New phase space calculations for β^+ and EC decay modes

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October 3, 2016

Definition of phase space factor $\beta^{\pm}$

For an n^{th}-forbidden β-branch, the probability per unit time that a nucleus will decay via this branch is [1]:

$$\lambda_n = \frac{g}{2\pi^3}\int_1^{W_0} pW(W_0 - W)^2 S_n(Z,W)dW \tag{1}$$

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For an n^{th}-forbidden β-branch, the transition between initial and final state of nucleus it's defined by: $\Delta J = n, n+1$ and $\Delta\pi = (-1)^n$

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The partial half-life of an n^{th}-forbidden β-branch is given by:

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So we have the expresion for **phase space factor**:

$$f_n = \int_1^{W_0} pW(W_0 - W)^2 \frac{S_n(Z,W)}{\eta^2} dW \tag{4}$$

Fermi transitions β^+

For Fermi transitions or allowed transitions $\Delta J = 0$ and η^2 is taken to be $\frac{S_0}{\lambda_1}$, so that:

$$\boxed{f_0 = \int_1^{W_0} pW(W_0 - W)^2 \lambda_1(Z, W) dW} \tag{5}$$

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$$\boxed{f_0 = \int_1^{W_0} pW(W_0 - W)^2 \lambda_1(Z,W) dW} \tag{5}$$

$$\lambda_1(Z,W) = [g_{-1}^2 + f_1^2]/2p^2 \tag{6}$$

$g_{-1}(Z,W)$ and $f_1(Z,W)$ are the Dirac radial components of the continuum electron or positron wave-function, solutions of:

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$g_{-1}(Z,W)$ and $f_1(Z,W)$ are the Dirac radial components of the continuum electron or positron wave-function, solutions of:

$$\left(\frac{d}{dr} + \frac{\kappa}{r}\right) g_\kappa(W,r) = [W + 1 + V(r)] f_\kappa(W,r) \tag{7}$$

$$\left(\frac{d}{dr} + \frac{\kappa}{r}\right) f_\kappa(W,r) = -[W - 1 + V(r)] g_\kappa(W,r)$$

where $\kappa = (l - j)(2j + 1)$ is relativistic quantum number

Fermi transitions *EC*

The energy carried off by the neutrino after a electron capture decay envolving a K-shell electron, is then given by:

$$q_K = Q - \epsilon_K \quad (8)$$

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$$\lambda^0_{EC,K} = \lambda^0_K B_K, \tag{9}$$

where

$$\lambda^0_K = \frac{g^2 |M_{0,1}|^2}{4\pi^2} q^2_K g^2_K, \tag{10}$$

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In analogy with Eq. (9), the L-shell total capture rate will be

$$\lambda^0_{EC,L_i} = \lambda^0_{L_i} B_{L_i}, \tag{11}$$

Fermi transitions EC

Taking into account the most important contributions from K and L_1, we can write the PSF expression of electron capture, for an allowed transition:

$$\boxed{F_{EC}^{K,L_1} = \frac{\pi}{2}\left(q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1}\right)} \tag{12}$$

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For the q_{K/L_1} quantities we used the expression

$$q_{K/L_1} = W_{EC} - \epsilon_{K/L_1} \tag{13}$$

were, W_{EC} is the Q value of the β^+ decay in $m_e c^2$ units. For the $1s_{1/2}$ and $2s_{1/2}$ electron orbitals, we use $g_K^2 = D_{0,-1}^2$ and $g_{L_1}^2 = D_{1,-1}^2$, respectively.

$$D_{n,\kappa}^2 = \frac{1}{(m_e c^2)^3}\left(\frac{\hbar c}{a_0}\right)^3\left(\frac{a_0}{R_A}\right)^2\left[g_{n,\kappa}^2(R_A) + f_{n,\kappa}^2(R_A)\right] \tag{14}$$

Coulomb potential

In specialty literature coulomb potential used is:

$$V(Z,r) = \begin{cases} -\frac{Z\alpha\hbar c}{r}, & r \geq R_A, \\ -Z(\alpha\hbar c)\left(\frac{3-(r/R_A)^2}{2R_A}\right), & r < R_A, \end{cases} \tag{15}$$

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In this program, we take into account the influence of the nuclear structure by deriving the potential V(r) from a realistic proton density distribution in the daughter nucleus [5]:

$$V(Z,r) = \alpha\hbar c \int \frac{\rho_e(\vec{r'})}{\mid \vec{r} - \vec{r'} \mid} d\vec{r'} \tag{16}$$

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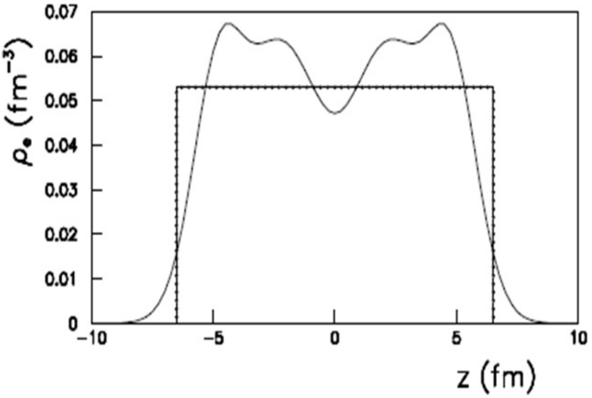
where the proton density is:

$$\rho_e(\vec{r}) = \sum_i (2j_i + 1) v_i^2 \mid \Psi_i(\vec{r}) \mid^2, \tag{17}$$

Realistic proton density



Profile of the realistic proton density for ^{150}Sm compared with that given with the constant density approximation

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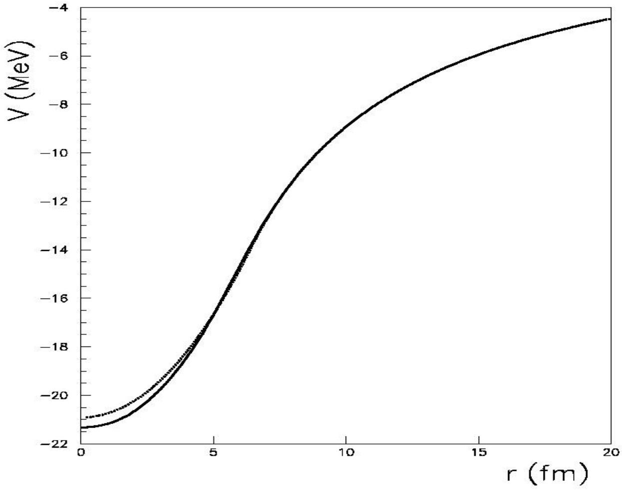
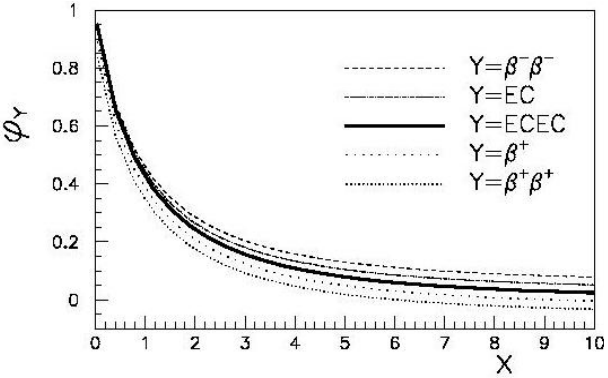
Results



Screening function

A further improvement in calculation can be done by multiplying the expression of $V(r)$ with screening function, with is the solution of the Thomas-Fermi equation: $d^2\phi/dx^2 = \phi^{3/2}/\sqrt{x}$, with $x = r/b$, $b \approx 0.8853 a_0 Z^{-1/3}$ and a_0 = Bohr radius.



For the β^+-**decay process**, the potential used to obtain the electron w.f. is

$$rV_{\beta^+}(Z,r) = (rV(Z,r)+1)\times\phi(r)-1 \qquad (18)$$

to take into account the fact that β^+-decay releases a final negative ion with charge -1. $V(Z,r)$ is positive.

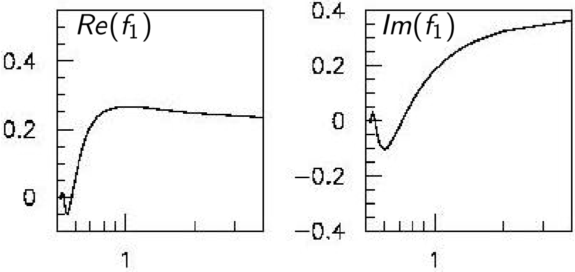
For the *EC* **process**, the potential used to obtain the electron w.f. is

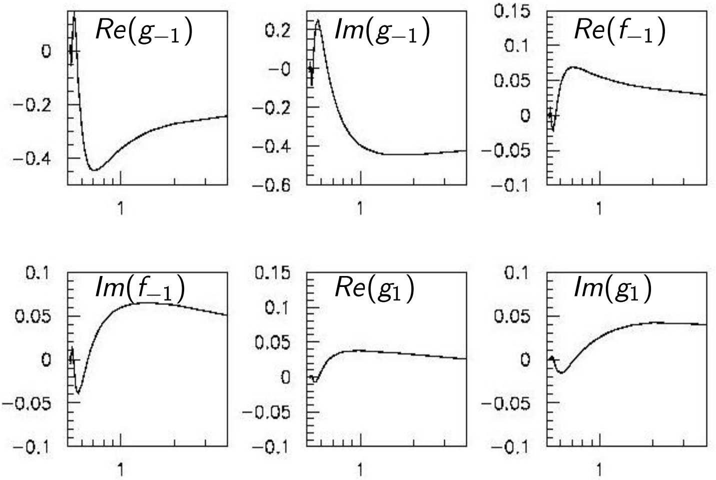
$$rV_{EC}(Z,r) = rV(Z,r)\phi(r), \qquad (19)$$

and the charge number $Z=Z_0$ corresponds to the parent nucleus. $V(Z,r)$ is negative.

Radial components of wave-function

For the PSF computation, all integrals in Eq. (7) were performed accurately with Gauss-Legendre quadrature in 32 points. We calculated up to 49 values of the radial functions in the Q value energy interval, that were interpolated with spline functions.




$Re(g_{-1})$
$Im(g_{-1})$
$Re(f_{-1})$
$Im(f_{-1})$
$Re(g_1)$
$Im(g_1)$
0
−0.2
−0.4
0.2
−0
−0.6
0.15
0.1
0.05
−0.05
−0.1
1

Results β^+.Light nuclei

Nucleus	W_0[7] (MeV)	F_{BP} [3]	F_{BP}[7]	F_{BP}[TW]	F_{BP}[1]
^{10}C	0.8884	2.361	2.361	2.325	2.326
^{14}O	1.8098	43.398	43.378	42.822	42.814
^{18}Ne	2.383	136.83	136.83	135.19	135.08
^{22}Mg	3.109	427.02	426.88	422.19	421.51
^{26}Al	3.211	483.84	483.68	478.3	477.43
^{26}Si	3.817	1036.8	1035.9	1025.51	1023.06
^{30}S	4.439	1990.2	1987.8	1969.24	1963.9
^{34}Cl	4.468	2014.7	2013.4	1993.13	1987.4
^{34}Ar	5.021	3388.3	3383.8	3351.58	3339.85
^{38}K	5.028	3346.9	3344.9	3312.82	3300.54
^{38}Ca	5.620	5515.9	5510.3	5457.95	5449.00
^{42}Sc	5.409	4533.5	4531.7	4490.19	4462.21
^{42}Ti	5.964	7025.4	7024.1	6934.9	6853.74
^{46}V	6.032	7285.9	7284.2	7186.04	7091.9
^{50}Mn	6.609	10818	10810	10492.76	10262
^{54}Co	7.227	15956	15951	14988.470	14412.5

Results β^+.Heavy Nuclei

Nucleus	W_0 [8] (Mev)	$F_{BP}[TW]$	F_{BP} [1]
^{52}Fe	1.3525	8.3403	8.4132
^{56}Ni	1.1109	3.4439	3.5250
^{62}Zn	0.5974	0.2344	0.2438
^{66}Ga	4.153	1125.6442	1132.5483
^{76}Br	3.9409	835.1982	843.3343
^{81}Rb	1.2161	4.3222	6.8878
^{88}Y	2.6006	120.2644	121.8624
^{90}Nb	5.0893	2503.0555	2533.7049
^{102}Cd	1.565	11.2214	11.5267
^{103}In	5.0005	2100.3727	2136.0153
^{105}Ag	0.325	0.0102	0.1127
^{107}Sb	6.837	8528.5047	8931.8197
^{113}Sb	2.8891	168.1487	172.0209
^{113}Te	5.048	2124.1816	2165.2927
^{115}I	4.7029	1517.2376	1549.2409
^{116}I	6.7547	7913.1790	8272.0244
^{116}Xe	3.235	352.3565	361.4082
^{120}Ba	3.98	678.0918	705.0294
^{120}Xe	0.5587	0.1047	0.1108
^{126}Cs	3.7731	542.4653	563.8184
^{182}Re	1.778	16.123	17.206
^{205}Bi	1.6835	12.3984	13.4576

Results electron capture.Light nuclei

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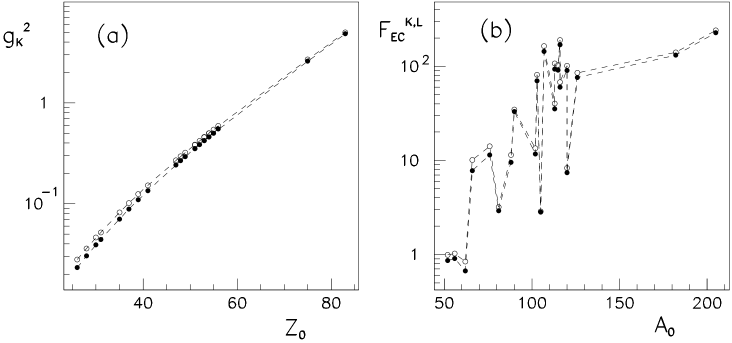
Nucleus	Q_{β^+} (MeV)	g_K^2 [2]	g_K^2 (TW)	$g_{L_1}^2/g_K^2$ [2]	$g_{L_1}^2/g_K^2$ (TW)	F_{EC}^{K,L_1} (TW)	F_{EC}^{K,L_1} [2]
^{10}C	1.9104	0.00031	0.00031	0.04930	0.02867	0.00703	0.00640
^{14}O	2.83186	0.00075	0.00065	0.05640	0.04420	0.03297	0.03786
^{18}Ne	3.405	0.00151	0.00118	0.05840	0.05794	0.08713	0.11005
^{22}Mg	4.131	0.00268	0.00199	0.06660	0.06811	0.218	0.29060
^{26}Al	4.2331	0.00344	0.00251	0.06990	0.07265	0.27558	0.39270
^{26}Si	4.839	0.00435	0.00312	0.07290	0.07661	0.47240	0.65060
^{30}S	5.461	0.00664	0.00467	0.07810	0.08342	0.90680	1.27140
^{34}Cl	5.4908	0.00807	0.00563	0.08040	0.08628	1.10727	1.56600
^{34}Ar	6.043	0.00970	0.00675	0.08240	0.08862	1.61130	2.28490
^{38}K	6.05	0.01156	0.00802	0.08440	0.09079	1.92311	2.73480
^{38}Ca	6.642	0.01367	0.00947	0.08620	0.09259	2.74237	3.90650
^{42}Sc	6.4311	0.01600	0.01113	0.08790	0.09430	3.02434	4.28930
^{42}Ti	6.986	0.01870	0.01300	0.08960	0.09579	4.17496	5.92320
^{46}V	7.0543	0.02170	0.01512	0.09100	0.09699	4.95575	7.02120
^{50}Mn	7.6311	0.02870	0.02016	0.09380	0.09920	7.74617	10.9103
^{54}Co	8.2498	0.03730	0.02651	0.09620	0.10077	11.91799	16.6144

Results electron capture.Heavy Nuclei

Nucleus	Q_{β^+} (MeV)	g_K^2 [2]	g_K^2 (TW)	$g_{L_1}^2/g_K^2$ [2]	$g_{L_1}^2/g_K^2$ (TW)	F_{EC}^{K,L_1} (TW)	F_{EC}^{K,L_1} [2]
^{52}Fe	2.374	0.0328	0.0232	0.0950	0.0987	0.859	1.203
^{56}Ni	2.136	0.0423	0.0303	0.0974	0.1013	0.907	1.258
^{62}Zn	1.626	0.0538	0.0390	0.0995	0.1025	0.675	0.926
^{66}Ga	5.175	0.0604	0.0410	0.1006	0.1029	7.80	10.61
^{76}Br	4.963	0.0935	0.0704	0.1035	0.1048	11.45	15.16
^{81}Rb	2.23815	0.1149	0.0883	0.1063	0.1080	9.069	11.74
^{88}Y	3.6226	0.1402	0.1091	0.1080	0.1174	9.528	12.11
^{90}Nb	6.111	0.170	0.1344	0.1098	0.1059	33.17	41.97
^{102}Cd	2.587	0.319	0.2663	0.1159	0.1102	11.66	14.02
^{103}In	6.050	0.348	0.2930	0.1168	0.1116	71.05	84.54
^{105}Ag	1.345	0.293	0.2423	0.1150	0.1086	2.816	3.426
^{107}Sb	7.920	0.413	0.3526	0.1187	0.1096	146.5	172.4
^{113}Sb	3.913	0.413	0.3516	0.1187	0.1096	35.38	41.80
^{113}Te	6.070	0.449	0.3844	0.1196	0.1113	93.70	109.9
^{115}I	5.729	0.488	0.4121	0.1205	0.1124	91.54	106.4
^{116}I	7.780	0.488	0.4215	0.1205	0.1124	169.3	196.75
^{116}Xe	4.450	0.529	0.4609	0.1215	0.1123	60.15	69.41
^{120}Ba	5.00	0.623	0.5496	0.1234	0.1130	90.65	103.5
^{120}Xe	1.617	0.529	0.4599	0.1215	0.1123	7.72	8.95
^{126}Cs	4.824	0.574	0.501	0.1224	0.112	76.88	88.69
^{182}Re	2.800	2.69	2.593	0.1448	0.128	22.86	24.15
^{205}Bi	2.708	4.88	4.837	0.1561	0.138	228.17	233.83


g_K^2
(a)
1
10^{-1}
40
60
80
Z_0
$F_{EC}^{K,L}$
(b)
10^2
10
1
50
100
150
200
A_0

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Thank you!