

Heavy hadrons spectra on lattice using NRQCD

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- Lattice methods are powerful techniques in analyzing the spectrum of hadrons. However for hadrons containing heavy quarks particularly bottom quark are difficult to analyze.
- For spectrum calculation it is necessary that $aM \ll 1$. For light quarks it is true but for charm quark $aM_c > 0.7$ and for bottom quark $aM_b > 2$ with lattice spacing $a = 0.12fm$.
- However in hadrons containing heavy quarks the velocities of heavy quarks are non-relativistic. One can use effective theories like NRQCD.
 $M_\Upsilon = 9390$ MeV where as $2 \times M_b = 8360$ MeV (\overline{MS} Scheme) and $M_{J/\psi} = 3096$ MeV where as $2 \times M_c = 2580$ MeV.

- The Dirac equation $H\psi = i\frac{\partial\psi}{\partial t}$ where

$$H = \vec{\alpha} \cdot (\vec{P} - e\vec{A}) + e\phi + m\beta$$

- Non-relativistic limit is reached by making the following transformation $\psi' = e^{iS}\psi$ where $S = -\frac{i}{2m}\beta\vec{\alpha} \cdot (\vec{P} - e\vec{A})$.
- We get $i\frac{\partial\psi'}{\partial t} = H'\psi'$ where

$$\begin{aligned} H' &= e^{iS}He^{-iS} - ie^{iS}\frac{\partial e^{-iS}}{\partial t} \\ &= H + i[S, H] - \frac{1}{2}[S, [S, H]] - \frac{i}{6}[S, [S, [S, H]]] + \dots \\ &\quad - \dot{S} - \frac{i}{2}[S, \dot{S}] + \frac{1}{6}[S, [S, \dot{S}]] + \dots \end{aligned}$$

- Defining $\theta = \vec{\alpha} \cdot (\vec{P} - e\vec{A})$ we get (up to $O(v^4/c^4)$)

$$\begin{aligned}
 H' = & \beta\left(m + \frac{\theta^2}{2m} - \frac{\theta^4}{8m^3}\right) + e\phi - \frac{e}{8m^2}[\theta, [\theta, \phi]] - \frac{i}{8m^2}[\theta, \dot{\theta}] \\
 & + \frac{e\beta}{2m}[\theta, \phi] + i\beta\frac{\dot{\theta}}{2m} - \frac{\theta^3}{3m^2}
 \end{aligned}$$

- writing

$$\psi' = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{aligned}
 i\frac{\partial u}{\partial t} = & \left[m - \frac{1}{2m} \sum_j D_j^2 - \frac{e}{2m} \sigma \cdot B - \frac{1}{8m^3} (\sum_j D_j^2)^2\right. \\
 & \left. + e\phi - \frac{e}{8m^2} \nabla \cdot E - \frac{ie}{8m^2} \sigma \cdot (\nabla \times E - E \times \nabla)\right] u
 \end{aligned}$$

- Similarly like QED we write NRQCD Lagrangian upto $O[(v/c)^6]$

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}_{v^4} + \delta\mathcal{L}_{v^6}$$

$$\mathcal{L}_0 = \psi(x)^\dagger (iD_0 + \frac{\vec{D}^2}{2m})\psi(x)$$

$$\begin{aligned} \delta\mathcal{L}_{v^4} = & c_1 \frac{1}{8m^3} \psi^\dagger D^4 \psi + c_2 \frac{g}{8m^2} \psi^\dagger (\vec{D} \cdot \vec{E} - \vec{E} \cdot \vec{D}) \psi \\ & + c_3 \frac{ie}{8m^2} \psi^\dagger \vec{\sigma} \cdot (\vec{D} \times \vec{E} - \vec{E} \times \vec{D}) \psi + c_4 \frac{g}{2m} \psi^\dagger \vec{\sigma} \cdot \vec{B} \psi \end{aligned}$$

$$\begin{aligned} \delta\mathcal{L}_{v^6} = & c_5 \frac{g}{m^3} \psi^\dagger \{ \vec{D}^2, \vec{\sigma} \cdot \vec{B} \} \psi + c_6 \frac{ig^2}{m^3} \psi^\dagger (\vec{\sigma} \cdot \vec{E} \times \vec{E}) \psi \\ & + c_7 \frac{ig}{m^4} \psi^\dagger \{ \vec{D}^2, \vec{\sigma} \cdot (\vec{D} \times \vec{E} - \vec{E} \times \vec{D}) \} \psi \end{aligned}$$

- \mathcal{L}_0 merely gives us Schrodinger equation.
- $c_1, c_2, c_3, c_4 = 1$ (tree level).

- To calculate c_7 let us consider the term $T_E = \bar{\psi}(q)\gamma^0 g\phi(q-p)\psi(p)$ with the positive energy spinor

$$\psi(p) = \left(\frac{E_p + m}{2E_p}\right)^{\frac{1}{2}} \begin{bmatrix} u \\ \frac{\sigma \cdot p}{E_p + m} u \end{bmatrix}$$

$$T_E = \sqrt{\frac{(E_p + m)(E_q + m)}{4E_p E_q}} \times u^\dagger \left[1 + \frac{p \cdot q + i\sigma \cdot q \times p}{(E_q + m)(E_p + m)} \right] g\phi(q-p)u$$

- Term containing σ

$$V(p, q) = \left[\frac{i}{4m^2} - \frac{3i}{32m^4}(p^2 + q^2) \right] u^\dagger \sigma \cdot (q \times p) g\phi(q-p)u$$

- $c_7 = \frac{3}{64}$.
- c_5 can be calculated from $T_B(p, q) = -\bar{\psi}(q)g\gamma \cdot A(q-p)\psi(p)$ and so on
- $c_5 = \frac{1}{8}, c_6 = -\frac{1}{8}$

- Replace continuum derivatives by lattice derivatives. For quark fields

$$a\Delta_{\mu}^{+}\psi(x) = U_{\mu}(x)\psi(x + a\hat{\mu}) - \psi(x)$$

$$a\Delta_{\mu}^{-}\psi(x) = \psi(x) - U_{\mu}^{\dagger}(x - a\hat{\mu})\psi(x - a\hat{\mu})$$

For gauge fields

$$a\Delta_{\rho}^{+}F_{\mu\nu}(x) = U_{\rho}(x)F_{\mu\nu}(x + a\hat{\rho})U_{\rho}^{\dagger}(x) - F_{\mu\nu}(x)$$

$$a\Delta_{\rho}^{-}F_{\mu\nu}(x) = F_{\mu\nu}(x) - U_{\rho}^{\dagger}(x - a\hat{\rho})F_{\mu\nu}(x)U_{\rho}(x - a\hat{\rho})$$

Here a is the lattice spacing and $U_{\mu}(x)$ is link variable.

- Symmetric derivative

$$\Delta^{\pm} = \frac{1}{2}(\Delta^{+} + \Delta^{-})$$

- Laplacian

$$\Delta^2 \equiv \sum_i \Delta_i^{+} \Delta_i^{-} = \sum_i \Delta_i^{-} \Delta_i^{+}$$

Improvement upto $O(a^4)$

- For $a = 0.12fm$ it is desirable to correct operators upto order $O(a^4)$.
- Symmetric derivative

$$\begin{aligned}\Delta_i^\pm f(x) &= \frac{1}{2a}[f(x + a\hat{i}) - f(x - a\hat{i})] \\ &= \partial_i f + \frac{a^2}{6}\partial_i^3 f \\ &= \partial_i f + \frac{a^2}{6}\Delta_i^\pm \Delta_i^\pm \Delta_i^\mp f \\ \partial_i f &= \Delta_i^\pm f - \frac{a^2}{6}\Delta_i^\pm \Delta_i^\pm \Delta_i^\mp f \\ \tilde{\Delta}_i^\pm f &= \Delta_i^\pm f - \frac{a^2}{6}\Delta_i^\pm \Delta_i^\pm \Delta_i^\mp f\end{aligned}$$

- Laplacian

$$\tilde{\Delta}^2 = \Delta^2 - \frac{a^2}{12} \sum_i [\Delta_i^+ \Delta_i^-]^2$$

- Gauge fields corrected upto $O(a^4)$ {using cloverleaf}

$$g\tilde{F}_{\mu\nu}(x) = gF_{\mu\nu}(x) - \frac{a^4}{6}[\Delta_\mu^+ \Delta_\mu^- + \Delta_\nu^+ \Delta_\nu^-]gF_{\mu\nu}(x)$$

- The Lagrangian has the following form

$$\mathcal{L} = \psi^\dagger(x, t)D_4\psi(x, t) + \psi^\dagger(x, t)H\psi(x, t)$$

- H contains spatial derivatives only. E.O.M. corresponding to ψ^\dagger

$$D_4\psi(x, t) + H\psi(x, t) = 0 \text{ after discretization}$$

$$U_t(x)\psi(x, t+1) - \psi(x, t) + aH\psi(x, t) = 0$$

Green's function obeys

$$U_t(x, t)G(x, t+1; 0, 0) - (1 - aH)G(x, t; 0, 0) = \delta_{x,0}\delta_{t,0}$$
$$\Rightarrow G(x, t+1; 0, 0) = U_t^\dagger(x, t)(1 - aH)G(x, t; 0, 0)$$

- From renormalization considerations

$$G(x, t + 1; 0, 0) = \left(1 - \frac{aH_0}{2}\right)\left(1 - \frac{a\delta H}{2}\right)U_t(x, t)^\dagger \\ \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2}\right)G(x, t; 0, 0)$$

H_0 and δH are related as $H = H_0 + \delta H$. For stability purpose we modify

$$G(x, t + 1; 0, 0) = \left(1 - \frac{aH_0}{2n}\right)^n\left(1 - \frac{a\delta H}{2}\right)U_t(x, t)^\dagger \\ \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)^n G(x, t; 0, 0)$$

with $G(x, t; 0, 0) = 0$ for $t < 0$ and $G(x, t; 0, 0) = \delta_{x,0}$ for $t = 0$. From the above equation it is evident that $n > \frac{3}{2m}$.

- Action for free Dirac field

$$S[\psi, \bar{\psi}] = \int d^4x \bar{\psi}(x)(i\gamma^\mu \partial_\mu - M)\psi(x)$$

Lattice version of the above action is

$$S = \sum_{n,m,\alpha,\beta} \bar{\psi}_\alpha(n) K_{\alpha\beta}(n,m) \hat{\psi}_\beta(m)$$

where $K_{\alpha\beta}(n,m)$ is given by

$$K_{\alpha\beta}(n,m) = \sum_{\mu} \frac{1}{2} (\gamma_\mu)_{\alpha\beta} [\delta_{m,n+\hat{\mu}} - \delta_{m,n-\hat{\mu}}] + \hat{M} \delta_{\alpha\beta} \delta_{m,n}$$

- This action has doubling problem.

$$\delta_{n,m} = \int_{-\pi}^{\pi} \frac{d^4 \hat{p}}{(2\pi)^4} e^{i\hat{p} \cdot (m-n)}$$

$$K_{\alpha\beta}(n,m) = \int_{-\pi}^{\pi} \frac{d^4 \hat{p}}{(2\pi)^4} [i\gamma_\mu \sin(\hat{p}_\mu) + \hat{M}] e^{i\hat{p} \cdot (n-m)}$$

- Add a term to the action such that it goes to zero in the continuum limit

$$S^W = S - \frac{r}{2} \sum_n \bar{\hat{\psi}}(n) \hat{\square} \hat{\psi}(n)$$

- r is the Wilson parameter, lattice Laplacean $\hat{\square}$ is

$$\hat{\square} \hat{\psi}_\alpha(na) = \sum_\mu [\hat{\psi}_\alpha(na + \hat{\mu}a) + \hat{\psi}_\alpha(na - \hat{\mu}a) - 2\hat{\psi}_\alpha(na)]$$

- Use $\int d^4x \rightarrow a^4 \sum_n$, $\hat{\square} = a^2 \square$ and $\psi_\alpha(x) \rightarrow \frac{1}{a^{3/2}} \hat{\psi}_\alpha(n)$.
- The additional term vanishes linearly with a .

$$S^W = \sum_{n,m} \bar{\hat{\psi}}(n) K^W(n, m) \hat{\psi}(m)$$

where $K^W(n, m)$ is given by

$$K^W(n, m) = \sum_\mu \frac{1}{2} [(\gamma_\mu - r)\delta_{m, n+\hat{\mu}} - (\gamma_\mu + r)\delta_{m, n-\hat{\mu}}] + (\hat{M} + 4r)\delta_{m, n}$$

- Momentum space representation of K^W

$$K^W(n, m) = \int_{-\pi}^{\pi} \frac{d^4 \hat{p}}{(2\pi)^4} [i\gamma_{\mu} \sin(\hat{p}_{\mu}) + \hat{M} + 2r\bar{p}_{\mu}\bar{p}_{\mu}] e^{i\hat{p} \cdot (n-m)}$$

with $\bar{p}_{\mu} = \sin(\frac{\hat{p}_{\mu}}{2})$

- This action is free from doubling problem but it breaks the chiral symmetry ($\psi \rightarrow e^{i\theta\gamma_5}\psi$) as the Wilson term does not contain $\gamma_{\mu} \Rightarrow$ "Overlap Fermions"
- The modified Dirac operator D satisfies "GW-relation"

$$\gamma_5 D + D \gamma_5 = a D \gamma_5 D$$

- Any D of the form

$$D = \frac{1}{a}(1 - V) \text{ with}$$

$$V^{\dagger} V = 1 \text{ and } V^{\dagger} = \gamma_5 V \gamma_5$$

satisfies GW-relation.

- An explicit solution for D was given by Neuberger

$$V = A(A^{\dagger}A)^{-\frac{1}{2}} \text{ where}$$

$$A = 1 - K^W$$

- Other fermions = HISQ, Domain Wall, Twisted mass

- In NRQCD Lagrangian the rest mass term is not included.
- In order to tune b-quark we calculated 'kinetic mass' of η_b meson

$$\begin{aligned}E(p) - E(0) &= \sqrt{p^2 + M^2} - M \\ \Rightarrow \Delta E + M &= \sqrt{p^2 + M^2} \text{ where } \Delta E = E(p) - E(0) \\ \Rightarrow (\Delta E)^2 + 2M\Delta E &= p^2 \\ \Rightarrow M &= \frac{p^2 - (\Delta E)^2}{2\Delta E}\end{aligned}$$

- For mesons containing both heavy quarks let the heavy quark and anti-quark are created by two component spinor ψ^\dagger and χ and their destruction operators are ψ and χ^\dagger . As anti-quarks transform by $\bar{3}$ under color rotation so it is convenient to rename the anti-quark spinor.

$$\begin{aligned}
 C(\vec{p}, t) &= \sum_x \langle 0 | e^{i\vec{p}\cdot\vec{x}} O(\vec{x}, t) O^\dagger(\vec{0}, 0) | 0 \rangle \\
 &= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | \chi^\dagger(x) \Gamma_{sk}(x) \psi(x) \psi^\dagger(0) \Gamma_{sc}^\dagger(0) \chi(0) | 0 \rangle \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | \chi(0) \chi^\dagger(x) \Gamma_{sk}(x) \psi(x) \psi^\dagger(0) \Gamma_{sc}^\dagger(0) | 0 \rangle \\
 &= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \text{Tr}[G^\dagger(x, 0) \Gamma_{sk}(x) G(x, 0) \Gamma_{sc}^\dagger(0)]
 \end{aligned}$$

- In the last line we have used $G^\dagger(x, 0) = -[\chi(x)\chi^\dagger(0)]^\dagger$. Here $\Gamma(x) = \Omega\phi(x)$. ϕ is the smearing operator and Ω is a 2×2 matrix in spin space. $\Omega = I$ for pseudoscalar particles and $\Omega = \sigma_i$ for vector particles.

- We ran our code on $40, 24^3 64$ milc lattices. Nr-loop $n = 3$ and mass is tuned to $m = 0.759$.
- Here we shown the correlators for η_b obtained at momenta $\vec{p} = \frac{2\pi}{L}(2, 0, 0)$ and $\vec{p} = \frac{2\pi}{L}(0, 0, 0)$ with $L = 24$. We find kinetic mass of $\eta_b = 9.42$ GeV.

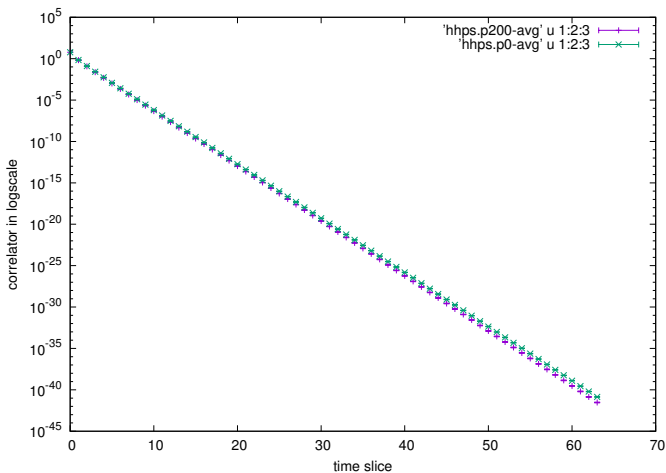


Figure: Heavy-heavy p200 vs p0

- The following plot shows the mass difference between Υ and η_b . For fit range 7-17 we find the splitting to be = 131 MeV.

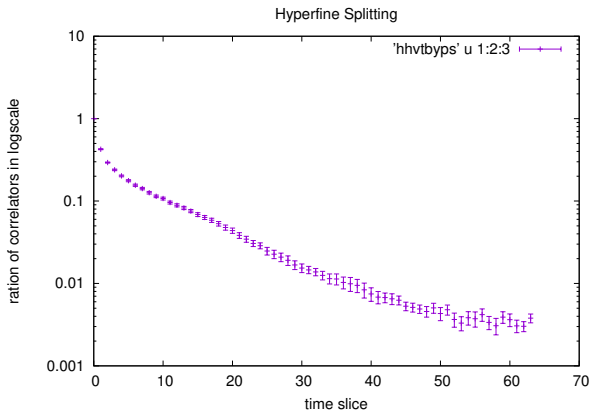


Figure: Hyperfine splitting

- For mesons containing one heavy quark and one light anti-quark the interpolating operator is $Q^\dagger(x)\Gamma(x)q(x)$.

$$\begin{aligned}
 C(\vec{p}, t) &= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | q^\dagger(x) \Gamma_{sk}^\dagger(x) Q(x) Q^\dagger(0) \Gamma_{sc}(0) q(0) | 0 \rangle \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | q(0) q^\dagger(x) \Gamma_{sk}^\dagger(x) Q(x) Q^\dagger(0) \Gamma_{sc}(0) | 0 \rangle \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \text{Tr}[M(0, x) \gamma_4 \Gamma_{sk}^\dagger(x) G(x, 0) \Gamma_{sc}(0)] \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \text{Tr}[\gamma_5 M(x, 0)^\dagger \gamma_5 \gamma_4 \Gamma_{sk}^\dagger(x) G(x, 0) \Gamma_{sc}(0)]
 \end{aligned}$$

- Plot for B_c meson correlators.

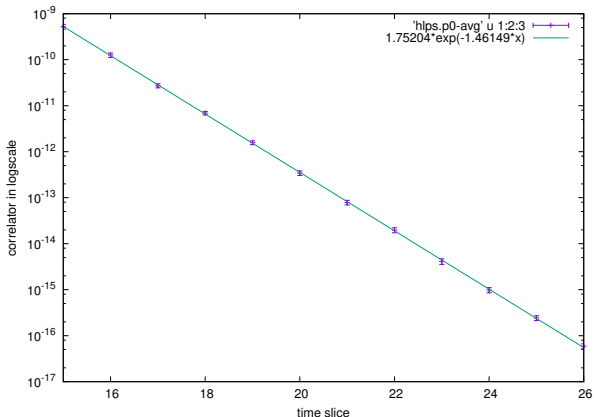


Figure: Heavy-light meson correlators obtained at zero momentum

- From fit $E_{B_c} = 1.46149 = 1.46149 \times 197.3/0.12 = 2402$ Mev

- As we used kinetic mass in tuning the bottom and charm masses we had to use the following formula to calculate the mass of B_c .

$$M_{B_c} = E_{B_c} + \frac{1}{2}(M_{\eta_b} - E_{\eta_b}) + \frac{1}{2}(M_{\eta_c} - E_{\eta_c})$$

Here E_{B_c} , E_{η_b} , E_{η_c} are the simulated masses and M_{η_b} , M_{η_c} are their pdg values.

- $M_{B_c} = 2402 + [(2980 - 2190)/2] + [(9391 - 2472)/2] = 6256.5 \text{ MeV}$
with error = $\pm 20 \text{ MeV}$

- s-quark has been directly tuned to produce $s\bar{s}$ pseudoscalar mass to be 686 MeV. Here we used the following formula to calculate the mass of pseudoscalar s-quarkonium state

$$M_{s\bar{s}} = \sqrt{2M_K^2 - M_\pi^2}$$

where M_K and M_π are kaon and pion masses.

- B_s meson mass is calculated as

$$M_{B_s} = E_{B_s} + \frac{1}{2}(M_{\eta_b} - E_{\eta_b})$$

$$M_{B_s} = 1640 + (9389 - 2476)/2 = 5096 \text{ MeV}$$

- $Q = \begin{pmatrix} \phi \\ 0 \end{pmatrix}$, $\Gamma = \gamma_5$ or $\Gamma = \gamma_k$

$$\begin{aligned}
 C(\vec{p}, t) &= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | q^\dagger(x) \Gamma_{sk}^\dagger(x) Q(x) Q^\dagger(0) \Gamma_{sc}(0) q(0) | 0 \rangle \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | q(0) q^\dagger(x) \Gamma Q(x) Q^\dagger(0) \Gamma | 0 \rangle \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \text{Tr}[M(0, x) \gamma_4 \Gamma G(x, 0) \Gamma] \\
 &= - \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \text{Tr}[\gamma_5 M(x, 0)^\dagger \gamma_5 \Gamma G(x, 0) \Gamma]
 \end{aligned}$$

- $G(x, 0)$ is now a 4×4 matrix in spinor space having vanishing lower components but it is in Dirac representation of gamma matrices. We can convert it to milc gamma representation by an unitary transformation $S = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_y & \sigma_y \\ -\sigma_y & \sigma_y \end{pmatrix}$

- Plot for B_s meson correlators.

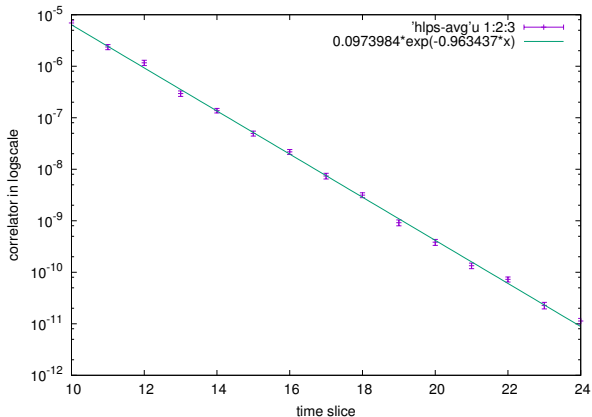


Figure: Heavy-light meson correlators obtained at zero momentum

- $M_{B_s} = 1584 + (9389 - 2476)/2 = 5040$ MeV

- Interpolator $(\mathcal{O}_k)_\alpha = \epsilon_{abc}(Q^a T C \gamma_k Q^b) Q_\alpha^c$ with $C = \gamma_4 \gamma_4$

$$\begin{aligned} C_{ij\alpha\beta}(t) &= \sum_{\vec{x}} \langle 0 | [\mathcal{O}_i(\vec{x}, t)]_\alpha [\mathcal{O}_j^\dagger(\vec{0}, 0)]_\beta | 0 \rangle \\ &= \sum_{\vec{x}} \epsilon_{abc} \epsilon_{fgh} G_{\alpha\beta}^{ch}(x, 0) \text{Tr}[C \gamma_i G^{bg}(x, 0) \overline{C} \gamma_j G^{af T}(x, 0)] \end{aligned}$$

- The correlator has overlap with both spin 3/2 and spin 1/2 states

$$C_{ij}(t) = Z_{3/2} e^{-E_{3/2} t} \Pi P_{ij}^{3/2} + Z_{1/2} e^{-E_{1/2} t} \Pi P_{ij}^{1/2}$$

$$\Pi = \frac{1}{2}(1 + \gamma_4), P_{ij}^{3/2} = \delta_{ij} - \frac{1}{3}\gamma_i \gamma_j, P_{ij}^{1/2} = \frac{1}{3}\gamma_i \gamma_j \text{ and } P_{ij}^{3/2} \cdot P_{jk}^{1/2} = 0.$$

- $P_{xx}^{3/2} \cdot C_{xx} + P_{xy}^{3/2} \cdot C_{yx} + P_{xz}^{3/2} \cdot C_{zx} = \frac{2}{3} Z_{3/2} \Pi e^{-E_{3/2} t}$

- Plot for Ω_{bbb} correlator

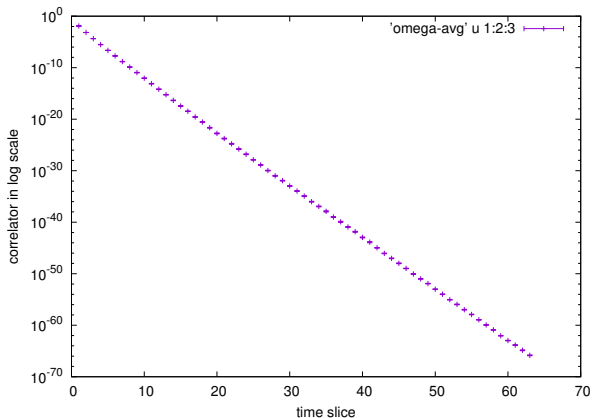


Figure: Omega 3/2

- $M_{\Omega_{bbb}} = E_{\Omega_{bbb}} + \frac{3}{2}(M_{\eta_b} - E_{\eta_b}) = 14.38 \text{ GeV}$ with error = $\pm 20 \text{ MeV}$

- Interpolator $(\mathcal{O}_k)_\alpha = \epsilon_{abc} (Q^{aT} C \gamma_k Q^b) s_\alpha^c$

$$\begin{aligned}
 C_{ij\alpha\beta}(t) &= \sum_{\vec{x}} \langle 0 | [\mathcal{O}_i(\vec{x}, t)]_\alpha [\mathcal{O}_j^\dagger(\vec{0}, 0)]_\beta | 0 \rangle \\
 &= \sum_{\vec{x}} \epsilon_{abc} \epsilon_{fgh} [M^{ch}(x, 0) \cdot \gamma_4]_{\alpha\beta} \text{Tr}[C \gamma_i G^{bg}(x, 0) \overline{C} \gamma_j G^{afT}(x, 0)]
 \end{aligned}$$

- Change $G(x, 0)$ into milc gamma representation.
- $P_{xx}^{3/2} \cdot C_{xx} + P_{xy}^{3/2} \cdot C_{yx} + P_{xz}^{3/2} \cdot C_{zx} = \frac{2}{3} Z_{3/2} \Pi e^{-E_{3/2} t}$

- Plot for Ω_{bbs} correlator

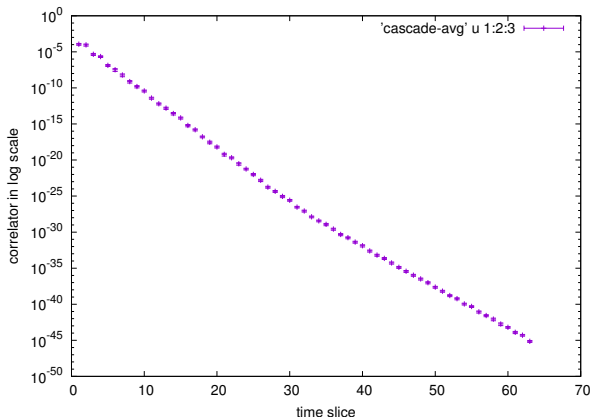


Figure: Ω_{bbs} 3/2

- $M_{\Omega_{bbs}} = E_{\Omega_{bbs}} + (M_{\eta_b} - E_{\eta_b}) = 9.81 \text{ GeV}$ with error = $\pm 40 \text{ MeV}$

THANK YOU

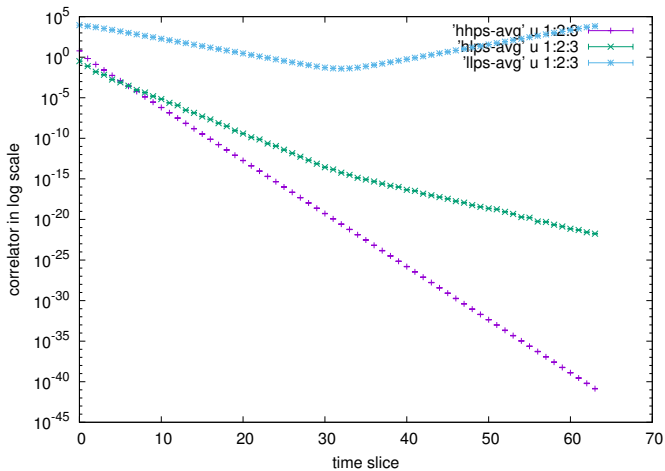


Figure: hh hl ll mesons

- Antiquarks transform as $\bar{3}$'s under color rotation i.e change $U_{x,\mu} \rightarrow U_{x,\mu}^*$. Replace ψ by $\tilde{\chi}$. To compare it with Dirac's theory we change the variable as $\chi = \tilde{\chi}^*$

$$\begin{aligned} \tilde{\chi}(x, t)^\dagger U_t^*(x) \tilde{\chi}(x, t+1) &= (\chi^*(x, t))^\dagger U_t^*(x) \chi^*(x, t+1) \\ &= (\chi(x, t))^T (U_t^\dagger)^T(x) (\chi^\dagger(x, t+1))^T \\ &= -\chi^\dagger(x, t+1) U_t^\dagger(x) \chi(x, t) \end{aligned}$$

We used the fact that it is 1×1 quantity so we can ignore the transpose sign altogether and we put the minus sign because χ 's are fermionic field they obey Grassmann algebra. So if the we write the quark action as $S_Q = \psi^\dagger K \psi$ then we for anti-quark we have $S_{\bar{Q}} = -\chi^\dagger K^\dagger \chi$.