sum_{i}\Delta^{-}_{i}\Delta^{+}_{i}$$

Improvement upto $O(a^4)$

- For $a = 0.12 fm$ it is desirable to correct operators upto order $O(a^4)$.
- Symmetric derivative

$$
\begin{aligned}
\Delta_i^{\pm} f(x) &= \frac{1}{2a}[f(x + a\hat{i}) - f(x - a\hat{i})] \\
&= \partial_i f + \frac{a^2}{6}\partial_i^3 f \\
&= \partial_i f + \frac{a^2}{6}\Delta_i^{\pm}\Delta_i^{+}\Delta_i^{-} f \\
\partial_i f &= \Delta_i^{\pm} f - \frac{a^2}{6}\Delta_i^{+}\Delta_i^{\pm}\Delta_i^{-} f \\
\tilde{\Delta}_i^{\pm} f &= \Delta_i^{\pm} f - \frac{a^2}{6}\Delta_i^{+}\Delta_i^{\pm}\Delta_i^{-} f
\end{aligned}
$$

- Laplacian

$$
\tilde{\Delta}^2 = \Delta^2 - \frac{a^2}{12}\sum_i [\Delta_i^{+}\Delta_i^{-}]^2
$$

- Gauge fields corrected upto $O(a^4)$ {using cloverleaf}

$$
g\tilde{F}_{\mu\nu}(x) = gF_{\mu\nu}(x) - \frac{a^4}{6}[\Delta_\mu^{+}\Delta_\mu^{-} + \Delta_\nu^{+}\Delta_\nu^{-}]gF_{\mu\nu}(x)
$$

Green's Function

- The Lagrangian has the following form

$$\mathcal{L} = \psi^\dagger(x,t)D_4\psi(x,t) + \psi^\dagger(x,t)H\psi(x,t)$$

- H contains spatial derivaties only. E.O.M. corrsponding to $\psi^\dagger$

$$\begin{aligned} D_4\psi(x,t) + H\psi(x,t) &= 0 \text{ after discretization} \\ U_t(x)\psi(x,t+1) - \psi(x,t) + aH\psi(x,t) &= 0 \end{aligned}$$

Green's function obeys

$$\begin{aligned} & U_t(x,t)G(x,t+1;0,0) - (1-aH)G(x,t;0,0) = \delta_{x,0}\delta_{t,0} \\ \Rightarrow \quad & G(x,t+1;0,0) = U_t^\dagger(x,t)(1-aH)G(x,t;0,0) \end{aligned}$$

- From renormalization considerations

$$
\begin{aligned}
G(x,t+1;0,0) &= (1-\frac{aH_0}{2})(1-\frac{a\delta H}{2})U_t(x,t)^\dagger \\
&\quad (1-\frac{a\delta H}{2})(1-\frac{aH_0}{2})G(x,t;0,0)
\end{aligned}
$$

H_0 and δH are related as $H = H_0 + \delta H$. For stability purpose we modify

$$
\begin{aligned}
G(x,t+1;0,0) &= (1-\frac{aH_0}{2n})^n(1-\frac{a\delta H}{2})U_t(x,t)^\dagger \\
&\quad (1-\frac{a\delta H}{2})(1-\frac{aH_0}{2n})^n G(x,t;0,0)
\end{aligned}
$$

with $G(x,t;0,0) = 0$ for $t < 0$ and $G(x,t;0,0) = \delta_{x,0}$ for $t = 0$. From the above equation it is evident that $n > \frac{3}{2m}$.

Relativistic fermions

- Action for free Dirac field

$$S[\psi, \bar{\psi}] = \int d^4x \bar{\psi}(x)(i\gamma^\mu \partial_\mu - M)\psi(x)$$

Lattice version of the above action is

$$S = \sum_{n,m,\alpha,\beta} \hat{\bar{\psi}}_\alpha(n) K_{\alpha\beta}(n,m) \hat{\psi}_\beta(m)$$

where $K_{\alpha\beta}(n,m)$ is given by

$$K_{\alpha\beta}(n,m) = \sum_\mu \frac{1}{2}(\gamma_\mu)_{\alpha\beta}[\delta_{m,n+\hat{\mu}} - \delta_{m,n-\hat{\mu}}] + \hat{M}\delta_{\alpha\beta}\delta_{m,n}$$

- This action has doubling problem.

$$\delta_{n,m} = \int_{-\pi}^{\pi} \frac{d^4\hat{p}}{(2\pi)^4} e^{i\hat{p}.(m-n)}$$

$$K_{\alpha\beta}(n,m) = \int_{-\pi}^{\pi} \frac{d^4\hat{p}}{(2\pi)^4} [i\gamma_\mu \sin(\hat{p}_\mu) + \hat{M}] e^{i\hat{p}.(n-m)}$$

- Add a term to the action such that it is goes to zero in the continuum limit

$$S^W = S - \frac{r}{2}\sum_n \bar{\hat{\psi}}(n)\hat{\Box}\hat{\psi}(n)$$

- r is the Wilson parameter, lattice Laplacean $\hat{\Box}$ is

$$\hat{\Box}\hat{\psi}_\alpha(na) = \sum_\mu [\hat{\psi}_\alpha(na+\hat{\mu}a) + \hat{\psi}_\alpha(na-\hat{\mu}a) - 2\hat{\psi}_\alpha(na)]$$

- Use $\int d^4x \to a^4\sum_n$, $\hat{\Box} = a^2\Box$ and $\psi_\alpha(x) \to \frac{1}{a^{3/2}}\hat{\psi}_\alpha(n)$.
- The additional term vanishes linearly with a.

$$S^W = \sum_{n,m} \bar{\hat{\psi}}(n)K^W(n,m)\hat{\psi}(m)$$

where $K^W(n,m)$ is given by

$$K^W(n,m) = \sum_\mu \frac{1}{2}[(\gamma_\mu - r)\delta_{m,n+\hat{\mu}} - (\gamma_\mu + r)\delta_{m,n-\hat{\mu}}] + (\hat{M}+4r)\delta_{m,n}$$

- Momentum space representation of K^W

$$K^W(n,m) = \int_{-\pi}^{\pi} \frac{d^4\hat{p}}{(2\pi)^4}[i\gamma_\mu \sin(\hat{p}_\mu) + \hat{M} + 2r\bar{p}_\mu\bar{p}_\mu]e^{i\hat{p}.(n-m)}$$

with $\bar{p}_\mu = \sin(\frac{\hat{p}_\mu}{2})$

- This action is free from doubling problem but it breaks the chiral symmetry ($\psi \rightarrow e^{i\theta\gamma_5}\psi$) as the Wilson term does not contain $\gamma_\mu \Rightarrow$ "Overlap Fermions"
- The modified Dirac operator D satisfies "GW-relation"

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

- Any D of the form

$$\begin{aligned} D &= \frac{1}{a}(1-V) \text{ with} \\ & V^\dagger V = 1 \text{ and } V^\dagger = \gamma_5 V \gamma_5 \end{aligned}$$

satisfies GW-relation.

- An explicit solution for D was given by Neuberger

$$\begin{aligned} V &= A(A^\dagger A)^{-\frac{1}{2}} \text{ where} \\ A &= 1 - K^W \end{aligned}$$

- Other fermions = HISQ, Domain Wall, Twisted mass .....

Tuning and mass calculation

- In NRQCD Lagrangian the rest mass term is not included.
- In order to tune b-quark we calculated 'kinetic mass' of η_b meson

$$\begin{aligned} E(p) - E(0) &= \sqrt{p^2 + M^2} - M \\ \Rightarrow \Delta E + M &= \sqrt{p^2 + M^2} \text{ where } \Delta E = E(p) - E(0) \\ \Rightarrow (\Delta E)^2 + 2M\Delta E &= p^2 \\ \Rightarrow M &= \frac{p^2 - (\Delta E)^2}{2\Delta E} \end{aligned}$$

Heavy-heavy correlator

- For mesons containing both heavy quarks let the heavy quark and anti-quark are created by two component spinor $\psi^\dagger$ and χ and their destruction operators are ψ and $\chi^\dagger$. As anti-quaks transform by $\bar{3}$ under color rotation so it is convinient to rename the anti-quark spinor.

$$
\begin{aligned}
C(\vec{p},t) &= \sum_{x} \langle 0| e^{i\vec{p}.\vec{x}} O(\vec{x},t) O^\dagger(\vec{0},0)|0\rangle \\
&= \sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle 0|\chi^\dagger(x)\Gamma_{sk}(x)\psi(x)\psi^\dagger(0)\Gamma^\dagger_{sc}(0)\chi(0)|0\rangle \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle 0|\chi(0)\chi^\dagger(x)\Gamma_{sk}(x)\psi(x)\psi^\dagger(0)\Gamma^\dagger_{sc}(0)0\rangle \\
&= \sum_{\vec{x}} e^{i\vec{p}.\vec{x}} Tr[G^\dagger(x,0)\Gamma_{sk}(x)G(x,0)\Gamma^\dagger_{sc}(0)]
\end{aligned}
$$

- In the last line we have used $G^\dagger(x,0) = -[\chi(x)\chi^\dagger(0)]^\dagger$. Here $\Gamma(x) = \Omega\phi(x)$. ϕ is the smearing operator and Ω is a 2×2 matrix in spin space. $\Omega = I$ for pseudoscalar particles and $\Omega = \sigma_i$ for vector particles.

- We ran our code on 40, $24^3 64$ milc lattices. Nr-loop $n = 3$ and mass is tuned to $m = 0.759$.
- Here we shown the correlators for η_b obtained at momenta $\vec{p} = \frac{2\pi}{L}(2,0,0)$ and $\vec{p} = \frac{2\pi}{L}(0,0,0)$ with $L = 24$. We find kinetic mass of $\eta_b = 9.42$ GeV.

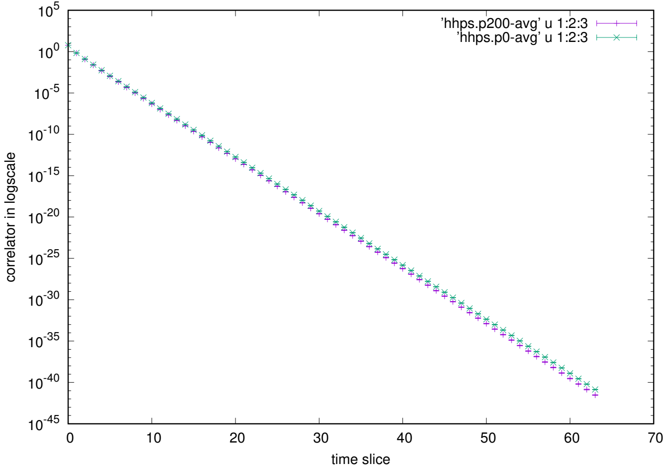

Figure: Heavy-heavy p200 vs p0

Mass difference of Υ and η_b

- The following plot shows the mass differencebetween Υ and η_b. For fit range 7-17 we find the splitting to be $= 131$ MeV.

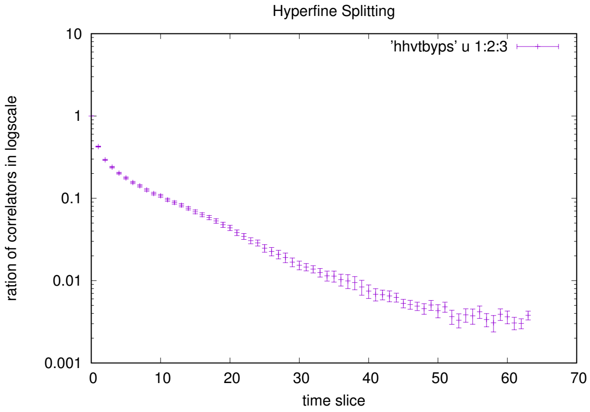

Figure: Hyperfine splitting

Heavy-light correlator(light = overlap)

- For mesons containing one heavy quark and one light anti-quark the interpolating operator is $Q^{\dagger}(x)\Gamma(x)q(x)$.

$$
\begin{aligned}
C(\vec{p},t) &= \sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle 0|q^{\dagger}(x)\Gamma^{\dagger}_{sk}(x)Q(x)Q^{\dagger}(0)\Gamma_{sc}(0)q(0)|0\rangle \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle 0|q(0)q^{\dagger}(x)\Gamma^{\dagger}_{sk}(x)Q(x)Q^{\dagger}(0)\Gamma_{sc}(0)|0\rangle \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} Tr[M(0,x)\gamma_4\Gamma^{\dagger}_{sk}(x)G(x,0)\Gamma_{sc}(0)] \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} Tr[\gamma_5 M(x,0)^{\dagger}\gamma_5\gamma_4\Gamma^{\dagger}_{sk}(x)G(x,0)\Gamma_{sc}(0)]
\end{aligned}
$$

B_c meson

- Plot for B_c meson correlators.

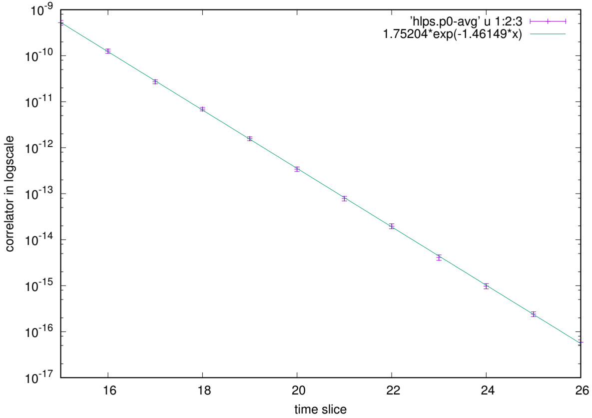

Figure: Heavy-light meson correlators obtained at zero momentum

- From fit $E_{B_c} = 1.46149 = 1.46149 \times\ 197.3/0.12 = 2402$ Mev

- As we used kinetic mass in tuning the bottom and charm masses we had to use the following formula to calculate the mass of B_c.

$$M_{B_c} = E_{B_c} + \frac{1}{2}(M_{\eta_b} - E_{\eta_b}) + \frac{1}{2}(M_{\eta_c} - E_{\eta_c})$$

  Here $E_{B_c}, E_{\eta_b}, E_{\eta_c}$ are the simulated masses and M_{η_b}, M_{η_c} are their pdg values.
- $M_{B_c} = 2402 + [(2980 - 2190)/2] + [(9391 - 2472)/2] = 6256.5$ Mev with error $= \pm$ 20 MeV

B_s meson

- s-quark has been directly tuned to produce $s\bar{s}$ pseudoscalar mass to be 686 MeV. Here we used the following formula to calculate the mass of pseudoscalar s-quarkonium state

$$M_{s\bar{s}} = \sqrt{2M_K^2 - M_\pi^2}$$

where M_K and M_π are kaon and pion masses.

- B_s meson mass is calculated as

$$M_{B_s} = E_{B_s} + \frac{1}{2}(M_{\eta_b} - E_{\eta_b})$$

$M_{B_s} = 1640 + (9389 - 2476)/2 = 5096$ MeV

Heavy-light correlator(light = hisq)

- $Q = \begin{pmatrix} \phi \\ 0 \end{pmatrix}$, $\Gamma = \gamma_5$ or $\Gamma = \gamma_k$

$$
\begin{aligned}
C(\vec{p}, t) &= \sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle 0| q^\dagger(x) \Gamma^\dagger_{sk}(x) Q(x) Q^\dagger(0) \Gamma_{sc}(0) q(0)|0\rangle \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle 0| q(0) q^\dagger(x) \Gamma Q(x) Q^\dagger(0) \Gamma|0\rangle \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} Tr[M(0,x)\gamma_4 \Gamma G(x,0)\Gamma] \\
&= -\sum_{\vec{x}} e^{i\vec{p}.\vec{x}} Tr[\gamma_5 M(x,0)^\dagger \gamma_5 \Gamma G(x,0)\Gamma]
\end{aligned}
$$

- $G(x,0)$ is now a 4×4 matrix in spinor space having vanishing lower components but it is in Dirac representation of gamma matrices. We can convert it to milc gamma representation by an unitary transformation $S = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_y & \sigma_y \\ -\sigma_y & \sigma_y \end{pmatrix}$

- Plot for B_s meson correlators.

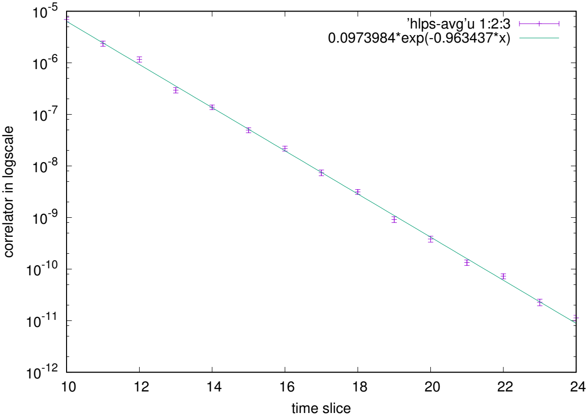

Figure: Heavy-light meson correlators obtained at zero momentum

- $M_{B_s} = 1584 + (9389 - 2476)/2 = 5040$ MeV

Ω_{bbb} baryon

- Interpolator $(\mathcal{O}_k)_\alpha = \epsilon_{abc}({Q^a}^T C\gamma_k Q^b)Q^c_\alpha$ with $C = \gamma_4\gamma_4$

$$C_{ij\alpha\beta}(t) = \sum_{\vec{x}} \langle 0|[\mathcal{O}_i(\vec{x},t)]_\alpha [\mathcal{O}_j^\dagger(\vec{0},0)]_\beta|0\rangle$$
$$= \sum_{\vec{x}} \epsilon_{abc}\epsilon_{fgh} G^{ch}_{\alpha\beta}(x,0)\, Tr[C\gamma_i G^{bg}(x,0)\overline{C\gamma_j}\, {G^{af}}^T(x,0)]$$

- The correlator has overlap with both spin 3/2 and spin 1/2 states

$$C_{ij}(t) = Z_{3/2}e^{-E_{3/2}t}\Pi P^{3/2}_{ij} + Z_{1/2}e^{-E_{1/2}t}\Pi P^{1/2}_{ij}$$

$\Pi = \frac{1}{2}(1+\gamma_4)$, $P^{3/2}_{ij} = \delta_{ij} - \frac{1}{3}\gamma_i\gamma_j$, $P^{1/2}_{ij} = \frac{1}{3}\gamma_i\gamma_j$ and $P^{3/2}_{ij}.P^{1/2}_{jk} = 0$.

- $P^{3/2}_{xx}.C_{xx} + P^{3/2}_{xy}.C_{yx} + P^{3/2}_{xz}.C_{zx} = \frac{2}{3}Z_{3/2}\Pi e^{-E_{3/2}t}$

- Plot for Ω_{bbb} correlator

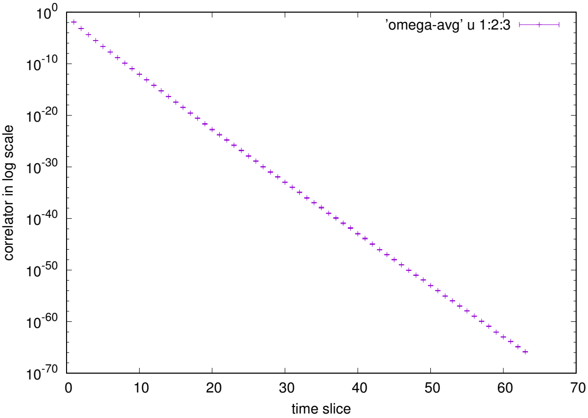

Figure: Omega 3/2

- $M_{\Omega_{bbb}} = E_{\Omega_{bbb}} + \frac{3}{2}(M_{\eta_b} - E_{\eta_b})$=14.38 GeV with error = $\pm$ 20 MeV

Ω_{bbs} baryon

- Interpolator $(\mathcal{O}_k)_\alpha = \epsilon_{abc}({Q^a}^T C\gamma_k Q^b)s^c_\alpha$

$$
\begin{aligned}
C_{ij\alpha\beta}(t) &= \sum_{\vec{x}} \langle 0|[\mathcal{O}_i(\vec{x},t)]_\alpha [\mathcal{O}_j^\dagger(\vec{0},0)]_\beta|0\rangle \\
&= \sum_{\vec{x}} \epsilon_{abc}\epsilon_{fgh}[M^{ch}(x,0).\gamma_4]_{\alpha\beta} Tr[C\gamma_i G^{bg}(x,0)\overline{C\gamma_j}{G^{af}}^T(x,0)]
\end{aligned}
$$

- Change $G(x,0)$ into milc gamma representation.
- $P^{3/2}_{xx}.C_{xx} + P^{3/2}_{xy}.C_{yx} + P^{3/2}_{xz}.C_{zx} = \frac{2}{3}Z_{3/2}\Pi e^{-E_{3/2}t}$

- Plot for Ω_{bbs} correlator

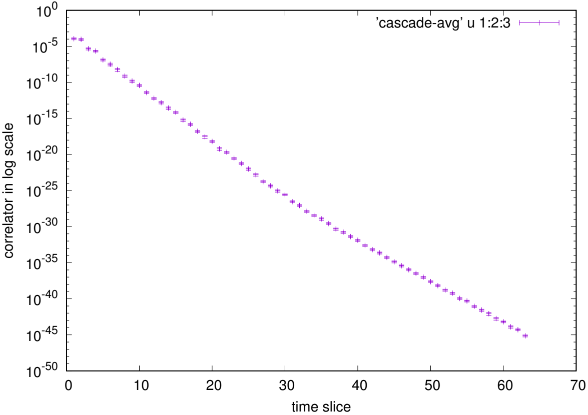

Figure: $Omega_{bbs}$ 3/2

- $M_{\Omega_{bbs}} = E_{\Omega_{bbs}} + (M_{\eta_b} - E_{\eta_b})$=9.81 GeV with error = ± 40 MeV

THANK YOU

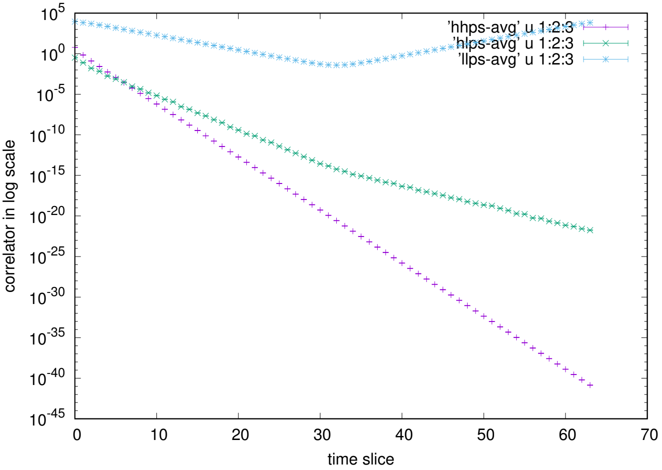

Figure: hh hl ll mesons

Green's Function for anti-quark

- Antiquarks transform as $\bar{3}$'s under color rotation i.e change $U_{x,\mu} \to U^*_{x,\mu}$. Replace ψ by $\tilde{\chi}$. To compare it with Dirac's theory we change the variable as $\chi = \tilde{\chi}^*$

$$\begin{aligned}\tilde{\chi}(x,t)^\dagger U_t^*(x)\tilde{\chi}(x,t+1) &= (\chi^*(x,t))^\dagger U_t^*(x)\chi^*(x,t+1) \\ &= (\chi(x,t))^T (U_t^\dagger)^T(x)(\chi^\dagger(x,t+1))^T \\ &= -\chi^\dagger(x,t+1)U_t^\dagger(x)\chi(x,t)\end{aligned}$$

We used the fact that it is 1×1 quantity so we can ignore the transpose sigh altogether and we put the minus sign because χ's are fermionic field they obey Grassmann algebra. So if the we write the quark action as $S_Q = \psi^\dagger K \psi$ then we for anti-quark we have $S_{\bar{Q}} = -\chi^\dagger K^\dagger \chi$.