Effective Potential Formalism at Finite Temperature in Dual QCD and Deconfinement Phase Transition

Garima Punetha
Kumaun University (Nainital) India

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Outline

Introduction

One of the most outstanding problems in theoretical physics is the confinement problem in QCD.

- **High energy regime:**

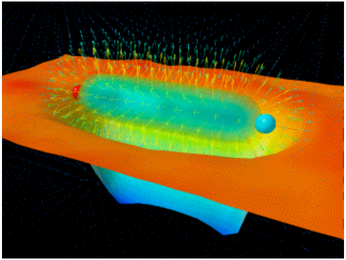
Short distances $\Rightarrow$ Asymptotic freedom $\Rightarrow$ Perturbative methods

- **Low energy regime:**

Large distances $\Rightarrow$ Confinement $\Rightarrow$ Non-perturbative methods

- In 1970's, Nambu, Mandelstam, 't Hooft proposed an interesting idea of quark confinement

1. **Magnetic monopole condensation**
2. **Color electric flux tube formation**
3. **Quark confinement**



Since, there is no well-established analytical method for non-perturbative phenomena, it is desirable to develop a gauge invariant approach based on the first principles of QCD, that would provide a clear understanding of the physical picture of QCD vacuum in non-perturbative regime.

Field Decomposition Formulation and Magnetic Symmetry

- The mathematical foundation for the dual gauge theory comes from the observation that the non-Abelian gauge symmetry allow an extra internal symmetry called magnetic symmetry which restricts and reduces the dynamical degrees of the theory.

$$D_\mu \hat{m} = 0, \; i.e. \; (\partial_\mu + g \mathbf{W}_\mu \times) \hat{m} = 0. \tag{1}$$

- The most general gauge potential which satisfies the above constraint is written as,

$$\mathbf{W}_\mu = A_\mu \hat{m} + A'_\mu \hat{m}' - g^{-1} (\hat{m} \times \partial_\mu \hat{m}) - g^{-1} (\hat{m}' \times \partial_\mu \hat{m}'), \tag{2}$$

where, A_μ and A'_μ are the Abelian component of $\mathbf{W}_\mu$ along $\hat{m}$ and $\hat{m}'$ respectively and are unrestricted by the constraint.

- The associated generalized field strength may then be written as,

$$\mathbf{G}_{\mu\nu} = (\, F_{\mu\nu} + B^{(d)}_{\mu\nu}\,)\,\hat{m} + (\, F^{'}_{\mu\nu} + B^{'}_{\mu\nu}{}^{(d)}\,)\,\hat{m}^{'}, \tag{3}$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ F^{'}_{\mu\nu} &= \partial_\mu A^{'}_\nu - \partial_\nu A^{'}_\mu, \\ B^{(d)}_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu = g^{-1}\,(\,\hat{m} \times \partial_\mu\,\hat{m}), \\ B^{'}_{\mu\nu}{}^{(d)} &= \partial_\mu B^{'}_\nu{}^{(d)} - \partial_\nu B^{'}_\mu{}^{(d)} = g^{-1}\,(\,\hat{m}^{'} \times \partial_\mu\,\hat{m}^{'}\,). \end{aligned} \tag{4}$$

- The topological structure may be brought into dynamics in a dual symmetric way by imposing magnetic symmetry and the λ_3-like multiplet $\hat{m}$ may be viewed to define the mapping,

$S^2_R \rightarrow SU(3)/U(1) \otimes U^{'}(1)$, where S^2_R is the two-dimensional sphere of three dimensional space and S^2 is the group coset space fixed by $\hat{m}$.

- Rotating the magnetic vector $\hat{m}$ to a fix time independent direction by a gauge transformation leads to the value of gauge potential as,

$$\mathbf{W}_\mu \xrightarrow{U} g^{-1}\left[\left((\partial_\mu\beta - \frac{1}{2}\partial_\mu\beta^{'})cos\alpha\right)\hat{\xi}_3 + \frac{1}{2}\sqrt{3}(\partial_\mu\beta^{'}cos\alpha)\hat{\xi}_8\right], \qquad (5)$$

and the associated field strength takes the form as

$$\mathbf{G}_{\mu\nu} \xrightarrow{U} (F_{\mu\nu} + B^{(d)}_{\mu\nu})\,\hat{\xi}_3 + (F^{'}_{\mu\nu} + B^{'}_{\mu\nu}{}^{(d)})\,\hat{\xi}_8. \qquad (6)$$

- In order to avoid the problem due to the pointlike structure and the singular behavior of the potential associated with the monopoles, we use the regular dual magnetic potential ($B_\mu^{(d)}$, $B_\mu^{'(d)}$) associated with the monopoles and introduce the complex scalar fields ($\phi(x)$, $\phi^{'}(x)$) for the monopole. Thus, we obtain the dual QCD Lagrangian in the following form,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}F_{\mu\nu}^{'}{}^2 - \frac{1}{4}B_{\mu\nu}^2 - \frac{1}{4}B_{\mu\nu}^{'}{}^2 + \bar{\psi}_r\gamma^\mu[i\partial_\mu + \frac{1}{2}g(A_\mu^{(d)} + B_\mu) +$$
$$\frac{1}{2\sqrt{3}}g(A_\mu^{'(d)} + B_\mu^{'})]\psi_r + \bar{\psi}_b\gamma^\mu[i\partial_\mu + \frac{1}{2}g(A_\mu^{(d)} + B_\mu) + \frac{1}{2\sqrt{3}}g(A_\mu^{'(d)} + B_\mu^{'})]\psi_b$$
$$+\bar{\psi}_y\gamma^\mu[i\partial_\mu - \frac{1}{\sqrt{3}}g(A_\mu^{'(d)} + B_\mu^{'})]\psi_y + |(\partial_\mu + i\frac{4\pi}{g}(A_\mu + B_\mu^{(d)}))\phi|^2 +$$
$$|(\partial_\mu + i\frac{4\pi\sqrt{(3)}}{g}(A_\mu^{'} + B_\mu^{'(d)}))\phi^{'}|^2 - m_0(\bar{\psi}_r\psi_r + \bar{\psi}_b\psi_b + \bar{\psi}_y\psi_y) - V. \quad (7)$$

- The confinement mechanism of the QCD vacuum can be understood in absence of color electric sources (quarks) and the Lagrangian may be reduced in the following form,

$$\mathcal{L}_d^{(m)} = -\frac{1}{4}B_{\mu\nu}^2 - \frac{1}{4}{B'_{\mu\nu}}^2 + |(\partial_\mu + i\frac{4\pi}{g}B_\mu^{(d)})\phi|^2 + |(\partial_\mu + i\frac{4\pi\sqrt{(3)}}{g}{B'_\mu}^{(d)})\phi'|^2 - V, \tag{8}$$

where, V is the effective potential responsible for the dynamical breaking of magnetic symmetry and is given below,

$$V = \frac{48\pi^2}{g^4}\lambda(\phi^*\phi - \phi_0^2)^2 + \frac{432\pi^2}{g^4}\lambda'(\phi^*\phi' - {\phi'_0}^2)^2, \tag{9}$$

- Using the cylinderically symmetric form of the potentials, the field equations associated with the Lagrangian (8) may be expressed as given below,

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\Big(\rho B(\rho)\Big)\right] - \frac{8\pi}{g}\left(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2}B(\rho)\right)\chi^2(\rho) = 0,$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\chi(\rho)}{d\rho}\right) - \left[\left(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2}B(\rho)\right)^2 + \frac{96\pi^2}{g^4}\lambda\left(\chi^2 - \phi_0^2\right)\right]\chi(\rho) = 0. \tag{10}$$

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\Big(\rho B'(\rho)\Big)\right] - \frac{8\pi\sqrt{3}}{g}\left(\frac{n'}{\rho} + \frac{4\pi\sqrt{3}}{g}B'(\rho)\right)\chi'^2(\rho) = 0,$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\chi'(\rho)}{d\rho}\right) - \left[\left(\frac{n'}{\rho} + \frac{4\pi\sqrt{3}}{g}B'(\rho)\right)^2 + \frac{864\pi^2}{g^4}\lambda'(\chi'^2 - \phi_0'^2)\right]\chi'(\rho) = 0. \tag{11}$$

- Utilizing the asymptotic solutions $B(\rho) = -\frac{ng}{4\pi\rho}[1 + F(\rho)]$ and $B^{'}(\rho) = -\frac{n^{'}g}{4\sqrt{3}\pi\rho}[1 + G(\rho)]$, the energy per unit length of the resulting flux tube configuration is obtained as,

$$k_{(n,n')} = 2\pi \int_0^\infty \rho d\rho \left[\frac{n^2 g^2}{32\pi^2\rho^2} \left(\frac{dF}{d\rho} \right)^2 + \frac{n^2}{\rho^2} F^2(\rho)\chi^2(\rho) + \left(\frac{d\chi}{d\rho} \right)^2 + 3\lambda\alpha_s^{-2}(\chi^2 - \phi_0^2)^2 \right] + 2\pi \int_0^\infty \rho d\rho \left[\frac{n^{'2} g^2}{96\pi^2\rho^2} \left(\frac{dG}{d\rho} \right)^2 + \frac{n^{'2}}{\rho^2} G^2(\rho)\chi^{'2}(\rho) + \left(\frac{d\chi^{'}}{d\rho} \right)^2 + 27\lambda^{'}\alpha_s^{-2}(\chi^{'2} - \phi_0^{'2})^2 \right], \quad (12)$$

where $F(\rho) \stackrel{\rho\rightarrow\infty}{\longrightarrow} C\sqrt{\rho}exp\left(-m_B\rho\right)$ and $G(\rho) \stackrel{\rho\rightarrow\infty}{\longrightarrow} C^{'}\sqrt{\rho}exp\left(-m_B^{'}\rho\right)$.

- Incorporating color reflection invariance the masses of the magnetic glueballs is estimated by evaluating the string tension $k_{(n,n')}$ of the resulting flux tube written as, $k_{(n,n')} = \frac{1}{2\pi\alpha'} = \gamma_{(n,n')}\phi_0^2$,

γ	α_s	$\bar{m}_\phi(GeV)$	$\bar{m}_B(GeV)$	$\lambda^{(d)}_{QCD}(fm)$	$\xi^{(d)}_{QCD}(fm)$	$\kappa^{(d)}_{QCD}$
5.617	0.25	1.20	1.75	0.57	0.83	0.69
6.828	0.24	1.69	1.62	0.61	0.59	0.99
8.093	0.23	2.17	1.52	0.65	0.46	1.42
9.833	0.22	2.90	1.41	0.70	0.34	2.05

Table: The masses of vector and scalar glueball as functions of α_s.

Effective Potential Formalism at Finite Temperature in Dual QCD

- The partition functional, for the dual QCD in thermal equilibrium at a constant temperature T, is expressed in the following form

$$Z[J] = \int D[\phi]D[B_{\mu}^{(d)}]D[\phi^{'}]D[B_{\mu}^{'(d)}]exp(-S^{(d)}), \tag{13}$$

where, $S^{(d)}$ is the dual QCD action and is given by,

$$S^{(d)} = -i \int d^4x(\mathcal{L}_d^{(m)} - J|\phi|^2 - J^{'}|\phi^{'}|^2). \tag{14}$$

- Using the mean-field treatment and separating the fluctuation part of the QCD-monopole field from its mean value as,

$$\phi \rightarrow (\phi + \tilde{\phi})exp(i\xi), \ \phi^{'} \rightarrow (\phi^{'} + \tilde{\phi}^{'})exp(i\xi^{'}), \tag{15}$$

The integrand in action may be expressed as

$$\mathcal{L}_d^{(m)} - J|\phi|^2 - J^{'}|\phi^{'}|^2 = -3\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)^2 - J\phi^2 - \frac{1}{4}B_{\mu\nu}^2 + \frac{1}{2}m_B^2(B_\mu^{(d)})^2$$
$$+[(\partial_\mu\tilde{\phi})^2 - (m_\phi\tilde{\phi})^2] + \left[4\pi\alpha_s^{-1}(B_\mu^{(d)})^2(2\phi\tilde{\phi} + \tilde{\phi}^2) - 3\lambda\alpha_s^{-2}(\tilde{\phi}^4 + 4\phi\tilde{\phi}^3)\right]$$
$$-\left[12\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)\phi + 2J\phi\right]\tilde{\phi} - 27\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi^{'2}_0)^2 - J^{'}\phi^{'2} - \frac{1}{4}B_{\mu\nu}^{'\,2}$$
$$+\frac{1}{2}m_B^{'\,2}(B_\mu^{'(d)})^2 + [(\partial_\mu\tilde{\phi}^{'})^2 - (m_\phi^{'}\tilde{\phi}^{'})^2] + [12\pi\alpha_s^{-1}(B_\mu^{'(d)})^2(2\phi^{'}\tilde{\phi}^{'}$$
$$+\tilde{\phi}^{'2}) - 27\lambda^{'}\alpha_s^{-2}(\tilde{\phi}^{'4} + 4\phi^{'}\tilde{\phi}^{'3})] - [108\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi_0^{'2})\phi^{'} + 2J^{'}\phi^{'}]\tilde{\phi}^{'}, \quad (16)$$

where $J = -6\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)$ and $J^{'} = -54\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi_0^{'2})$

- The corresponding partition function may be written as

$$Z[J] = exp[i \int d^4x(-3\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)^2 - J\phi^2 - 27\lambda^{'}\alpha_s^{-2}(\phi^{'2} - J^{'}\phi^{'2})]$$
$$\times[Det(iD_{\mu\nu})^{-1}(\mathbf{B},\mathbf{k})]^{-1}[Det(i \triangle^{-1} (\phi, k))]^{-1/2}$$
$$\times[Det(iD_{\mu\nu})^{-1}(\mathbf{B}^{'},\mathbf{k})]^{-1}[Det(i \triangle^{-1} (\phi^{'}, k))]^{-1/2}. \quad (17)$$

where $D(\mathbf{B},\mathbf{k})$ and $\Delta(\phi, k)$ are the propagators of $\mathbf{B}_\mu$ and ϕ in the QCD-monopole condensed vacuum,

$$D_{\mu\nu}(\mathbf{B},\mathbf{k}) = \frac{i}{k^2 - m_B^2 + i\epsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_B^2}\right), \quad \Delta(\phi, k) = -\frac{i}{k^2 - m_\phi^2 + i\epsilon},$$
$$D_{\mu\nu}(\mathbf{B}^{'},\mathbf{k}) = \frac{i}{k^2 - m_B^{'2} + i\epsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_B^{'2}}\right), \quad \Delta(\phi^{'}, k) = -\frac{i}{k^2 - m_\phi^{'2} + i\epsilon}. \quad (18)$$

- The effective action and potential at finite temperature leads to the following form,

$$
\begin{aligned}
S_{eff} = \int d^4x\{-3\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)^2 - 27\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi^{'2}_0)^2\} + \\
i ln Det(iD_{\mu\nu}^{-1}(\mathbf{B},\mathbf{k})) + \frac{i}{2} ln Det(i\triangle^{-1}(\phi,k)) + \\
i ln Det(iD_{\mu\nu}^{-1}(\mathbf{B}^{'},\mathbf{k})) + \frac{i}{2} ln Det(i\triangle^{-1}(\phi^{'},k)).
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
V_{eff}(\phi) = -\frac{S_{eff}}{\int d^4x} = 3\lambda\alpha_s^{-2}(\phi^2 - \phi_0^2)^2 + 27\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi^{'2}_0)^2 + \\
3\int \frac{d^4k}{i(2\pi)^4} ln(m_B^2 - k^2 - i\epsilon) + \frac{3}{2}\int \frac{d^4k}{i(2\pi)^4} ln(m_\phi^2 - k^2 - i\epsilon) + \\
3\int \frac{d^4k}{i(2\pi)^4} ln(m_B^{'2} - k^2 - i\epsilon) + \frac{3}{2}\int \frac{d^4k}{i(2\pi)^4} ln(m_\phi^{'2} - k^2 - i\epsilon).
\end{aligned}
\tag{20}
$$

- The effective potential at finite temperature in dual QCD reduces to the following form,

$$V_{eff}(\phi, T) = 3\lambda\alpha^{-2}(\phi^2 - \phi_0^2)^2 + 27\lambda^{'}\alpha_s^{-2}(\phi^{'2} - \phi^{'2}_0)^2 + 3\Big[-\frac{2\pi^2 T^4}{45} + \frac{T^4}{12}\Big(\frac{m_B^2 + m_B^{'2}}{T^2}\Big)\Big] + \frac{1}{2}\Big[-\frac{2\pi^2 T^4}{45} + \frac{T^4}{12}\Big(\frac{m_\phi^2 + m_\phi^{'2}}{T^2}\Big)\Big]. \tag{21}$$

- Incorporating color reflection invariance the following expression for effective potential at finite temperature is obtained,

$$V_{eff}(\phi, T) = 4\lambda\alpha^{-2}(\phi^2 - \phi_0^2)^2 - \frac{7\pi^2 T^4}{45} + \frac{T^2}{12}[m_\phi^2 + 6m_B^2]. \tag{22}$$

- The vacuum expectation value of monopole condensate as a function of temperature T is obtained as follows,

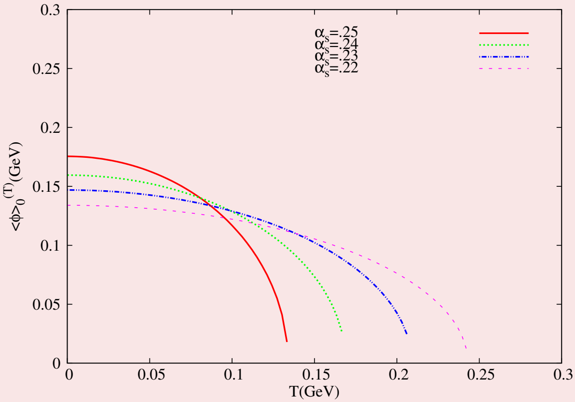
$$< \phi >_0^{(T)} = \sqrt{\phi_0^2 - \left(\frac{4\pi\alpha_s + \lambda}{\lambda}\right)\frac{T^2}{8}}. \tag{23}$$

- The temperature dependent expression of scalar and vector glueball masses,

$$m_\phi^{(T)} = 2\sqrt{3\lambda}\alpha_s^{-1}\sqrt{\phi_0^2 - \left(\frac{4\pi\alpha + \lambda}{\lambda}\right)\frac{T^2}{8}},$$

$$m_B^{(T)} = \sqrt{8\pi\alpha_s^{-1}}\sqrt{\phi_0^2 - \left(\frac{4\pi\alpha + \lambda}{\lambda}\right)\frac{T^2}{8}}. \tag{24}$$

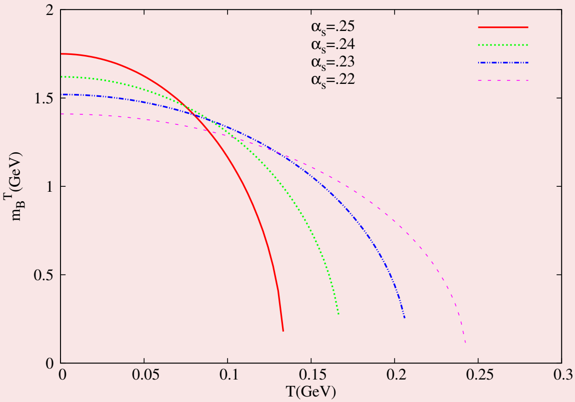
The vacuum expectation value $<\phi_0^{(T)}>$ of monopole condensate as a function of temperature T for different couplings.



The numerical estimate of minimum and maximum value of magnetic condensate at T_{min} and $T = 0$ for different values of coupling.

α_s	$< \phi >_0 {}^{(T)}_{min}(T_{min})(GeV)$	$< \phi >_0 {}^{(T)}_{max}(T = 0)(GeV)$	T_c
0.22	0.0245 (0.133)	0.174	0.135
0.23	0.0113 (0.169)	0.159	0.170
0.24	0.0172 (0.209)	0.147	0.210
0.25	0.00124 (0.245)	0.133	0.245

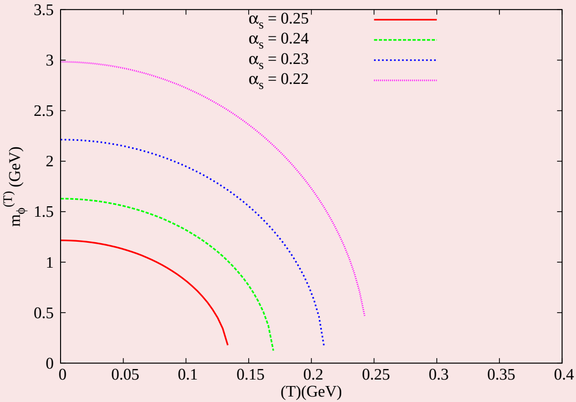
The variation of vector glueball mass $m_B^{(T)}$ with temperature T for different couplings.



The numerical estimate of minimum and maximum value of thermal vector glueball masses at T_{min} and $T = 0$ for different values of coupling.

α_s	$m_{B(min)}^{(T)}(T_{min})(GeV)$	$m_{B(max)}^{(T)}(T=0)(GeV)$	T_c
0.22	0.245 (0.133)	1.75	0.135
0.23	0.119 (0.169)	1.63	0.170
0.24	0.178 (0.209)	1.53	0.210
0.25	0.0183 (0.245)	1.43	0.245

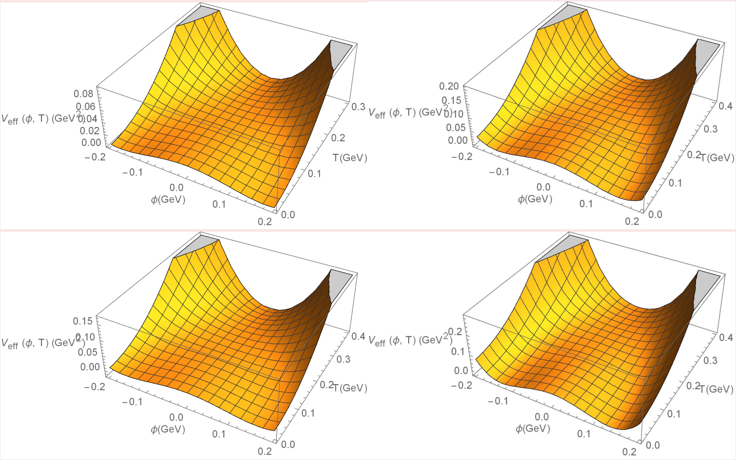
The variation of scalar glueball mass $m_\phi^{(T)}$ with temperature T for different couplings.



The numerical estimate of minimum and maximum value of thermal scalar glueball masses at T_{min} and $T = 0$ for different values of coupling.

α_s	$m^{(T)}_{\phi(min)}(T_{min})(GeV)$	$m^{(T)}_{\phi(max)}(T = 0)(GeV)$	T_c
0.22	0.177 (0.133)	1.21	0.135
0.23	0.104 (0.169)	1.62	0.170
0.24	0.156 (0.209)	2.20	0.210
0.25	0.466 (0.242)	2.99	0.245

The behavior of $V_{eff}(\phi, T)$ as a function of ϕ and T for the coupling $\alpha_s = 0.25, 0.24, 0.23, 0.22.$

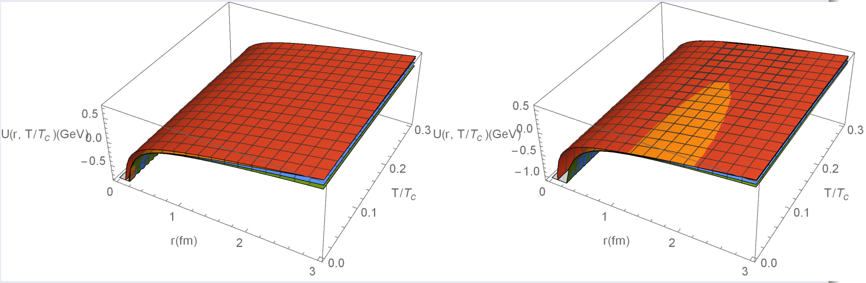


The thermalization then modify the inter-quark confining potential in the following way,

$$U(r,T) = -\frac{Q^2}{4\pi}\left[\frac{exp(-m_B^{(T)}r)}{r} + \frac{1}{2}(m_B^{(T)})^2\left(\frac{1-exp(-cm_B^{(0)}r)}{cm_B^{(0)}}\right)\right.$$
$$\left. ln\left(\frac{(m_B^{(0)})^2(1+(\kappa_{QCD}^{(d)})^2) - (\frac{4\pi\alpha_s+\lambda}{\lambda})T^2[\pi\alpha_s^{-1}+\frac{3}{2}\lambda\alpha_s^{-2}]}{(m_B^{(0)})^2 - \pi\alpha_s^{-1}(\frac{4\pi\alpha_s+\lambda}{\lambda})T^2}\right)\right] \tag{25}$$

$$U^{SC}(r,T) = -\frac{Q^2}{4\pi}\left[\frac{exp(-m_B^{(T)}r)}{r} + \frac{1}{2}(m_B^{(T)})^2\left(\frac{1-exp(-cm_B^{(0)}r)}{cm_B^{(0)}}\right)\right.$$
$$\left. ln\left(\frac{(m_B^{(0)})^2(1+(\kappa_{QCD}^{(d)})^2 - c^2) - (\frac{4\pi\alpha_s+\lambda}{\lambda})T^2[\pi\alpha_s^{-1}+\frac{3}{2}\lambda\alpha_s^{-2}]}{(m_B^{(0)})^2(1-c^2) - \pi\alpha_s^{-1}(\frac{4\pi\alpha_s+\lambda}{\lambda})T^2}\right)\right] \tag{26}$$

The variation of temperature dependent quark potential including color screening effect for several values of c with $\alpha_s = 0.25$ and $\alpha_s = 0.24$ for $SU(3)$ color gauge theory.



Conclusion

- A dual QCD formulation based on magnetic symmetry has been shown to lead to a dual dynamics between color isocharges and topological charges which enforces the confinement of colored sources in a dynamical way.
- Utilizing the path-integral formalism, dual QCD has been extended to the thermal domain by undertaking the mean field approach.
- The effective potential at finite temperature has been derived to compute the critical temperature for phase transition.
- A large reduction of color monopole condensate and glueball masses near the critical point has been shown to lead to a first order deconfinement phase transition in QCD.
- The evaporation of color monopole condensate and the release of the magnetic degrees of freedom in high temperature domain in QCD vacuum has been shown to lead the restoration of magnetic symmetry which might have its link with the quark-gluon phase of QCD.

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