Hidden attractors in bubble contrast agent model

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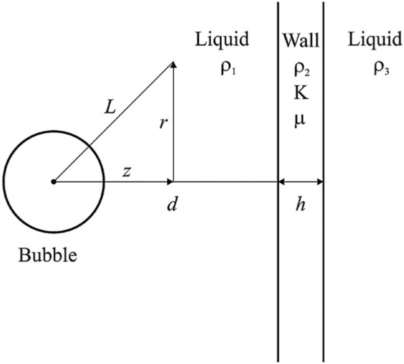
Bubble contrast agent



$$R_0 = 10\ \mu\text{m},\ P_{stat} = 100 - 101.325\ \text{kPa},$$
$$P_v = 2.33\ \text{kPa},\ P_0 = P_{stat} - P_v,$$
$$\sigma = 0.0725\ \text{N/m},\ \rho_1 = 1000\ \text{kg/m}^3,$$
$$\rho_2 = 1060\ \text{kg/m}^3,\ \rho_3 = 1000\ \text{kg/m}^3,$$
$$\eta = 0.001\ \text{Ns/m}^3,\ c = 1500\ \text{m/s},$$
$$\gamma = 4/3,\ \beta = \rho_2\nu/(1-\nu),\ \nu = 0.5,$$
$$d \sim R_0,\ h = 1\ \text{mm}.$$

Model

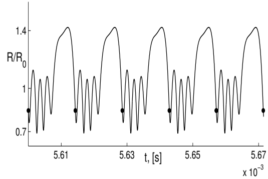
Let us consider the following dynamical system, based on the well-known Rayleigh-Plesset equation (*Parlitz*, *J.Acoust.Soc.Am.*, 1990; *Doinikov*, *Phys.Med.Biol.*, 2011; *Doinikov*, *J.App.Mech.Tech.Phys.*, 2013):

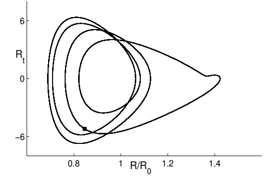
$$
\begin{aligned}
&\dot{R} = U,\\
&\dot{U} = \Bigg(\Bigg(\Bigg(\left(1 + \tfrac{(1-3\gamma)U}{c}\right)\left(P_0 + 2\,\tfrac{\sigma}{R_0}\right)\left(\tfrac{R_0}{R}\right)^{3\gamma} - 2\,\tfrac{\sigma}{R} - 4\,\tfrac{\eta U}{R} -\\
&- \left(1 + \tfrac{U}{c}\right)\left(P_0 + P_{ac}\sin(t\omega)\right) - \tfrac{R\omega P_{ac}\cos(t\omega)}{c}\Bigg)\rho_1{}^{-1} -\\
&- \tfrac{1}{2}U^2\left(3 - 2\,\tfrac{(\rho_1-\beta)R}{(\rho_1+\beta)d} - 2\,\tfrac{(\beta-\rho_3)R}{(\beta+\rho_3)(d+h)} + 2\,\tfrac{(\rho_1-\beta)(\beta-\rho_3)R}{(\rho_1+\beta)(\beta+\rho_3)h} - \tfrac{U}{c}\right)\Bigg)\times\\
&\times\Bigg(R\left(1 - \tfrac{1}{2}\,\tfrac{(\rho_1-\beta)R}{(\rho_1+\beta)d} - \tfrac{1}{2}\,\tfrac{(\beta-\rho_3)R}{(\beta+\rho_3)(d+h)} + \tfrac{1}{2}\,\tfrac{(\rho_1-\beta)(\beta-\rho_3)R}{(\rho_1+\beta)(\beta+\rho_3)h} - \tfrac{U}{c}\right) +\\
&+ 4\,\tfrac{\eta}{\rho_1 c}\Bigg)^{-1},\\
&\dot{t} = 1,
\end{aligned}
$$

Perpetual points: $\ddot{U} = 0, \ddot{R} = 0.$

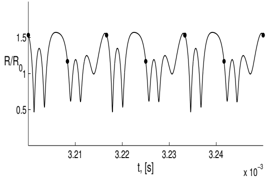
Single attractor

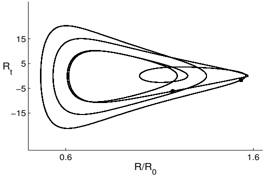
$P_{ac} = 80$ kPa,
$\omega/2\pi = 70$ kHz,
1-periodic motion



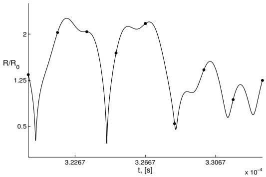


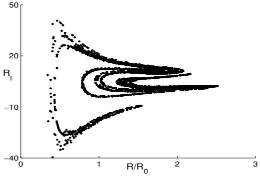
$P_{ac} = 80$ kPa,
$\omega/2\pi = 120$ kHz,
2-periodic motion





$P_{ac} = 300$ kPa,
$\omega/2\pi = 600$ kHz,
Chaotic attractor.





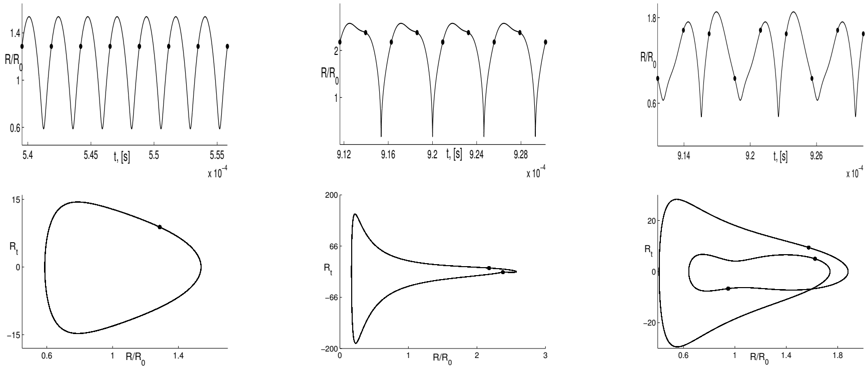
Coexisting periodic attractors

$$P_{ac} = 170 \text{ kPa}, \ \omega/2\pi = 430 \text{ kHz}$$

1-periodic attractor. $R(0) = 0.838 \cdot 10^{-4} \cdot R_0$, $U(0) = 500$.

2-periodic attractor. $R(0) = 15.4 \cdot R_0$, $U(0) = 4500$.

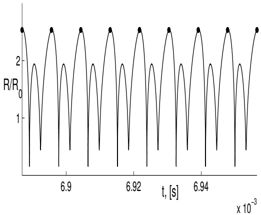
3-periodic attractor. $R(0) = 0.903 \cdot R_0$, $U(0) = -6.162$

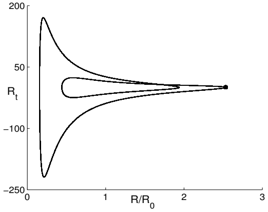


Hidden chaotic attractor

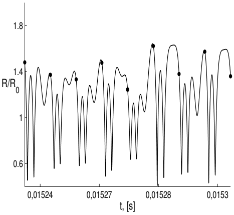
$$P_{ac} = 90 \text{ kPa}, \ \omega/2\pi = 115 \text{ kHz}.$$

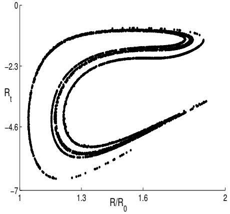
1-periodic attractor





chaotic attractor

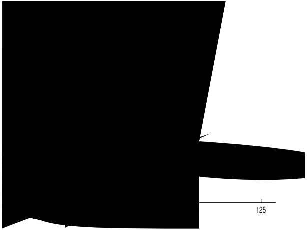




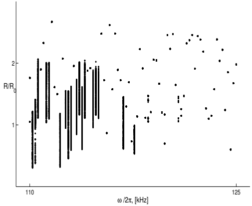
Hidden chaotic attractor

$$P_{ac} = 90 \text{ kPa}, \; \omega/2\pi \in [110, 125] \text{ kHz}$$

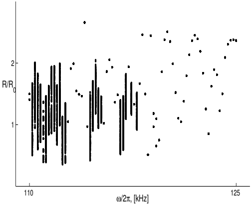
Period doubling



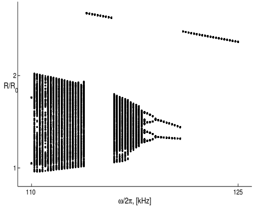
Increasing frequency



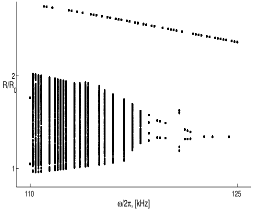
Decresing frequency



$R(0) \ll R_0$

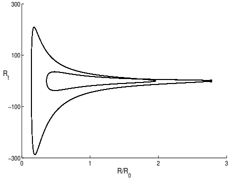


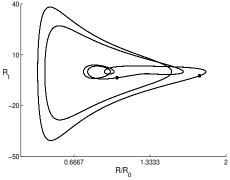
$R(0) \gg R_0$



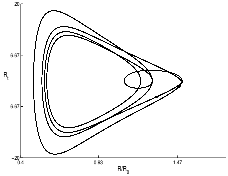
Hidden chaotic attractor

$P_{ac} = 90$ kPa,
$\omega/2\pi = 110$ kHz.
1- and 2- periodic
attractors

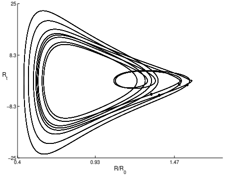




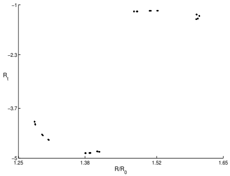
$P_{ac} = 90$ kPa,
$\omega/2\pi =$
$= 120$ kHz.



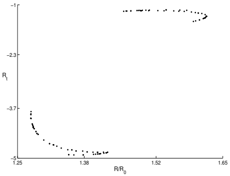
$P_{ac} = 90$ kPa,
$\omega/2\pi =$
$= 118.8$ kHz.



$P_{ac} = 90$ kPa,
$\omega/2\pi =$
$= 118.2$ kHz.



$P_{ac} = 90$ kPa,
$\omega/2\pi =$
$= 118.0$ kHz.



Thank you for your attention!