Grasmannians and form factors

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   - Wilson line operator
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   - Momentum twistors
   - Gluing operation in momentum twistor space
3. Off-shell BCFW Conjecture

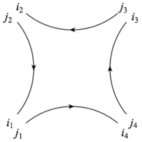
- Formfactors $\Leftrightarrow \langle 0|\mathcal{O}(x)|1, ..., n\rangle$
- On-shell diagram $\Leftrightarrow$ integral over Grassmannian manifold
- Generalization for form factors
- Gluing operation

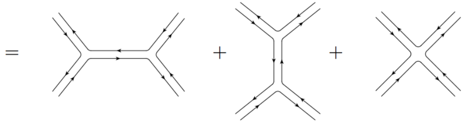
Preliminaries

Basics of spinor helicity formalism

$$(p_i^\mu)^{a\dot{a}} = \begin{pmatrix} p_i^0 + p_i^3 & p_i^1 - ip_i^2 \\ p_i^1 + ip_i^2 & p_i^0 - p_i^3 \end{pmatrix} \Leftrightarrow p_i^{\dot{a}a} = \lambda_i^a \tilde{\lambda}_i^{\dot{a}}$$

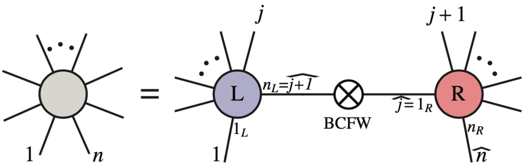
$$\langle ij \rangle := \epsilon_{ab} \lambda_i^a \lambda_j^b, [ij] := \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}}$$





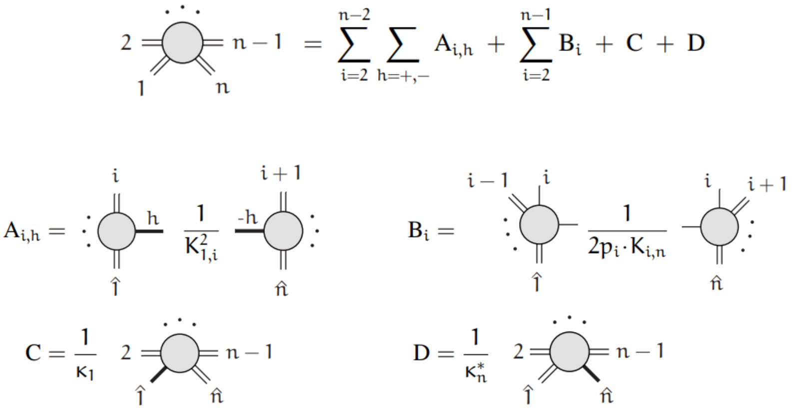
On-shell BCFW

- Deformation of momenta: $\hat{p}_i = p_i - zq$, $\hat{p}_j = p_j + zq$
- Cauchy theorem: $0 = \oint_{\mathcal{C}} dz \frac{\hat{A}(z)}{z} = A + \sum_{z \neq 0} \mathrm{res}\left(\frac{\hat{A}(z)}{z}\right) \Rightarrow$
  $A = \sum_k \hat{A}_L(z_k) \frac{1}{P_k^2} \hat{A}_R(z_k)$



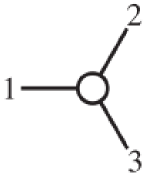
Preliminaries

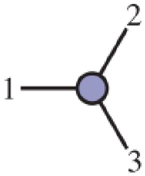
Off-shell BCFW



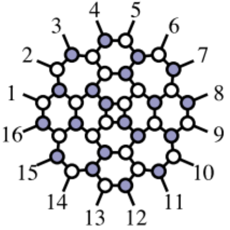
Preliminaries

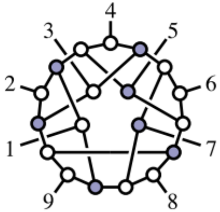
Novel methods





$$\sum_I \int d^4\tilde{\eta}_I \int \frac{d^2\lambda_I d^2\tilde{\lambda}_I}{vol[GL(1)]}$$





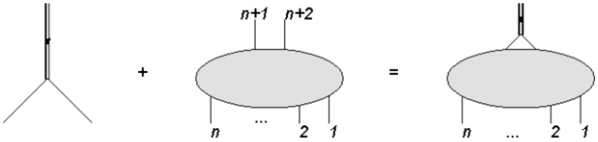
- Non-local operator
- From factors of Wilson line operator ⇔ Reggeon amplitudes

$$\mathcal{W}_p^c(k) = \int d^4x e^{ik\cdot x} \mathrm{Tr}\left\{ \frac{1}{\pi g} t^c \mathcal{P}\exp\left[\frac{ig}{\sqrt{2}} \int_{-\infty}^{+\infty} ds p \cdot A_b(x+sp) t^b \right] \right\}$$

$$A^*_{m+n}(\Omega_1, ..., \Omega_m, g^*_{m+1}, ..., g^*_{m+n}) = \langle \Omega_1 ... \Omega_m | \prod_{i=1}^{n} \mathcal{W}_{p_{m+i}}^{c_{m+i}}(k_{m+i}) | 0 \rangle$$

i-th off-shell gluon

- k_T - parametrization: $k_i^\mu = x p_i^\mu + k_{Ti}^\mu$
- 2 on-shell degrees of freedom: direction $p_i^2 = 0$ and auxiliary on-shell vector q

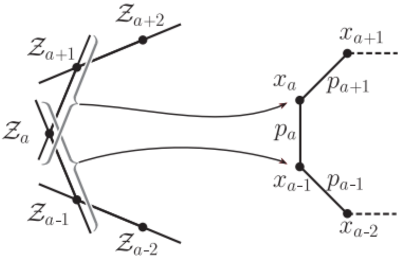


$$A^*_{k,n+1} = \frac{\langle \xi p \rangle}{\kappa^*} \int \frac{d^{k\times(n+2)}C}{Vol[GL(k)]} \frac{\hat{\delta}^{2\times k}(C \cdot \underline{\underline{\tilde{\lambda}}})\hat{\delta}^{4\times k}(C \cdot \underline{\underline{\tilde{\eta}}})\hat{\delta}^{2\times(n+2-k)}(C^{\perp} \cdot \underline{\underline{\lambda}})}{\prod_{i=1}^{n} M_i}$$

Wilson line operator form factors

Momentum twistors

- Needed to simplify momentum conservation and 0-mass condition
- Null-rays in space-time ⇔ points in twistor space



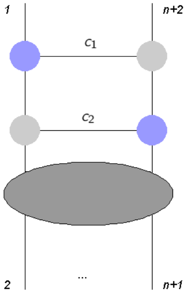
Incidence relations:

$$\mu_{\dot\alpha} = x_{\alpha\dot\alpha}\lambda^{\alpha},\ x_{\alpha\dot\alpha} = (p-q)_{\mu}\sigma^{\mu}_{\alpha\dot\alpha} \Rightarrow Z = (\lambda^{\alpha}, \mu_{\dot\alpha})$$

Claim:

Gluing operation in momentum twistor space is represented by attaching two consecutive BCFW bridges times some regulator:

$$\tilde{\mathcal{G}}^{m.tw.}_{i-1,i}[...] = N(\{\lambda\})\mathrm{Br}(\hat{i}, i+1) \circ \mathrm{Br}(\widehat{i+1}, i)[M(\{\lambda\}) \cdot ...]$$



Corollary

In momentum twistor space gluing operation amounts to shifting i-th twistor:

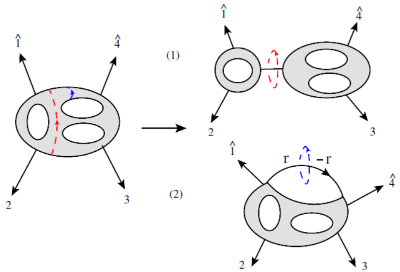
$$\tilde{\mathcal{G}}^{m.tw.}_{i-1,i}[\mathcal{P}^{4(k-2)}_{n+2}] = \mathcal{P}^{4(k-2)}_{n+2}(..., \mathcal{Z}_i - \frac{\langle i-1 i\rangle}{\langle i-1 i+1\rangle}\mathcal{Z}_{i+1}, \mathcal{Z}_{i+1}, ...)$$

BCFW for on-shell integrands

On-shell BCFW:

$$\mathcal{A}_n^L = \mathcal{A}^0_{n,\mathrm{MHV}}\Big(Y^L_{n-1} + \sum_{j=3}^{n-2}[j-1, j, n-1, n, 1]\, Y^{L_1}_{left}\, Y^{L_2}_{right} +$$

$$+ \int_{AB}[A, B, n-1, n, 1]\, Y^{L-1}_{n+2}(\ldots, \hat{\mathcal{Z}}_{n_{AB}}, \mathcal{Z}_A, \mathcal{Z}_B)\Big)$$



BCFW via gluing operation

Observation:

- Gluing operator maps terms of on-shell BCFW decomposition exactly to to terms of the off-shell recursion
- Depending on legs, to which the off-shell vertex is attached, the operator maps BCFW poles to poles of the form $p_{ij}^2 = 0$ (ordinary BCFW poles), or to eikonal ones
- By appropriate choice of the BCFW shift, eikonal poles can be eliminated from consideration

Off-shell BCFW:

$$\mathcal{I}^{*,L}_{(n-2)+1} = \mathcal{I}^{*,L}_{n-1} + \sum_{j=3}^{n-2}[j-1, j, n-1, n^*, 1]\mathcal{I}^{*,L_1}_{left}\mathcal{I}^{*,L_2}_{right} +$$

$$+ \int_{AB}[A, B, n-1, n*, 1]\mathcal{I}^{*,L-1}_{n+2}(..., \hat{\mathcal{Z}}^*_{n_{AB}}, \mathcal{Z}_A, \mathcal{Z}_B)$$

Summary

- A method for deriving Grassmannian integral representation is obtained
- The new method is valid for different representations of external data
- Results derived by means of new and conventional methods agree
- Derivation of BCFW recursion relations via gluing operation for off-shell amplitudes at arbitrary loop-level and for arbitrary number of external legs

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