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Elastic Imaging using Multiprocessor Computing Systems

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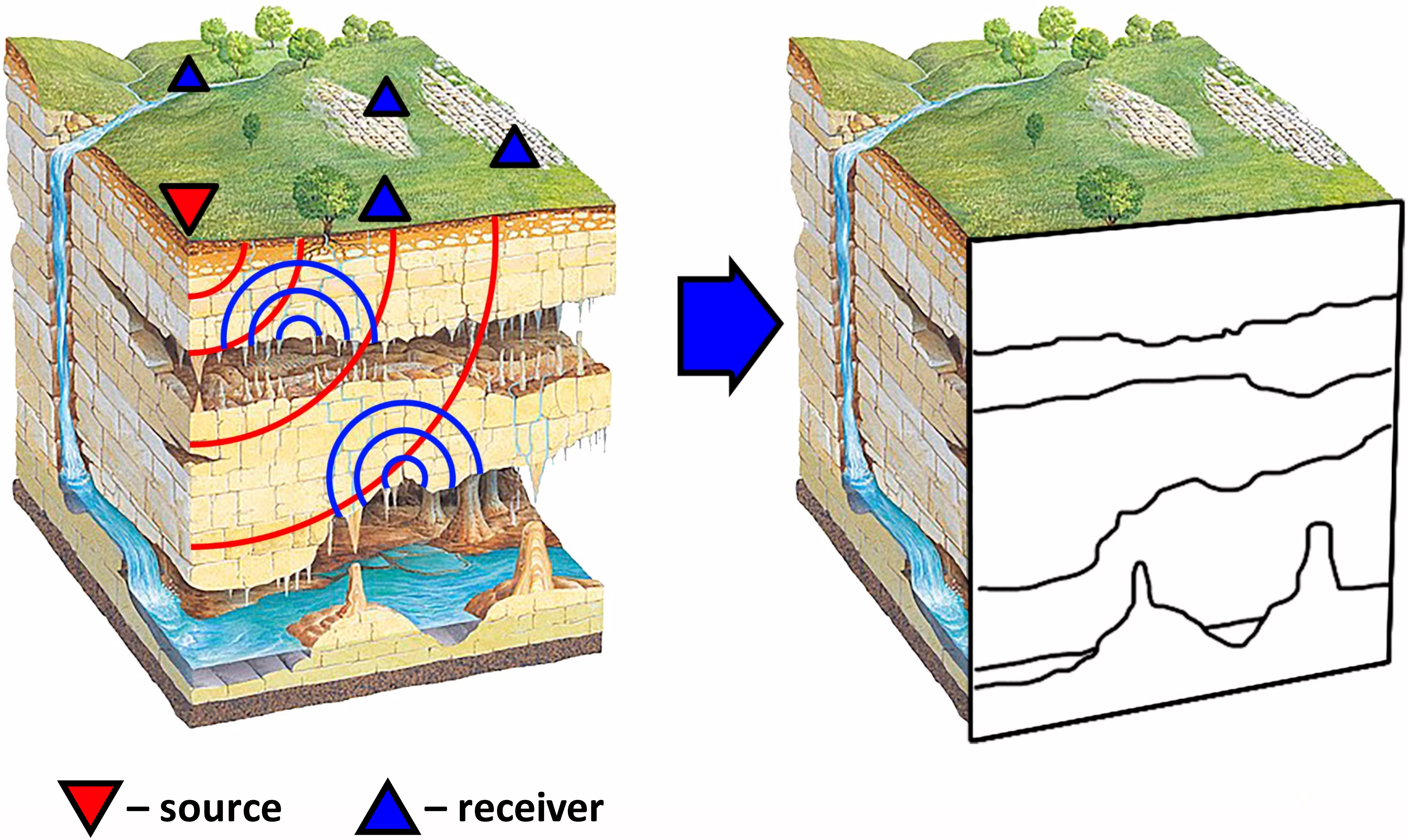
Seismic migration imaging



Formulae

Lame equation:

$$\hat{\Lambda}\vec{u} - \frac{\partial^2 \vec{u}}{\partial t^2} = -\frac{1}{\rho}\vec{F}, \qquad \hat{\Lambda} = c_p^2 \nabla\nabla \cdot - c_s^2 \nabla\times\nabla\times$$

Background and anomalous parts:

$$c_\alpha^2 = c_{\alpha,b}^2 + \Delta c_\alpha^2, \qquad \Delta c_\alpha^2\Big|_{r\notin V} = 0, \qquad \alpha \in \{p, s\},$$

$$\hat{\Lambda} = \hat{\Lambda}_b + \Delta\hat{\Lambda}, \qquad \vec{u} = \vec{u}^i + \vec{u}^s$$

Equations for incident and scattered fields:

Born approximation

$$\hat{\Lambda}_b\vec{u}^i - \frac{\partial^2 \vec{u}^i}{\partial t^2} = -\frac{1}{\rho}\vec{F}, \qquad \hat{\Lambda}_b\vec{u}^s - \frac{\partial^2 \vec{u}^s}{\partial t^2} = -\Delta\hat{\Lambda}(\vec{u}^i + \vec{u}^s)$$

Homogeneous space: $c_{\alpha,b} = const,\ V = \mathbb{R}^3$

$$s = c^{-1},$$

$$\widehat{D}_p^i = \mathrm{grad}^i \mathrm{div}^i, \qquad \widehat{D}_s^i = -\mathrm{rot}^i \mathrm{rot}^i,$$

$$\nabla^i = \left(\partial_{x^i}\, \partial_{y^i}\, \partial_{z^i}\right)^{\mathrm{T}}$$

Green's tensor:

$$\hat{G}_\alpha^L = \widehat{D}_\alpha \hat{g}_\alpha = \widehat{D}'_\alpha \hat{g}_\alpha,$$

$$\hat{g}_\alpha = \left\{\chi\left(t' - t - s_{\alpha,b}|\vec{r}' - \vec{r}|\right) - \chi(t' - t)\right\} \frac{\hat{I}}{4\pi|\vec{r}' - \vec{r}|},$$

$$\chi(t) = \max(0, t)$$

Permanently polarized point source :

$$\vec{F}(\vec{r},t) = \delta(\vec{r}-\vec{r}_0)f''(t)\vec{f},$$

$$\lim_{t\to+\infty} f(-t) = \lim_{t\to+\infty} f'(-t)t = \lim_{t\to0} f'(t'-t)t = 0$$

Forward modeling (whole space):

$$\vec{u}_\alpha^{s,B}(\vec{r}',t') = \sum_\beta \frac{1}{\rho(\vec{r}_0)}\widehat{D}'_\alpha\widehat{D}^0_\beta\int_V \Delta c_\beta^2(\vec{r})\frac{f\left(t'-s_{\beta,b}|\vec{r}_0-\vec{r}|-s_{\alpha,b}|\vec{r}'-\vec{r}|\right)}{16\pi^2 c_{\beta,b}^2|\vec{r}_0-\vec{r}||\vec{r}'-\vec{r}|}\vec{f}\,dV$$

Migration (whole space):

$$\Delta c_{\beta,\mathrm{migr}}^2(\vec{r}) = \sum_\alpha \int_S\int_T \frac{\vec{d}(\vec{r}',t')}{\rho(\vec{r}_0)}\cdot\widehat{D}'_\alpha\widehat{D}^0_\beta\frac{f\left(t'-s_{\beta,b}|\vec{r}_0-\vec{r}|-s_{\alpha,b}|\vec{r}'-\vec{r}|\right)}{16\pi^2 c_{\beta,b}^2|\vec{r}_0-\vec{r}||\vec{r}'-\vec{r}|}\vec{f}\,dt'dS$$

Homogeneous half-space with free surface:

$$c_{\alpha,b} = const, \qquad V = \{(x, y, z): z \geq 0\},$$

$$2\mu \frac{\partial u_z}{\partial z} + \lambda \operatorname{div} \vec{\mathrm{u}} = \mu \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) = \mu \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right) = 0$$

Green's tensor:

$$\hat{G}_\alpha^{L,H} = \hat{D}'_\alpha \left[\hat{g}_\alpha - \underline{\hat{g}_\alpha} \right],$$

$$\underline{\hat{g}_\alpha} = \left\{ \chi\left(t' - t - s_{\alpha,b} \left| \vec{r}' - \underline{\vec{r}} \right| \right) - \chi(t' - t) \right\} \frac{\hat{I}}{4\pi \left| \vec{r}' - \underline{\vec{r}} \right|},$$

$$\underline{\vec{r}} = (x, y, -z)^{\mathrm{T}}$$

Forward modeling:

$$\vec{u}_{\alpha}^{s,B}(\vec{r}\,',t') = \sum_{\beta} \frac{1}{\rho(\vec{r}_0)} \widehat{D}'_{\alpha} \int_V \Delta c_{\beta}^2(\vec{r}) \left\{ \begin{array}{l} \widehat{D}_{\beta}^0 \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c_{\beta,b}^2 |\vec{r}_0 - \vec{r}||\vec{r}\,' - \vec{r}|} \\ -\widehat{D}_{\underline{\beta}}^{\underline{0}} \dfrac{f\left(t' - s_{\beta,b}\left|\vec{r}_0 - \underline{\vec{r}}\right| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c_{\beta,b}^2 \left|\vec{r}_0 - \underline{\vec{r}}\right||\vec{r}\,' - \vec{r}|} \\ +\widehat{D}_{\underline{\beta}}^{\underline{0}} \dfrac{f\left(t' - s_{\beta,b}\left|\vec{r}_0 - \underline{\vec{r}}\right| - s_{\alpha,b}\left|\vec{r}\,' - \underline{\vec{r}}\right|\right)}{16\pi^2 c_{\beta,b}^2 \left|\vec{r}_0 - \underline{\vec{r}}\right|\left|\vec{r}\,' - \underline{\vec{r}}\right|} \\ -\widehat{D}_{\beta}^0 \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}\left|\vec{r}\,' - \underline{\vec{r}}\right|\right)}{16\pi^2 c_{\beta,b}^2 |\vec{r}_0 - \vec{r}|\left|\vec{r}\,' - \underline{\vec{r}}\right|} \end{array} \right\} \vec{f}\, dV\,,$$

$$\widehat{D}_{\beta}^0 \rightarrow \widehat{D}_{\underline{\beta}}^{\underline{0}} \sim \partial_{z_0} \rightarrow -\partial_{z_0}$$

Migration:

$$\Delta c^2_{\beta,\mathrm{migr}}(\vec{r}) = \sum_{\alpha} \int_S \int_T \frac{\vec{d}(\vec{r}', t')}{\rho(\vec{r}_0)} \cdot \widehat{D}'_\alpha \left\{ \begin{array}{l} \widehat{D}^0_\beta \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}' - \vec{r}|} \\ -\widehat{D}^{\underline{0}}_\beta \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}' - \vec{r}|} \\ +\widehat{D}^{\underline{0}}_\beta \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}' - \underline{\vec{r}}|} \\ -\widehat{D}^{\underline{0}}_\beta \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}' - \underline{\vec{r}}|} \end{array} \right\} \vec{f} dt' dS,$$

$$\widehat{D}^0_\beta \rightarrow \widehat{D}^{\underline{0}}_\beta \sim \partial_{z_0} \rightarrow -\partial_{z_0}$$

$$\Delta c^2_{\beta,\mathrm{migr}}(\vec{r}) = \sum_\alpha \int_S \int_T \frac{\vec{d}(\vec{r}\,',t')}{\rho(\vec{r}_0)} \cdot \widehat{D}'_\alpha \left\{ \begin{array}{l} \widehat{D}^0_\beta \frac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \vec{r}|} \\ -\widehat{D}^0_\beta \frac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}\,' - \vec{r}|} \\ +\widehat{D}^0_\beta \frac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}\,' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}\,' - \underline{\vec{r}}|} \\ -\widehat{D}^0_\beta \frac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \underline{\vec{r}}|} \end{array} \right\} \vec{f}\, dt'\, dS$$

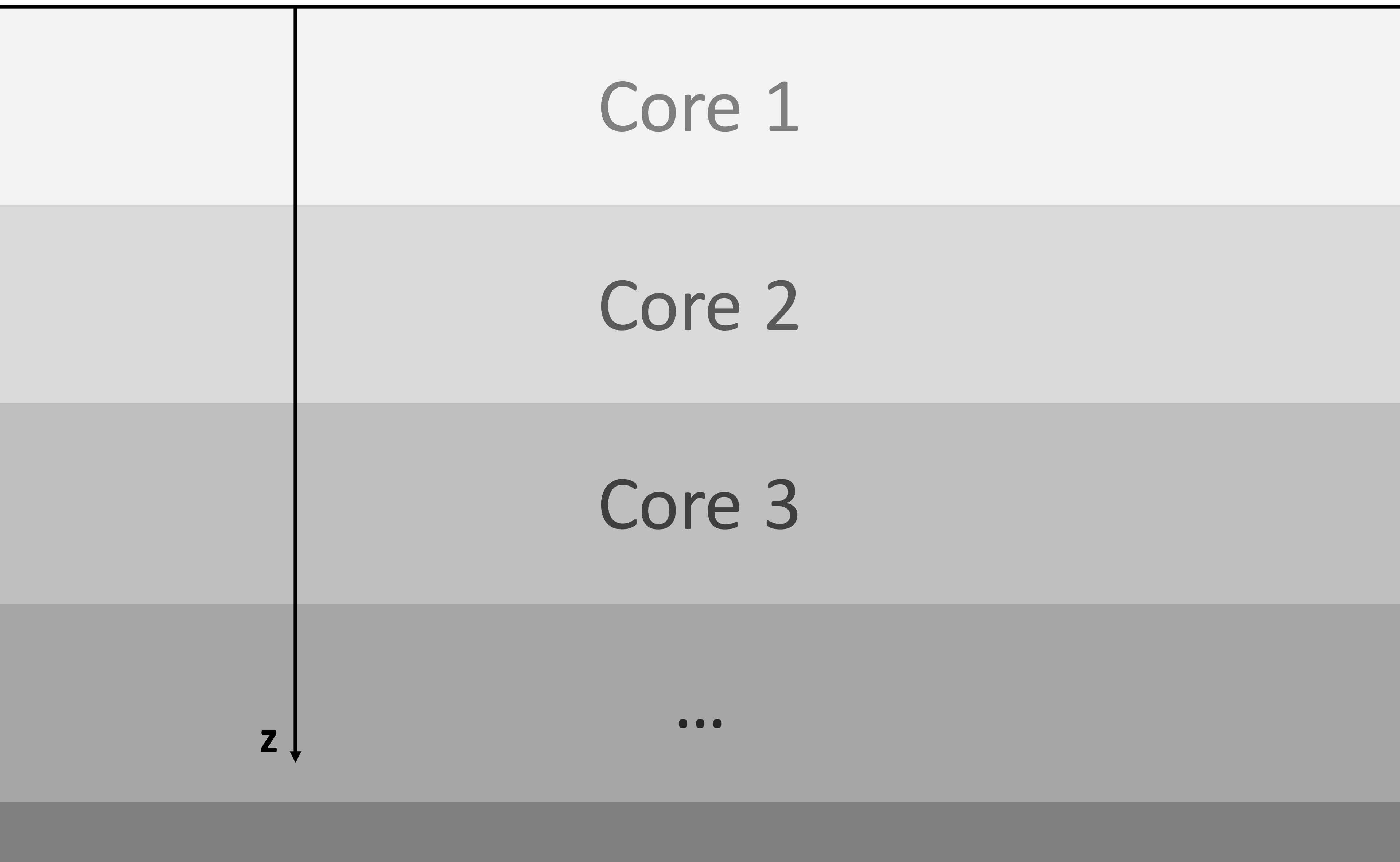
$$N_x \times N_y \times N_z \qquad\qquad N_t$$

$$\text{Complexity} \sim O\left(N_x N_y N_z N_t \log(N_x) \log(N_y)\right)$$

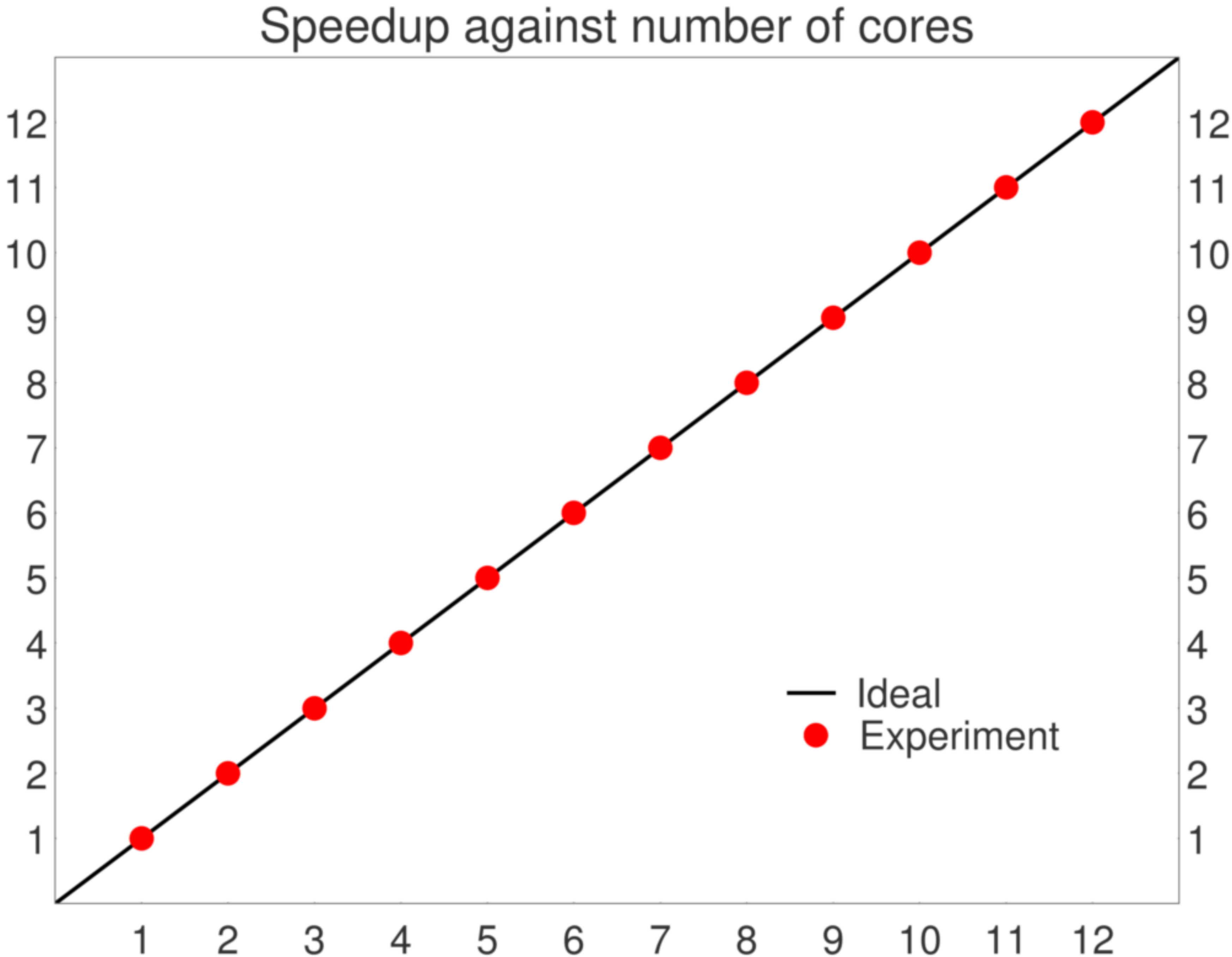
Parallelization

$$\vec{r} = (x, y, z)^{\mathrm{T}}$$

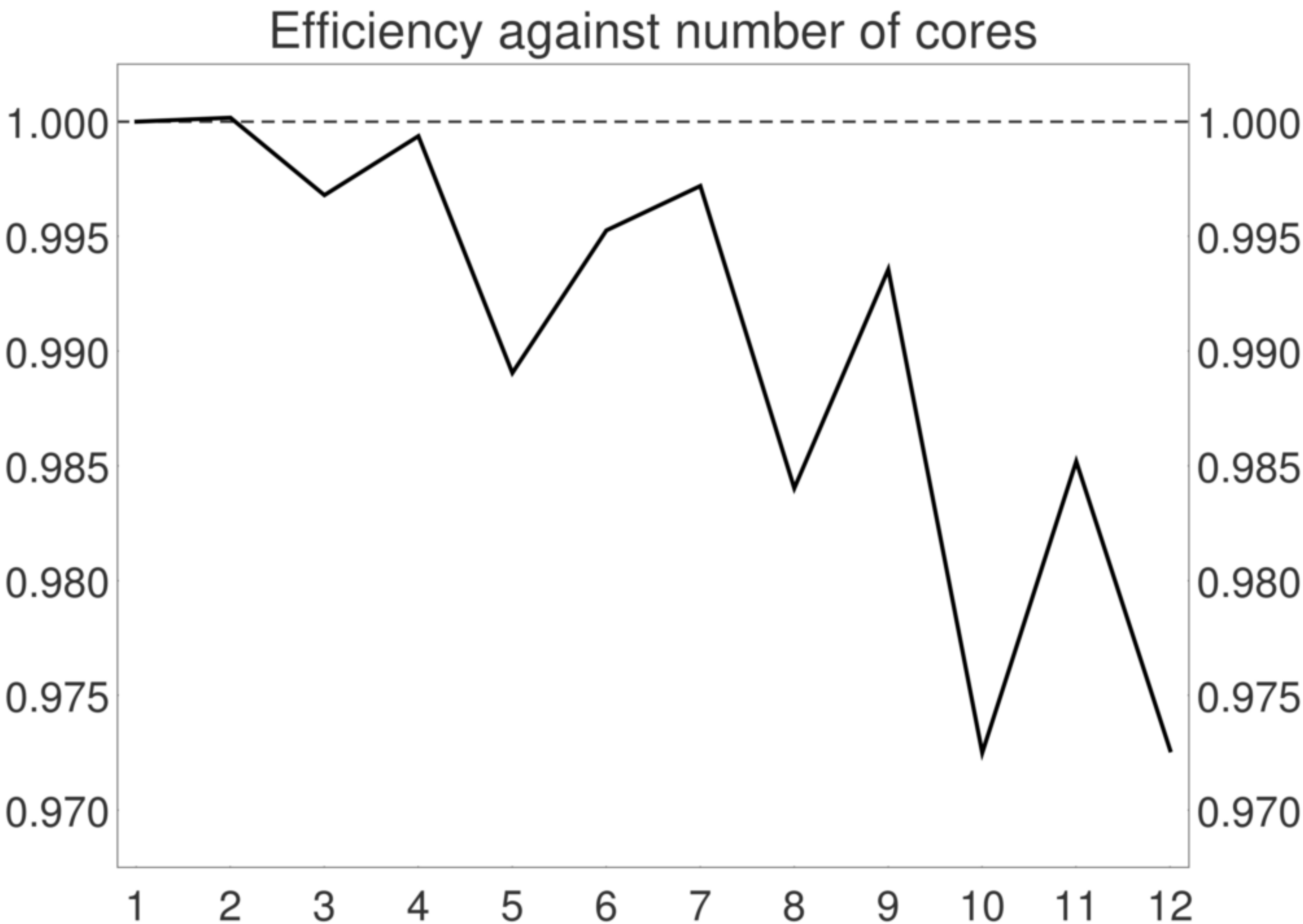
$$\Delta c^2_{\beta,\mathrm{migr}}(\vec{r}) = \sum_{\alpha} \int_S \int_T \frac{\vec{d}(\vec{r}\,', t')}{\rho(\vec{r}_0)} \cdot \widehat{D}'_{\alpha} \left\{ \begin{array}{l} \widehat{D}^0_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \vec{r}|} \\ -\widehat{D}^0_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}\,' - \vec{r}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}\,' - \vec{r}|} \\ +\widehat{D}^0_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}| - s_{\alpha,b}|\vec{r}\,' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \underline{\vec{r}}||\vec{r}\,' - \underline{\vec{r}}|} \\ -\widehat{D}^0_{\beta} \dfrac{f\left(t' - s_{\beta,b}|\vec{r}_0 - \vec{r}| - s_{\alpha,b}|\vec{r}\,' - \underline{\vec{r}}|\right)}{16\pi^2 c^2_{\beta,b}|\vec{r}_0 - \vec{r}||\vec{r}\,' - \underline{\vec{r}}|} \end{array} \right\} \vec{f}\, dt'\, dS$$



Core 1
Core 2
Core 3
...
z



Speedup against number of cores
Ideal
Experiment
1
2
3
4
5
6
7
8
9
10
11
12



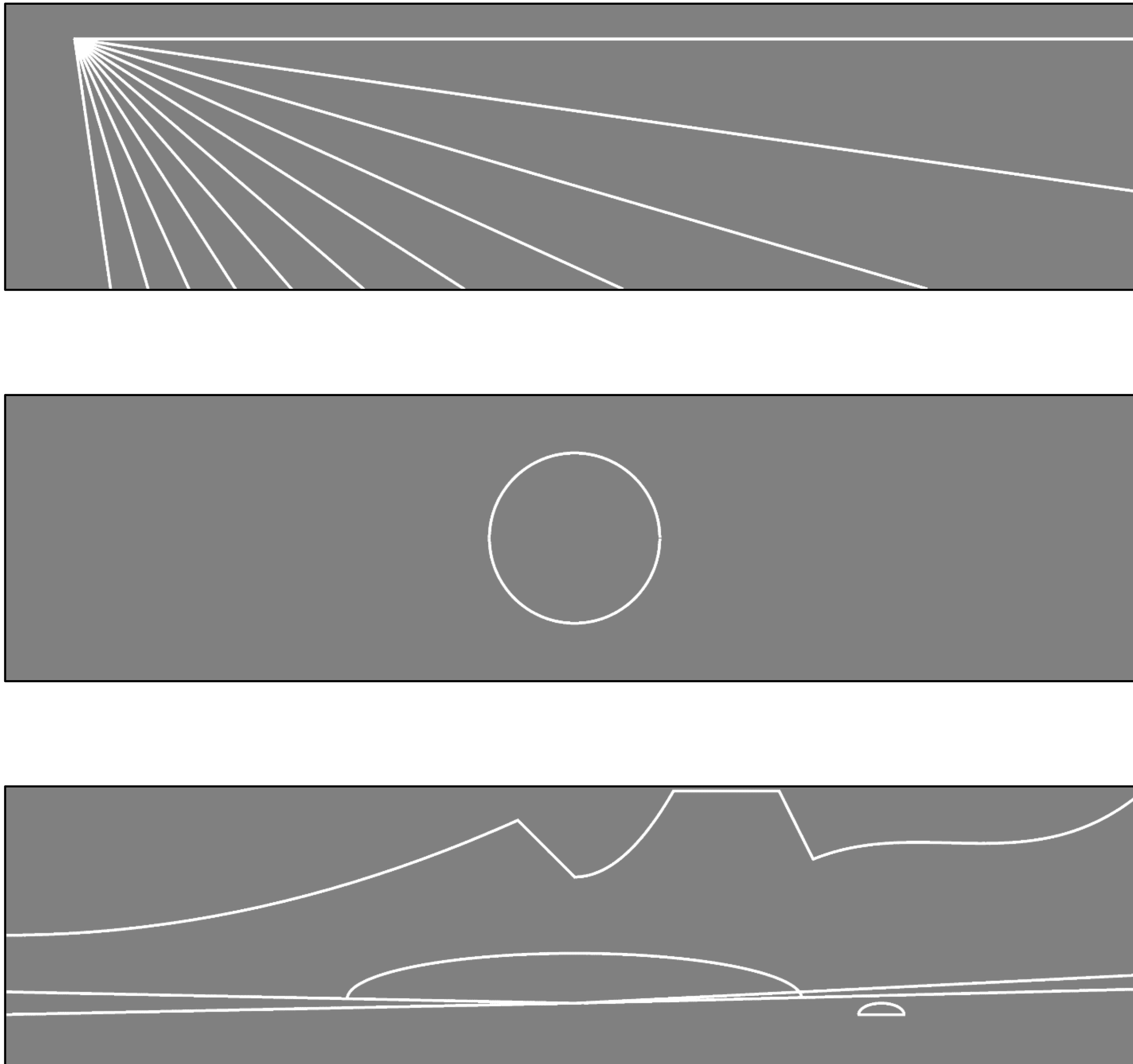
Efficiency against number of cores
1.000
0.995
0.990
0.985
0.980
0.975
0.970
1
2
3
4
5
6
7
8
9
10
11
12

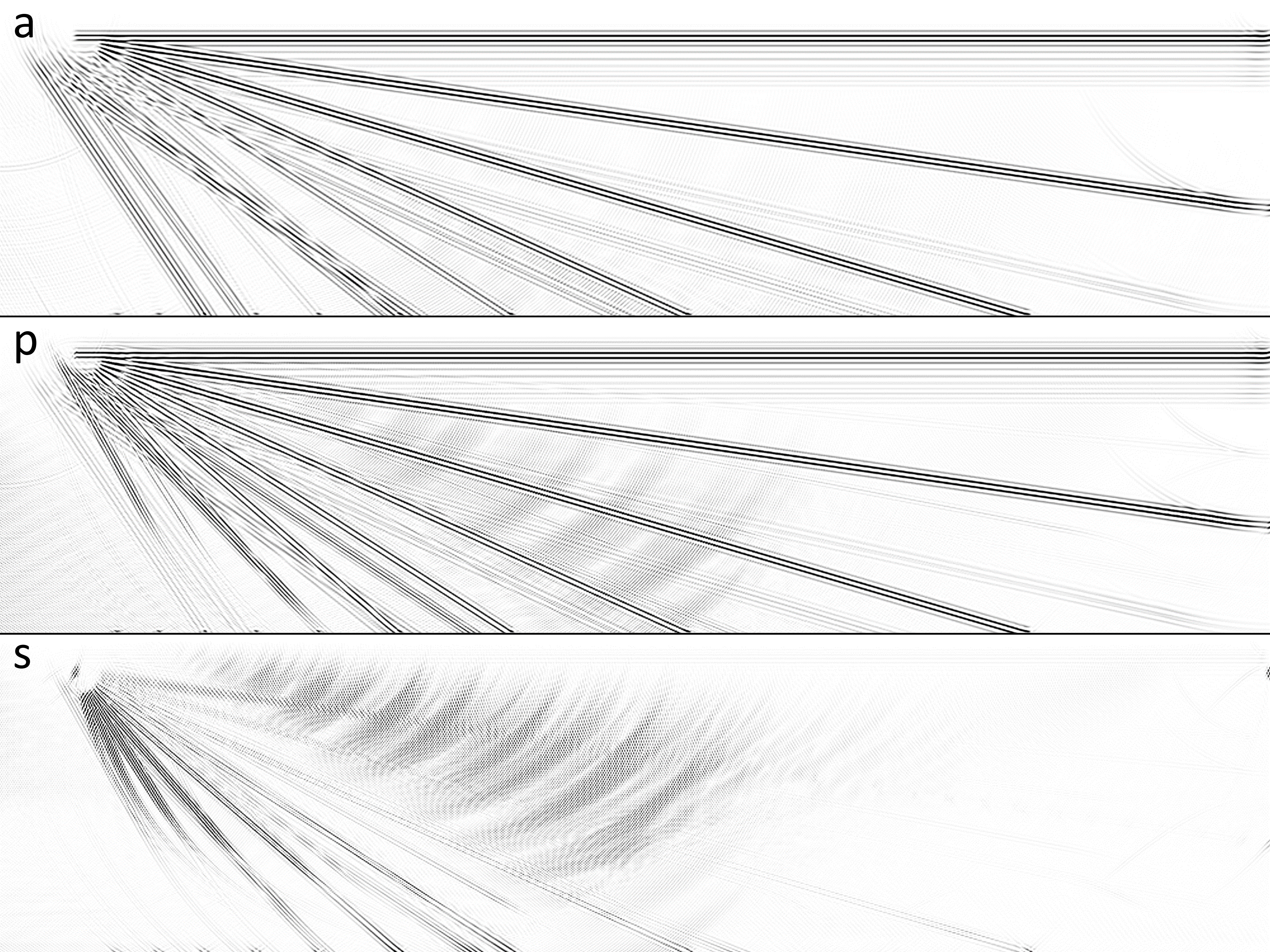
Number of cores	Time of calculation, secs	Memory used, GB
1	17437	0.52
2	8717	0.83
3	5831	1.07
4	4362	1.36
5	3526	1.62
6	2920	1.96
7	2498	2.17
8	2215	2.39
9	1950	2.72
10	1793	3.05
11	1609	3.34
12	1494	3.59

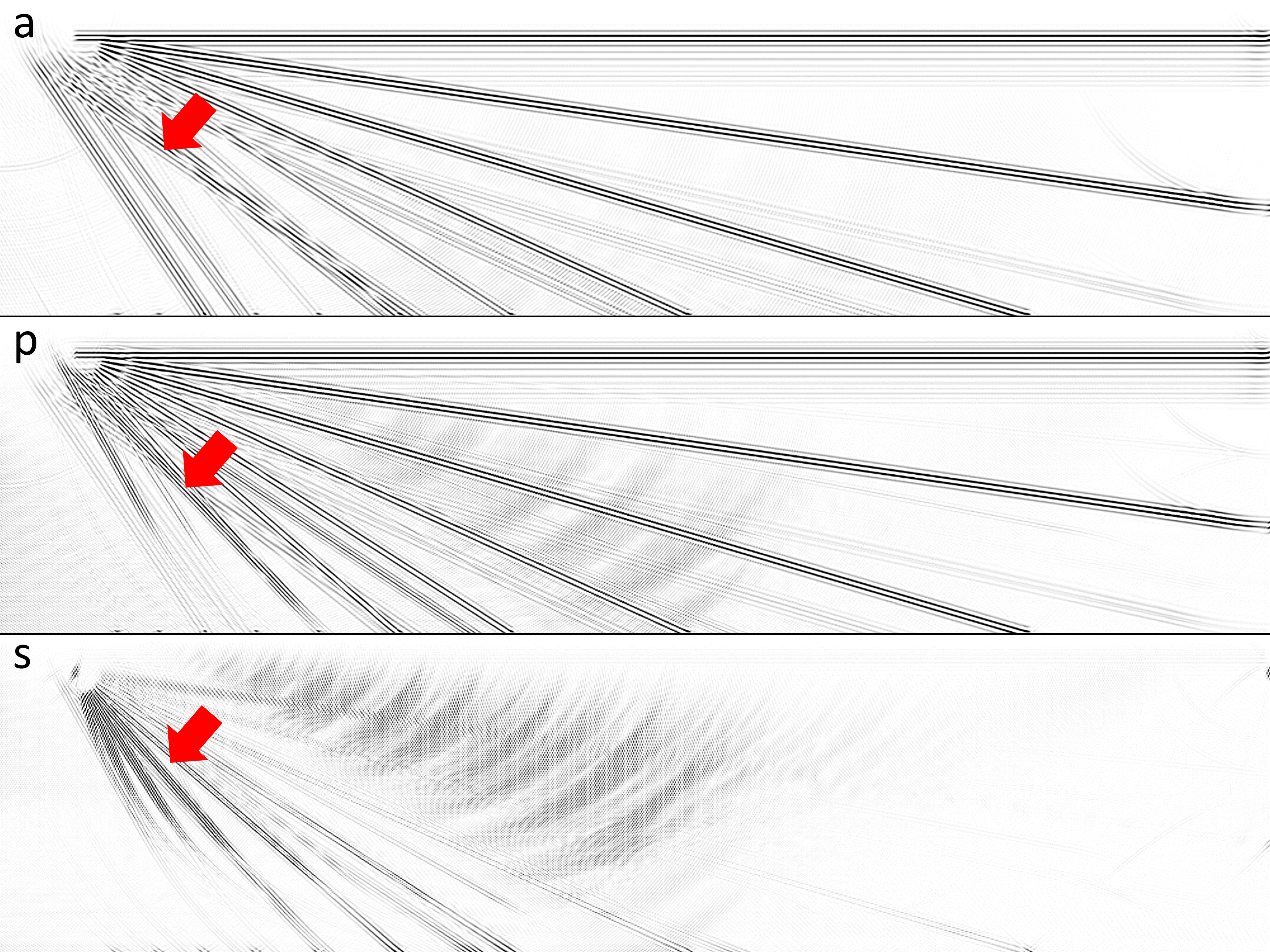
Computation results

Media
$x \times z = 10 \times 2.5$ km,
$y = const$,
$c_{p,b} = 2.5$ km/s,
$c_{s,b} = 1.25$ km/s,
$\Delta c_\alpha^2 / c_{\alpha,b}^2 = 0.01$,
$\rho = 2.5$ t/m^3

Data
(only z-component of scattered field)
$z = 15$ m,
$\Delta x = 10$ m,
$\Delta t = 2$ ms,
$t \in [0, 4]$ s,
$F(t) =$
$(1 - 2\pi^2 f_M^2 t^2) \cdot$
$\cdot\, e^{-\pi^2 f_M^2 t^2}$,
$f_M = 25$ Hz,
$\vec{f} = (0\ 0\ 1)^T$

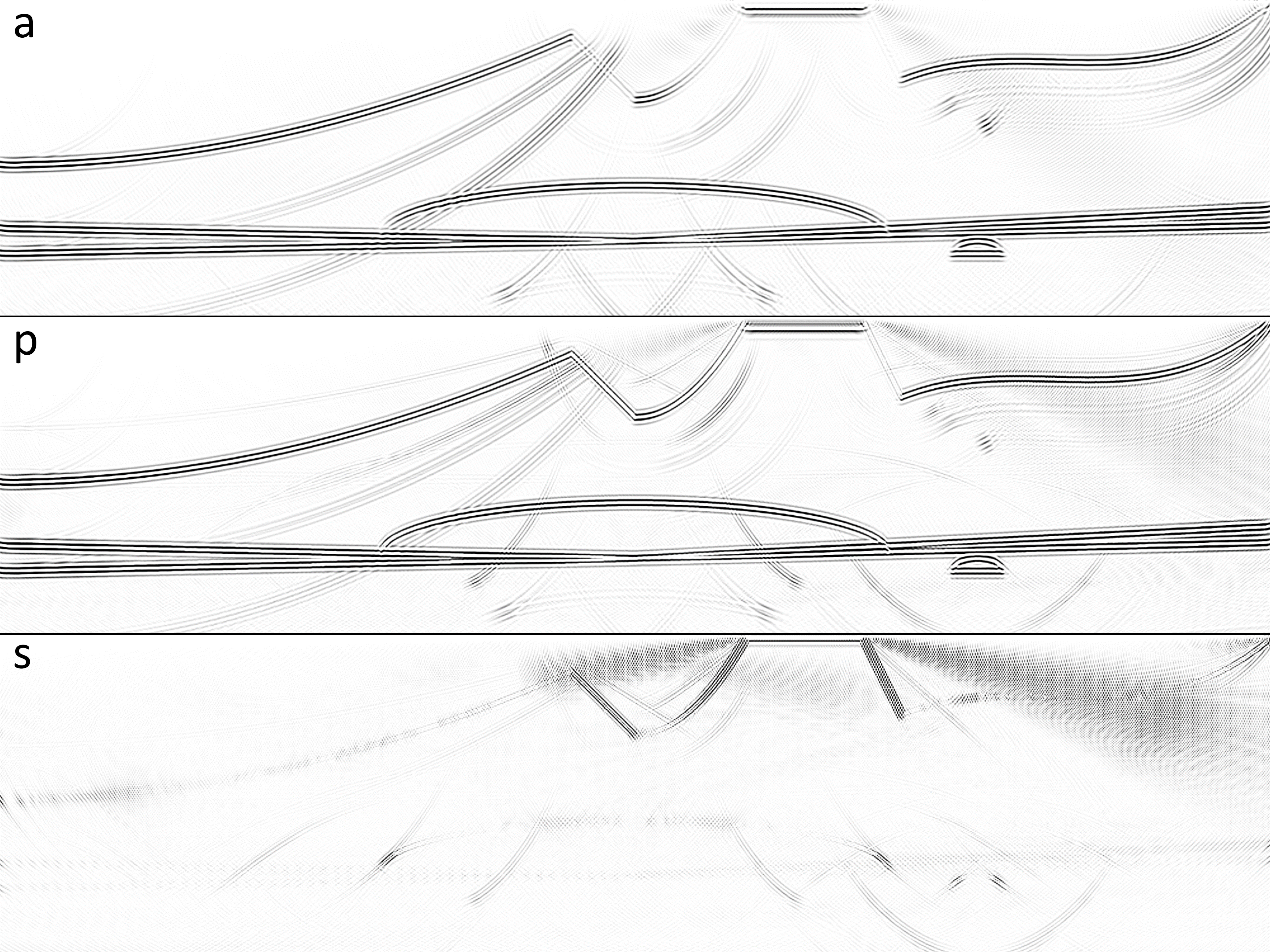



a
p
s


a
p
s


a
p
s


a
p
s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p
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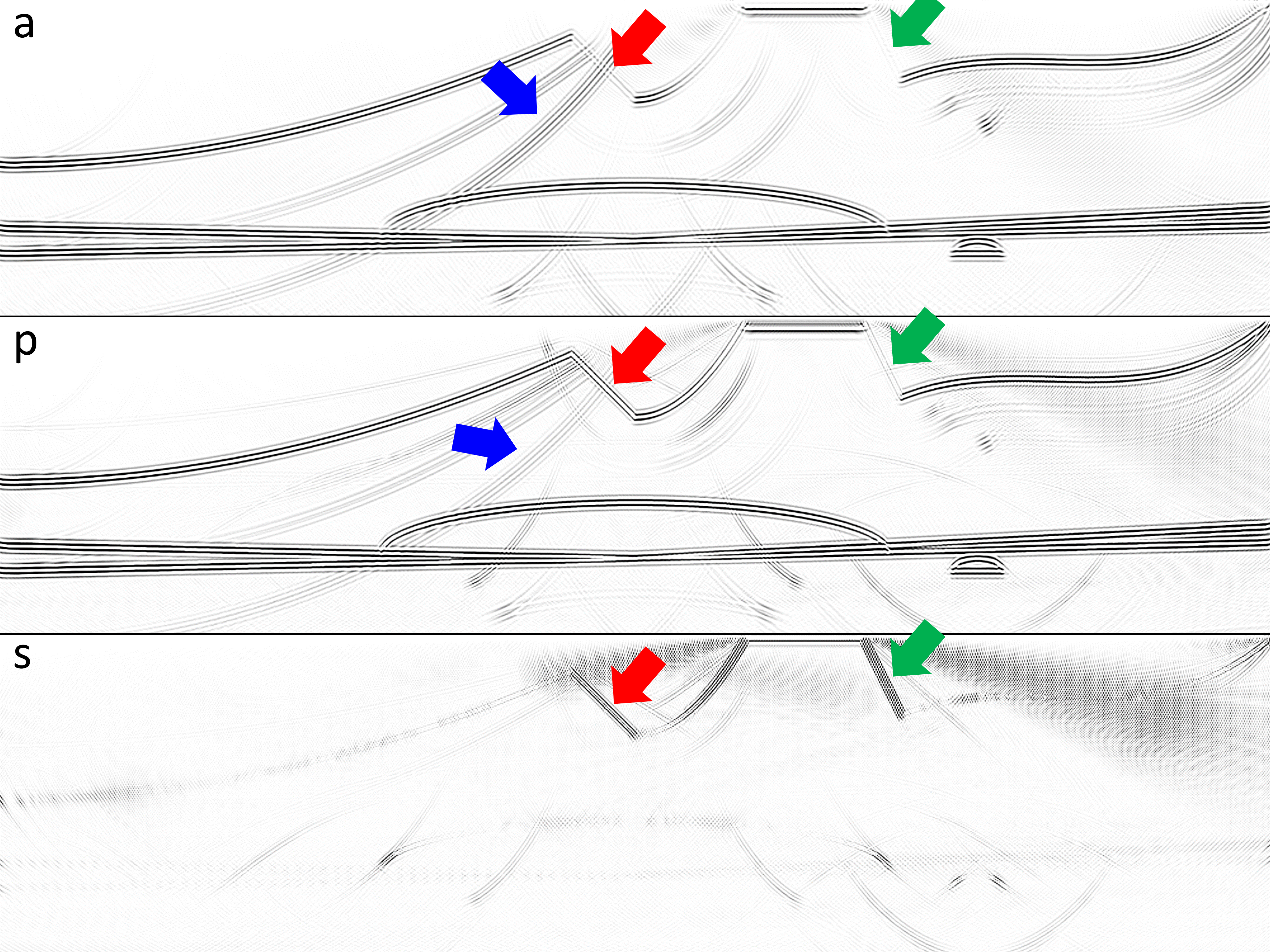
a
p
s

Results

- Algorithm of elastic migration based on Born approximation was proposed and developed
- It has been shown to locate steep interfaces better than acoustic algorithm and to have less strongly pronounced false boundaries
- Algorithm has been parallelized for 12-core shared memory system with efficiency close to 100 %