NUMERICAL VALUES OF f^F, f^D, f^S COUPLING CONSTANTS IN SU(3) INVARIANT INTERACTION LAGRANGIAN OF VECTOR-MESON NONET WITH $1/2^+$ OCTET BARYONS.

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Introduction

- Strong interaction of the vector meson nonet with the $1/2^+$ octet baryons is described by the SU(3) invariant Lagrangian

$$
\begin{aligned}
L_{VB\bar{B}} &= \frac{\mathrm{i}}{\sqrt{2}} f^F [\bar{B}^\alpha_\beta \gamma_\mu B^\beta_\gamma - \bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta](V_\mu)^\gamma_\alpha \\
&\quad + \frac{\mathrm{i}}{\sqrt{2}} f^D [\bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta + \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\gamma](V_\mu)^\gamma_\alpha + \frac{\mathrm{i}}{\sqrt{2}} f^S \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\alpha \omega^0_\mu,
\end{aligned}
$$

  where $B, \bar{B}$ are baryon, anti-baryon octet matrices, V is vector-meson octet matrice. ω^0_μ is omega-meson singlet. and f^F,f^D,f^S are coupling constants.

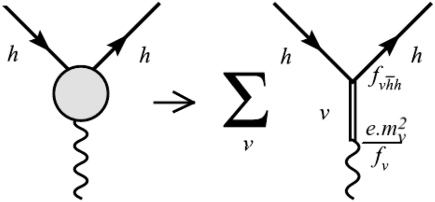
Baryon octet and vector-meson octet matrices B,V

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}$$

INTRODUCTION II

- In order to determine numerical values of the f^F, f^D, f^S coupling constants, one need to analyze the experimental information on the interaction of the $1/2^+$ baryons with the vector-mesons.
- According to the vector meson dominance (VMD) hypothesis, the electromagnetic interaction between virtual photon and hadrons is mediated by the vector-mesons ρ, ω, ϕ.



- processes: $e^- p \to e^- p \quad , \quad e^- e^+ \to p\bar{p}$

ADVANCED U&A NUCLEON ELECTROMAGNETIC STRUCTURE MODEL

- We have analyzed known experimental data on the nucleon electromagnetic form factors using the advanced Unitary and Analytic (U&A) nucleon electromagnetic structure model.
- Such model is inspired by the VMD model $(\rho, \omega, \phi, \rho', \omega', \phi', \rho'', \omega'', \phi'')$ and it contains some ratios of the coupling constants $f_{v\bar{h}h}/f_v$ as free parameters of the model, while others can be fixed from the asymptotic conditions of the nucleon EM FFs.
- It allows to calculate isoscalar and isovecto Dirac and Pauli EM FFs of the nucleon – $F_{1s}^N, F_{2s}^N, F_{1v}^N, F_{2v}^N$

$$F_{2v}^N(t) = \left(\frac{1-X^2}{X_N^2}\right)^6 \left(\frac{1}{2}(\mu_p - \mu_n - 1)L_\rho(X)L_{\rho'}(X)H_{\rho''}(X)\right)$$

ADVANCED U&A NUCLEON ELECTROMAGNETIC STRUCTURE MODEL

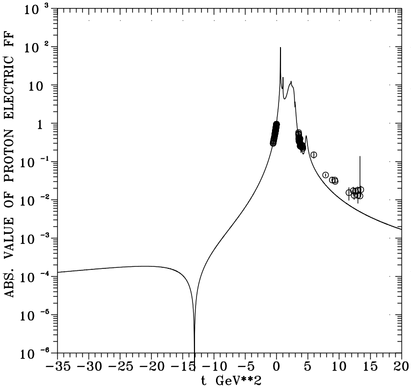
$$X(t) = \mathrm{i}\frac{\sqrt{\left(\frac{t_{in}^{2v}-t_0^v}{t_0^v}\right)^{1/2}+\left(\frac{t-t_0^v}{t_0^v}\right)^{1/2}}-\sqrt{\left(\frac{t_{in}^{2v}-t_0^v}{t_0^v}\right)^{1/2}-\left(\frac{t-t_0^v}{t_0^v}\right)^{1/2}}}{\sqrt{\left(\frac{t_{in}^{2v}-t_0^v}{t_0^v}\right)^{1/2}+\left(\frac{t-t_0^v}{t_0^v}\right)^{1/2}}+\sqrt{\left(\frac{t_{in}^{2v}-t_0^v}{t_0^v}\right)^{1/2}-\left(\frac{t-t_0^v}{t_0^v}\right)^{1/2}}}$$

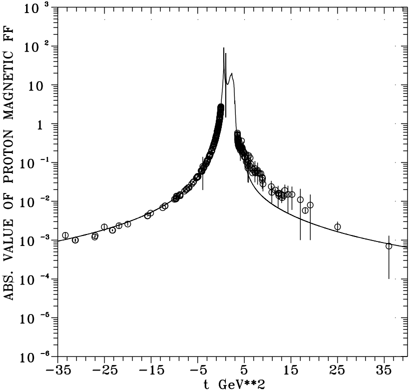
and

$$\begin{aligned}
G_E^p(t) &= [F_{1s}^N(t)+F_{1v}^N(t)] + \tfrac{t}{4m_p^2}[F_{2s}^N(t)+F_{2v}^N(t)] \\
G_M^p(t) &= [F_{1s}^N(t)+F_{1v}^N(t)] + 4m_p^2[F_{2s}^N(t)+F_{2v}^N(t)] \\
G_E^n(t) &= [F_{1s}^N(t)-F_{1v}^N(t)] + \tfrac{t}{4m_n^2}[F_{2s}^N(t)-F_{2v}^N(t)] \\
G_M^n(t) &= [F_{1s}^N(t)-F_{1v}^N(t)] + [F_{2s}^N(t)-F_{2v}^N(t)].
\end{aligned}$$

The behavior of the nucleon EM form factors

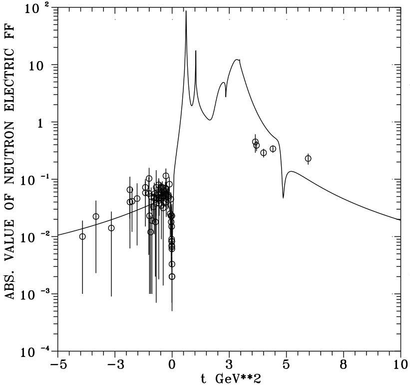
The proton EM form factors behavior as predicted by the advanced U&A model of the nucleon.

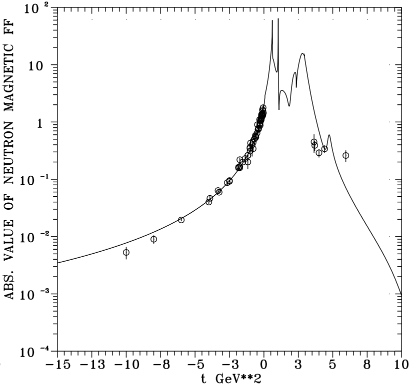




The behavior of the nucleon EM form factors

The neutron EM form factors behavior as predicted by the advanced U&A model of the nucleon.





Fitted ratios of the coupling constants

The advanced U&A model of the nucleon has 12 free parameter – the effective inelastic thresholds $t_{in}^{1s}, t_{in}^{1v}, t_{in}^{2s}, t_{in}^{2v}$ and ratios of the coupling constants

$$\frac{f^{(1)}_{\omega NN}}{f_\omega} = (1.5717 \pm 0.0022) \quad ; \quad \frac{f^{(1)}_{\phi NN}}{f_\phi} = (-1.1247 \pm 0.0011)$$

$$\frac{f^{(1)}_{\omega' NN}}{f_{\omega'}} = (0.0418 \pm 0.0065) \quad ; \quad \frac{f^{(1)}_{\phi' NN}}{f_{\phi'}} = (0.1879 \pm 0.0010)$$

$$\frac{f^{(2)}_{\omega NN}}{f_\omega} = (-0.2096 \pm 0.0067) \quad ; \quad \frac{f^{(2)}_{\phi NN}}{f_\phi} = (0.2657 \pm 0.0067)$$

$$\frac{f^{(2)}_{\phi' NN}}{f_{\phi'}} = (0.1781 \pm 0.0029) \quad ; \quad \frac{f^{(1)}_{\rho NN}}{f_\rho} = (0.3747 \pm 0.0022)$$

- In order to calculate the f^F, f^D, f^S coupling constants later, we need also numerical values of other coupling constant ratios

$$\frac{f^{(1)}_{\rho' NN}}{f_{\rho'}}, \frac{f^{(2)}_{\rho NN}}{f_{\rho}}, \frac{f^{(2)}_{\rho' NN}}{f_{\rho'}}, \frac{f^{(2)}_{\omega' NN}}{f_{\omega'}}.$$

- The can be calculated within the advanced U&A model as

$$(f^{(1)}_{\rho' NN}/f_{\rho'}) = \frac{1}{2}\frac{C^{1v}_{\rho''}}{C^{1v}_{\rho''}-C^{1v}_{\rho'}} - \frac{C^{1v}_{\rho''}-C^{1v}_{\rho}}{C^{1v}_{\rho''}-C^{1v}_{\rho'}}(f^{(1)}_{\rho NN}/f_{\rho}) = 0.7635$$

$$(f^{(2)}_{\rho NN}/f_{\rho}) = \frac{1}{2}(\mu_p - \mu_n - 1)\frac{C^{2v}_{\rho''}C^{2v}_{\rho'}}{(C^{2v}_{\rho'}-C^{2v}_{\rho})(C^{2v}_{\rho''}-C^{2v}_{\rho})} = 2.8956$$

$$(f^{(2)}_{\rho' NN}/f_{\rho'}) = \frac{1}{2}(\mu_p - \mu_n - 1)\frac{C^{2v}_{\rho''}C^{2v}_{\rho}}{(C^{2v}_{\rho''}-C^{2v}_{\rho'})(C^{2v}_{\rho}-C^{2v}_{\rho'})} = -1.3086$$

$$
\begin{aligned}
(f^{(2)}_{\omega' NN}/f_{\omega'}) =& \frac{1}{2}(\mu_p + \mu_n - 1)\frac{C^{2s}_{\omega''}C^{2s}_{\phi''}}{(C^{2s}_{\phi''} - C^{2s}_{\omega'})(C^{2s}_{\omega''} - C^{2s}_{\omega'})} \\
&- \frac{(C^{2s}_{\phi''} - C^{2s}_{\omega})(C^{2s}_{\omega''} - C^{2s}_{\omega})}{(C^{2s}_{\phi''} - C^{2s}_{\omega'})(C^{2s}_{\omega''} - C^{2s}_{\omega'})}(f^{(2)}_{\omega NN}/f_{\omega}) \\
&- \frac{(C^{2s}_{\phi''} - C^{2s}_{\phi})(C^{2s}_{\omega''} - C^{2s}_{\phi})}{(C^{2s}_{\phi''} - C^{2s}_{\omega'})(C^{2s}_{\omega''} - C^{2s}_{\omega'})}(f^{(2)}_{\phi NN}/f_{\phi}) \\
&- \frac{(C^{2s}_{\phi''} - C^{2s}_{\phi'})(C^{2s}_{\omega''} - C^{2s}_{\phi'})}{(C^{2s}_{\phi''} - C^{2s}_{\omega'})(C^{2s}_{\omega''} - C^{2s}_{\omega'})}(f^{(2)}_{\phi' NN}/f_{\phi'}) = -0.5771.
\end{aligned}
$$

VALUES OF THE f_V COUPLING CONSTANTS

- As we are interested in the coupling constants between vector-meson and baryon octet, we need to eliminate the vector-meson photon coupling f_V ($V \in \{\rho, \omega, \phi, \rho', \omega', \phi'\}$) from the coupling constant ratios obtained from the analysis of the nucleon EM form factors.
- The f_ρ,f_ω,f_ϕ can be calculated from the existing data on lepton width $\Gamma(V \to e^+e^-)$ as

$$f_\rho = 4.9582,\ f_\omega = 17.062,\ f_\phi = -13.4428$$

- The couplings for the excited vector mesons can be calculated from the theoretical lepton width estimation as

$$f_{\rho'} = 13.6491,\ f_{\omega'} = 47.6022,\ f_{\phi'} = -33.6598$$

VECTOR MESON TO NUCLEON COUPLINGS f_{VNN}

We get following coupling constants

$$
\begin{array}{lll}
f^{(1)}_{\rho NN} = 1.8578 & f^{(1)}_{\omega NN} = 26.8163 & f^{(1)}_{\phi NN} = 15.1191 \\
f^{(2)}_{\rho NN} = 14.357 & f^{(2)}_{\omega NN} = -3.5762 & f^{(2)}_{\phi NN} = -3.5718 \\
f^{(1)}_{\rho' NN} = 10.4211 & f^{(1)}_{\omega' NN} = 1.99 & f^{(1)}_{\phi' NN} = -6.3247 \\
f^{(2)}_{\rho' NN} = -17.8612 & f^{(2)}_{\omega' NN} = -27.4712 & f^{(2)}_{\phi' NN} = -5.9948
\end{array}
$$

f^F, f^D, f^S COUPLING CONSTANTS

- Considering the SU(3) invariant Lagrangian $L_{VB\bar{B}}$ and $\omega - \phi$ mixing, we get a relation between f^F, f^D, f^S and the vector-meson to baryon (nucleon) coupling constants $f_{VN\bar{N}}$ as

$$\begin{aligned}
f_{\rho N\bar{N}} &= \tfrac{1}{2}(f^D + f^F), \\
f_{\omega N\bar{N}} &= \tfrac{1}{\sqrt{2}} f^S \cos\theta - \tfrac{1}{2\sqrt{3}}(3f^F - f^D)\sin\theta, \\
f_{\phi N\bar{N}} &= \tfrac{1}{\sqrt{2}} f^S \sin\theta + \tfrac{1}{2\sqrt{3}}(3f^F - f^D)\cos\theta
\end{aligned}$$

- The relation for the excited states ρ', ω', ϕ' are the same, but with a different mixing angle θ'

$$\theta = 43.8^\circ \quad , \quad \theta' = 50.3^\circ$$

f^F, f^D, f^S COUPLING CONSTANTS FORMULAS

The inverse relation can be derived for the ground and excited states of the vector mesons and for both Dirac and Pauli ((1),(2)) types of the coupling constants

$$
\begin{aligned}
f^F_{(i)} &= \tfrac{1}{2}\left[\sqrt{3}(f^{(i)}_{\phi NN}\cos\theta - f^{(i)}_{\omega NN}\sin\theta) + f^{(i)}_{\rho NN}\right],\\
f^D_{(i)} &= \tfrac{1}{2}\left[-\sqrt{3}(f^{(i)}_{\phi NN}\cos\theta - f^{(i)}_{\omega NN}\sin\theta) + f^{(i)}_{\rho NN}\right],\\
f^S_{(i)} &= \sqrt{2}\left(f^{(i)}_{\phi NN}\sin\theta + f^{(i)}_{\omega NN}\cos\theta\right),\\
f^{F'}_{(i)} &= \tfrac{1}{2}\left[\sqrt{3}(f^{(i)}_{\phi' NN}\cos\theta' - f^{(i)}_{\omega' NN}\sin\theta') + f^{(i)}_{\rho' NN}\right],\\
f^{D'}_{(i)} &= \tfrac{1}{2}\left[-\sqrt{3}(f^{(i)}_{\phi' NN}\cos\theta' - f^{(i)}_{\omega' NN}\sin\theta') + f^{(i)}_{\rho' NN}\right],\\
f^{S'}_{(i)} &= \sqrt{2}\left(f^{(i)}_{\phi' NN}\sin\theta' + f^{(i)}_{\omega' NN}\cos\theta'\right),
\end{aligned}
$$

where $i \in \{1, 2\}$.

Numerical values of the f^F, f^D, f^S coupling constants

$$
\begin{array}{lll}
f^F_{(1)} = -5.6947 & f^D_{(1)} = 9.4103 & f^S_{(1)} = 42.1706 \\
f^F_{(2)} = 7.08952 & f^D_{(2)} = 21.6245 & f^S_{(2)} = -7.1465 \\
f^{F'}_{(1)} = 0.3858 & f^{D'}_{(1)} = 20.4564 & f^{S'}_{(1)} = -5.0842 \\
f^{F'}_{(2)} = 6.0577 & f^{D'}_{(2)} = -41.7801 & f^{S'}_{(2)} = -31.339
\end{array}
$$

Conclusions

- Using the U&A description of the nucleon electromagnetic structure we were able to extract the coupling constants of the vector mesons with nucleons f_{VNN}.
- These were used to calculate the coupling constants of the SU(3) Lagrangian $L_{VB\bar{B}} - f^F, f^D, f^S$.
- In the same manner f^F, f^D, f^S can be used to express the coupling constants of the vector mesons with hyperons $\Lambda^0, \Sigma^0, \Sigma^+, \Sigma^-, \Xi^-, \Xi^0$.
- In such way the U&A model can be used to predict hyperons' electromagnetic structure without using any additional data.