Neutrinoless Double-Beta Decay with Emission of Single Electron

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New Trends in High-Energy Physics
Budva, 02.10.–08.10.2016

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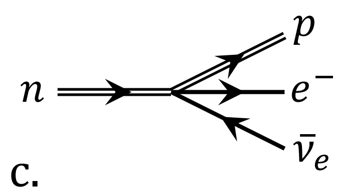
Double-Beta Decay

Fermi's effective QFT of beta decay is still valid at energy scales $\ll m_W$:

$$\mathcal{H}_\beta(x) = \frac{G_\beta}{\sqrt{2}}\, \bar{e}(x)\, \gamma^\mu\, (1-\gamma^5)\, \nu_e(x)\, j_\mu(x) + \mathrm{H.c.}$$

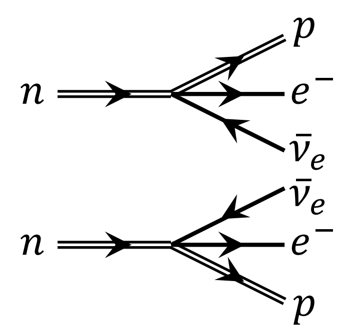
G_β: $G_\mathrm{F} \cos\theta_\mathrm{C}$

$j_\mu(x)$: $\bar{p}(x)\, \gamma_\mu\, (g_V - g_A\, \gamma^5)\, n(x)$, with g_V: 1, g_A: 1.27

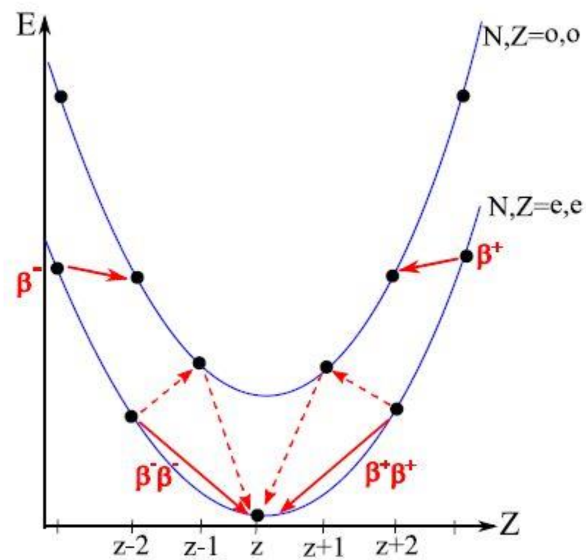


Double-beta decay ($2\nu\beta^-\beta^-$) is a rare 2nd-order process which can occur even if single-beta transition is forbidden or suppressed:

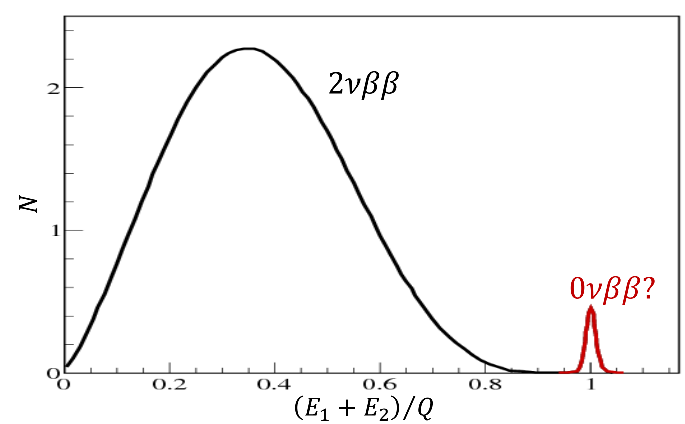
$${}^{A}_{Z}\mathrm{X} \longrightarrow {}_{Z+2}^{\;\;\;A}\mathrm{Y} + 2e^- + 2\bar{\nu}_e$$



^{A}X	$T_{1/2}^{2\nu\beta\beta}$ [y]
^{48}Ca	4.4×10^{19}
^{76}Ge	1.65×10^{21}
^{82}Se	0.92×10^{20}
^{96}Zr	2.3×10^{19}
^{100}Mo	7.1×10^{18}
^{116}Cd	2.87×10^{19}
^{128}Te	2.0×10^{24}
^{130}Te	6.9×10^{20}
^{136}Xe	2.19×10^{21}
^{150}Nd	8.2×10^{18}
^{238}U	2.0×10^{21}

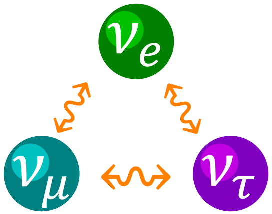


Half-lives $T_{1/2}^{2\nu\beta\beta}$:
[A. S. Barabash, Nucl. Phys. A, **935**, 52 (2015)]



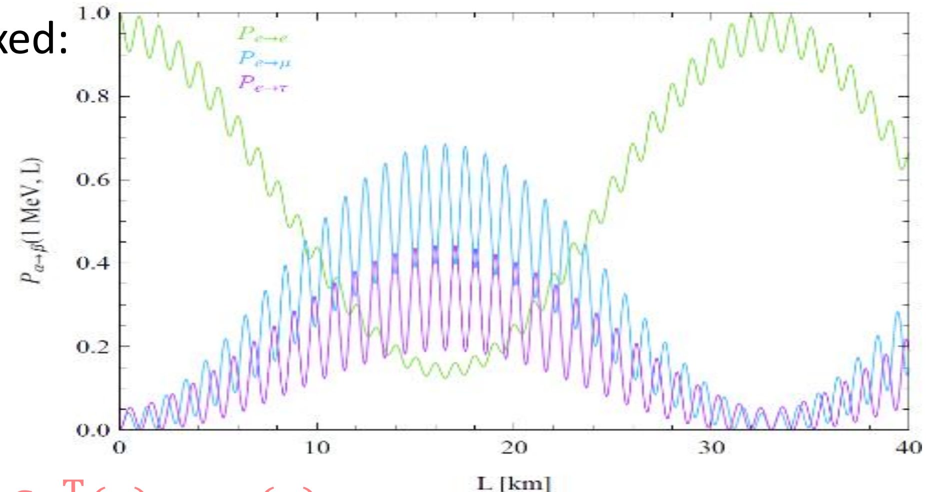
Neutrinoless Double-Beta Decay

Neutrino oscillations ⇒ ν are massive and mixed:



$$\nu_{\alpha L}(x) = \sum_i U_{\alpha i}\, \nu_{iL}(x)$$

α: e, μ, τ ; i: $1, 2, 3$

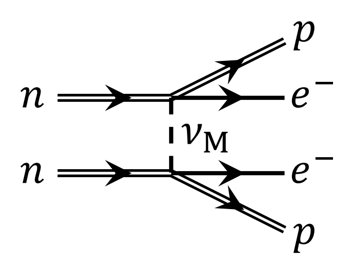


If massive neutrinos are Majorana fermions:

- Neutrinos $\nu_\alpha \equiv$ antineutrinos $\bar{\nu}_\alpha$ — $C\bar{\nu}_i^{\mathrm{T}}(x) = \nu_i(x)$
- Total lepton number L is not strictly conserved
- Neutrinoless double-beta decay mode ($0\nu\beta^-\beta^-$) exists:

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{\;\;A}_{Z+2}\mathrm{Y} + 2e^-$$

$\Delta L = 2$



$0\nu\beta\beta$ decay rate:

$$\Gamma^{0\nu\beta\beta} = g_A^4\, G^{0\nu\beta\beta}(Z,Q)\, \left|M^{0\nu\beta\beta}\right|^2 \frac{\left|m_{\beta\beta}\right|^2}{m_e^2}$$

$\Gamma^{0\nu\beta\beta}$: $\ln 2/T_{1/2}$; $\left|m_{\beta\beta}\right|^2$: $\propto 1/T_{1/2}$

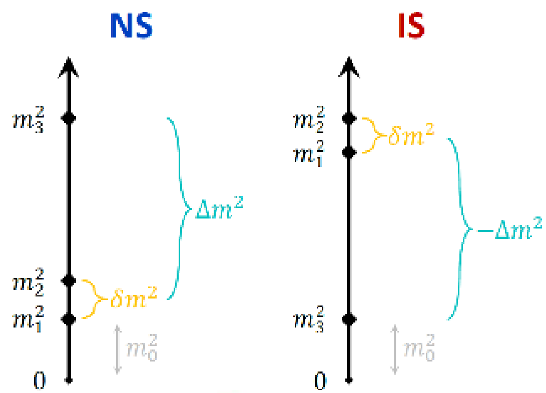
- Phase-space factor $G^{0\nu\beta\beta}(Z,Q)$ → particle kinematics (model-independent)
- Nuclear matrix element $M^{0\nu\beta\beta}$ → nuclear structure & dynamics (model-dependent)
- Effective Majorana neutrino mass $m_{\beta\beta}$ → neutrino physics (unknown)

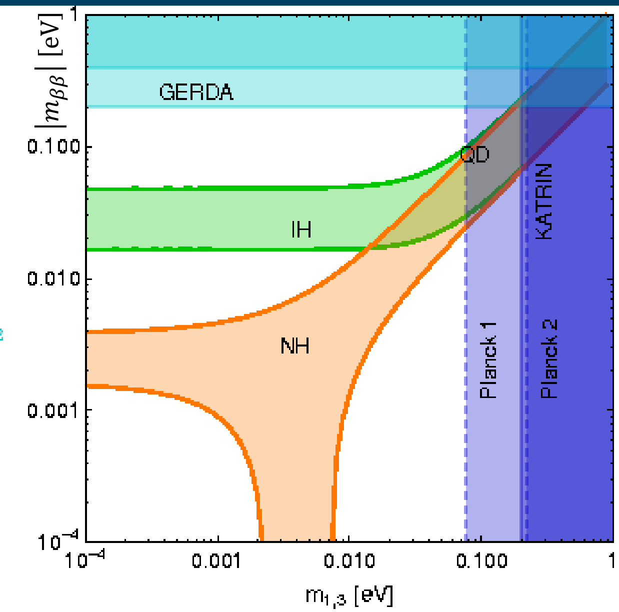
Effective Majorana Neutrino Mass

Effective Majorana ν mass:

$$m_{\beta\beta} = \sum_i U_{ei}^2 \, m_i$$

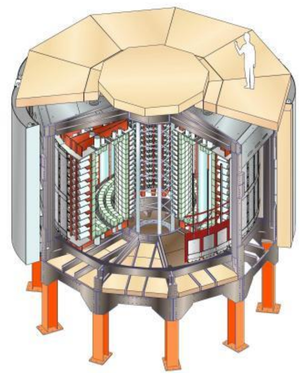
- Absolute scale of ν masses m_i
- Majorana phases in U_{ei} → leptonic CP violation (baryon asymmetry of the Universe)





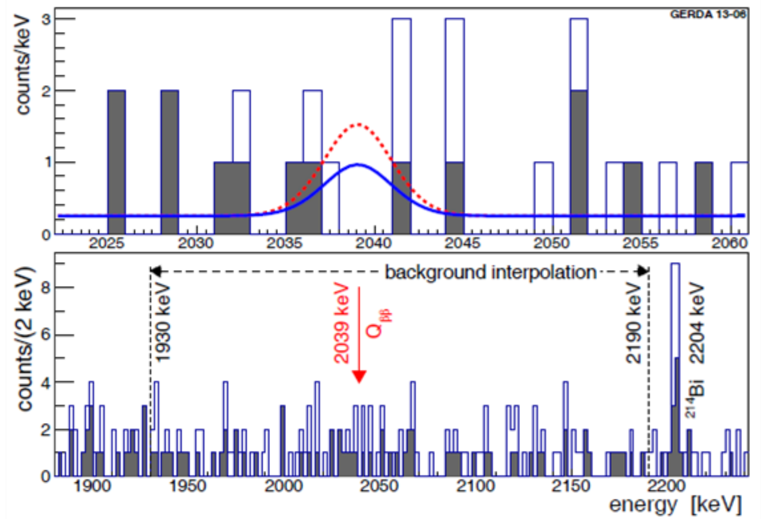
NEMO-3 @ LSM:

- Tracking calorimeter detector
- Thin source foils of ^{100}Mo, ^{82}Se, ^{48}Ca, etc.



GERDA @ LNGS:

- ^{76}Ge of HPGe in liquid Ar



[B. Majorovits (GERDA Collaboration), AIP Conf. Proc., **1672**, 110003 (2015)]

KamLAND-Zen @ Kamioka:

- ^{136}Xe-loaded liquid scintillator
- Most stringent bound:

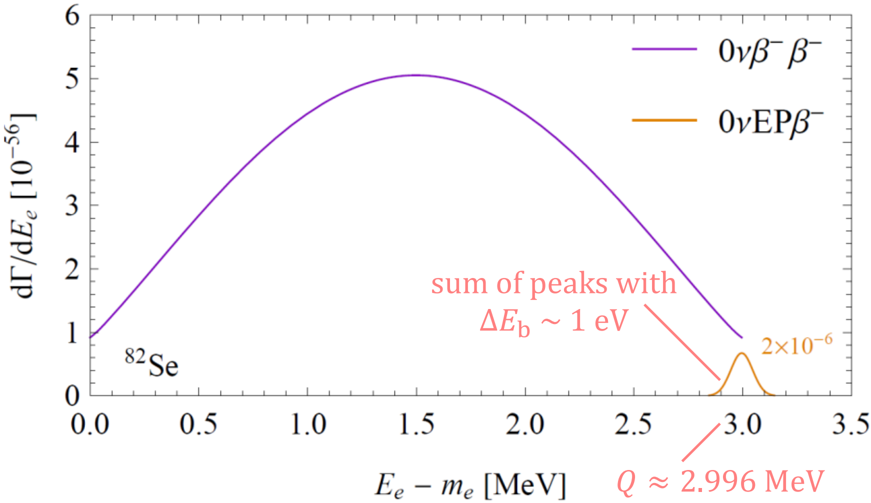
$$|m_{\beta\beta}| < 61 - 165 \text{ meV}$$

Single-Electron Mode of $0\nu\beta^-\beta^-$

$0\nu\mathrm{EP}\beta^-$: Nucleus is always surrounded by electron shells. What if one e_b^- remains bound and the other e^- carries away entire K.E. Q?

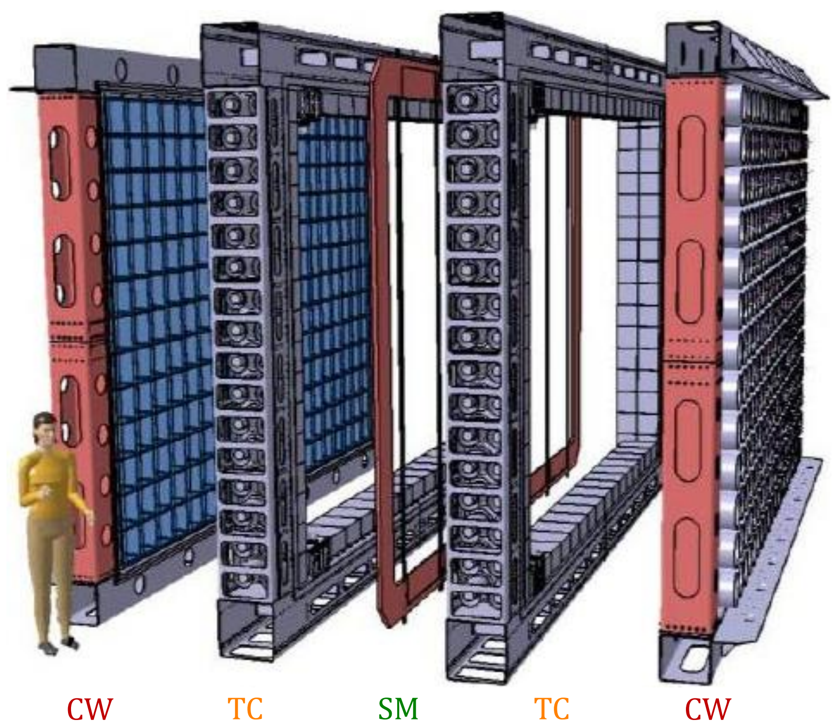
$$^A_Z\mathrm{X} \longrightarrow {}^{\;\;A}_{Z+2}\mathrm{Y} + e_b^- + e^-$$

- *Electron production (EP)* in an available $s_{1/2}$ or $p_{1/2}$ subshell of the daughter ion ${}^{\;\;A}_{Z+2}\mathrm{Y}^{2+}$
- Peak near the endpoint of single-electron spectrum



SuperNEMO:

- Source modules (SM): 20×5 kg thin foils of enriched and purified ^{82}Se, ^{150}Nd or ^{48}Ca
- Tracking chamber (TC): 9 planes of high-granularity drift cells in magnetic field → particle ID and vertex reconstruction → improved background rejection, angular correlations and single-electron spectra
- Calorimeter walls (CW): segmented organic scintillators + PMT: $\mathrm{FWHM}/E \approx 7\%/\sqrt{E/\mathrm{MeV}}$



Relativistic Electron Wave Functions

Solutions to the stationary Dirac equation with Coulomb potential (point-like source):

$$\psi_{\kappa\mu}(\vec{r}) = \begin{pmatrix} f_\kappa(r)\,\Omega_{\kappa\mu}(\hat{r}) \\ \mathrm{i}g_\kappa(r)\,\Omega_{-\kappa\mu}(\hat{r}) \end{pmatrix} \qquad \Omega_{\kappa\mu}(\hat{r}) = \sum_{s=\pm 1/2} C^{j\mu}_{l,\mu-s,1/2,s}\, Y_{l,\mu-s}(\hat{r})\,\chi^s \qquad V(r) = -\alpha Z/r$$

$$\kappa = (l-j)(2j+1) = \pm 1, \pm 2, \ldots$$

$$\mu = -j, \ldots, +j$$

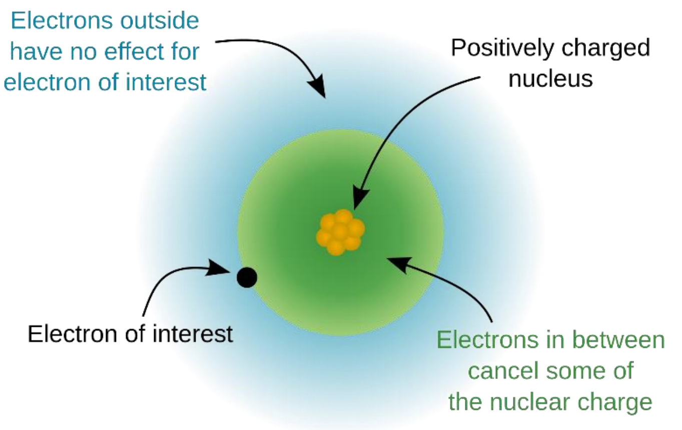
$p_{1/2}$, $s_{1/2}$

$j = |l \pm 1/2|$

Shielding effect → reduction of nuclear charge in the daughter ion ${}^{A}_{Z+2}\mathrm{Y}^{2+}$:

$$Z_\mathrm{b} \approx 2$$

$$Z_e \approx Z + 2$$



Continuous spectrum → dominant term from partial-wave expansion:

$$\psi^s_{s_{1/2}}(\vec{p},\vec{r}) = \begin{pmatrix} f_{-1}(r,E)\,\chi^s \\ g_{+1}(r,E)\,(\vec{\sigma}\cdot\hat{p})\,\chi^s \end{pmatrix}$$

Neglecting the electron-energy difference $(E_\mathrm{b} - E_e)/2$ with respect to the sum of nuclear masses $(M_i + M_f)/2$ and (in case of 0ν mode) the energy q^0 transferred by the Majorana neutrino, the NME for $0\nu\mathrm{EP}\beta^-$ and $2\nu\mathrm{EP}\beta^-$ remain essentially unchanged:

$$M^{0\nu\mathrm{EP}\beta} \approx M^{0\nu\beta\beta}$$

$$M^{2\nu\mathrm{EP}\beta} \approx M^{2\nu\beta\beta}$$

$0\nu\mathrm{EP}\beta^-$ decay rate (g.s. $0^+ \longrightarrow 0^+$):

$$\Gamma^{0\nu\mathrm{EP}\beta} = g_A^4 \frac{G_\beta^4 m_e^2}{(2\pi)^5 R^2} \left|M^{0\nu\beta\beta}\right|^2 \frac{\left|m_{\beta\beta}\right|^2}{m_e^2} \pi \sum_{n=n_\mathrm{F}+1}^{\infty} B(Z_\mathrm{b}, E_\mathrm{b})\, F(Z_e, E_e)\, E_e\, p_e$$

$p = \sqrt{E^2 - m_e^2}$

$R \approx 1.2\ \mathrm{fm}\ A^{1/3}$

$E_{2(e)} = E_{1(\mathrm{b})} + M_i - M_f$

$Q = M_i - M_f - 2m_e$

Fermi functions:

$s_{1/2}$, $p_{1/2}$

$$B(Z_\mathrm{b}, E_\mathrm{b}) = f_{n,-1}^2(R) + g_{n,+1}^2(R)$$

$$F(Z, E) = f_{-1}^2(R, E) + g_{+1}^2(R, E)$$

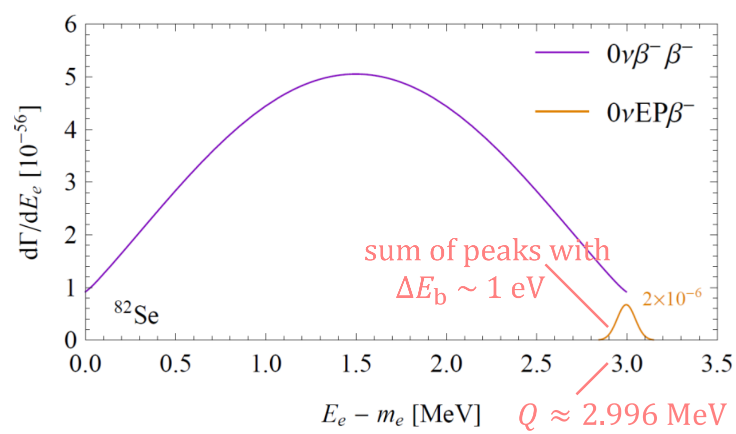
Summation runs over all energy levels above the valence shell (with $n = n_\mathrm{F}$); for the required precision, we summed numerically up to $n = 10^3$

$$B(Z_\mathrm{b}, E_\mathrm{b}) \xrightarrow{\text{non-relativistic}} \left|\psi_{nlm}^{\text{Schrödinger}}(R)\right|^2$$

$0\nu\beta^-\beta^-$ decay rate:

$$\Gamma^{0\nu\beta\beta} = g_A^4 \frac{G_\beta^4 m_e^2}{(2\pi)^5 R^2} \left|M^{0\nu\beta\beta}\right|^2 \frac{\left|m_{\beta\beta}\right|^2}{m_e^2} \int_{m_e}^{Q+m_e} \mathrm{d}E_1\, F(Z_e, E_1)\, E_1\, p_1\, F(Z_e, E_2)\, E_2\, p_2$$

Ratios $\Gamma^{0\nu\mathrm{EP}\beta}/\Gamma^{0\nu\beta\beta}$ do not depend on $M^{0\nu\beta\beta}$ and $m_{\beta\beta}$ $\rightarrow$ purely kinematic quantity, which can be evaluated for all $\beta^-\beta^-$ candidates



$\beta^-\beta^-$ Candidates

${}^{A}_{Z}X$	*Q* [MeV]	${}^{A}_{Z}X$	*Q* [MeV]	${}^{A}_{Z}X$	*Q* [MeV]
${}^{46}_{20}Ca$	0.990	${}^{110}_{46}Pd$	2.000	${}^{150}_{60}Nd$	3.368
${}^{48}_{20}Ca$	4.272	${}^{114}_{48}Cd$	0.537	${}^{154}_{62}Sm$	1.251
${}^{70}_{30}Zn$	1.001	${}^{116}_{48}Cd$	2.805	${}^{160}_{64}Gd$	1.730
${}^{76}_{32}Ge$	2.039	${}^{122}_{50}Sn$	0.366	${}^{170}_{68}Er$	0.654
${}^{80}_{34}Se$	0.134	${}^{124}_{50}Sn$	2.287	${}^{176}_{70}Yb$	1.087
${}^{82}_{34}Se$	2.996	${}^{128}_{52}Te$	0.867	${}^{186}_{74}W$	0.488
${}^{86}_{36}Kr$	1.256	${}^{130}_{52}Te$	2.529	${}^{192}_{76}Os$	0.414
${}^{94}_{40}Zr$	1.144	${}^{134}_{54}Xe$	0.830	${}^{198}_{78}Pt$	1.047
${}^{96}_{40}Zr$	3.350	${}^{136}_{54}Xe$	2.468	${}^{204}_{80}Hg$	0.416
${}^{98}_{42}Mo$	0.112	${}^{142}_{58}Ce$	1.417	${}^{232}_{90}Th$	0.842
${}^{100}_{42}Mo$	3.034	${}^{146}_{60}Nd$	0.070	${}^{238}_{92}U$	1.145
${}^{104}_{44}Ru$	1.300	${}^{148}_{60}Nd$	1.929		

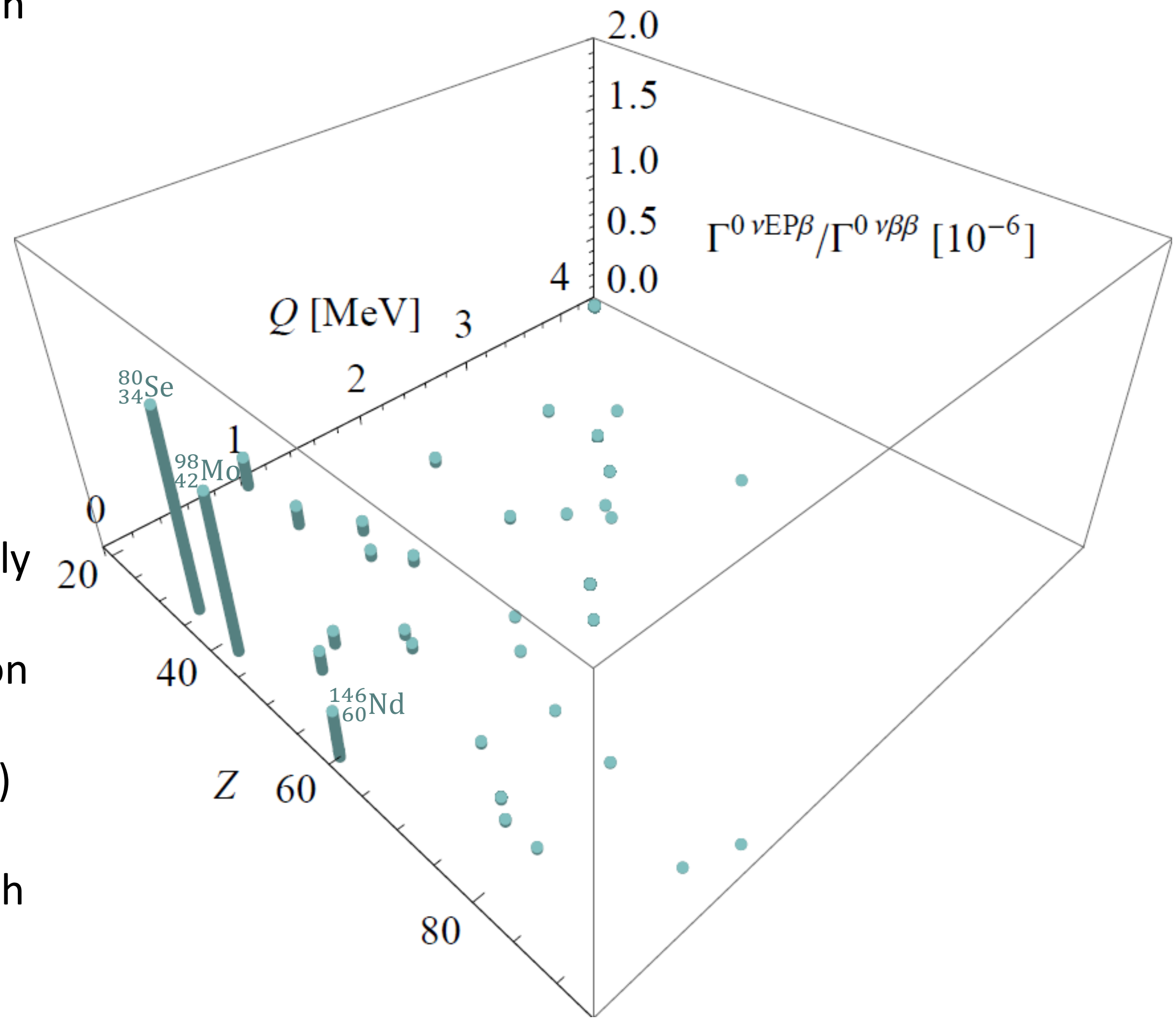
***Q* values:**
[V. I. Tretyak and Y. G. Zdesenko, Atom. Data Nucl. Data Tabl. **80**, 83 (2002)]

Ratios $\Gamma^{0\nu\text{EP}\beta}/\Gamma^{0\nu\beta\beta}(Z,Q)$

- $0\nu\text{EP}\beta^-$ is relatively more significant for isotopes with small Z and Q; top 3: $^{80}_{34}\text{Se}$, $^{98}_{42}\text{Mo}$ and $^{146}_{60}\text{Nd}$
- Ratio decreases rapidly with both Z and Q and ranges from 1.50×10^{-9} to 1.37×10^{-6}

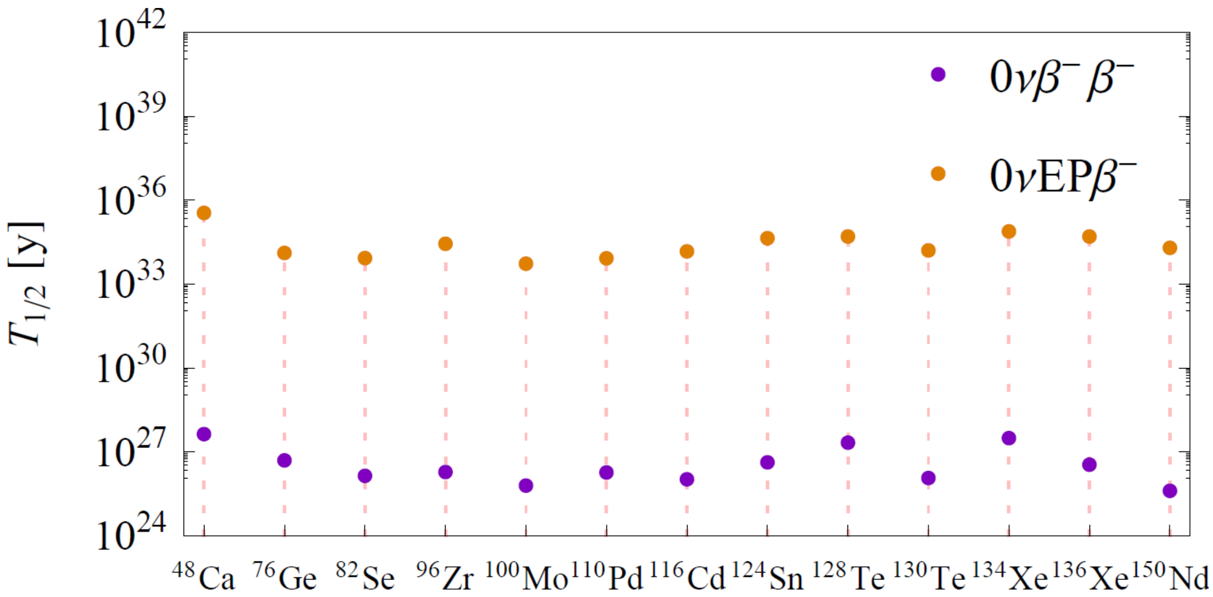
Overall suppression:

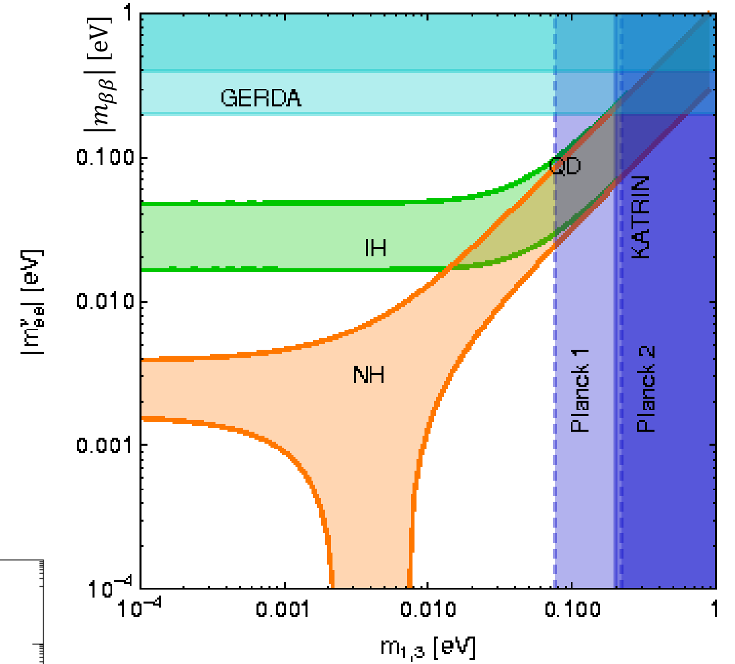
- Electron shielding of nuclear charge substantially reduces the bound-state wave function $B(Z_\text{b}, E_\text{b})$ on the surface of the nucleus (~ normalization constant)
- Discrete "phase space" of the bound electron is much more restricted when compared to all possible configurations of $\text{d}^3\vec{p}$

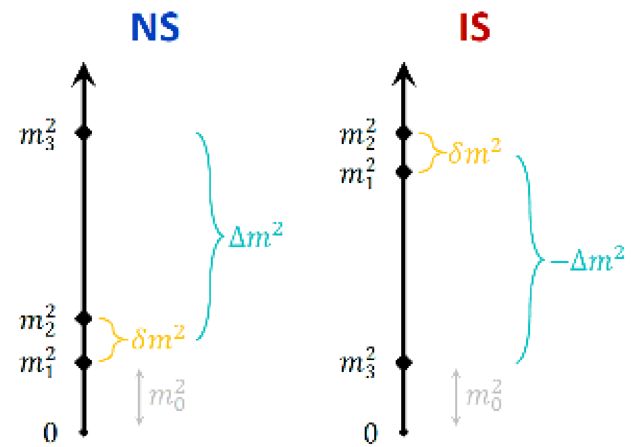


Estimation of Half-Lives $T_{1/2}^{0\nu\beta\beta}$ and $T_{1/2}^{0\nu EP\beta}$

- Within the inverted hierarchy of ν masses (and excluding the possibility of sterile neutrinos), the effective Majorana neutrino mass: $|m_{\beta\beta}| \approx (20-50)$ meV
- Assuming $|\boldsymbol{m_{\beta\beta}}| = \mathbf{50\ meV}$ and values of $M^{0\nu\beta\beta}$ from literature (QRPA with CD-Bonn potential), we estimated the $0\nu\beta^-\beta^-$ and $0\nu EP\beta^-$ half-lives $T_{1/2}^{0\nu\beta\beta}$ and $T_{1/2}^{0\nu EP\beta}$:







Nuclear matrix elements $\boldsymbol{M^{0\nu\beta\beta}}$**:**

[F. Šimkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C, **87**, 045501 (2013)]

[D.-L. Fang, A. Faessler, and F. Šimkovic, Phys. Rev. C, 92, 044301 (2015)]

Decay Rates $\Gamma^{2\nu\mathrm{EP}\beta}$ and $\Gamma^{2\nu\beta\beta}$

$$\Gamma^{2\nu\beta\beta} = g_A^4\, G^{2\nu\beta\beta}(Z,Q)\, \left|M^{2\nu\beta\beta}\right|^2$$

The results can be readily extended to $2\nu\beta^-\beta^-$:

- No unknown parameters of neutrino physics
- Possibility to test various nuclear-theoretical methods by comparing the calculated values of NME $M^{2\nu\beta\beta}$ (and hence $M^{0\nu\beta\beta}$) to the measured half-lives $T_{1/2}^{2\nu\beta\beta}$
- Once again, ratios $\Gamma^{2\nu\mathrm{EP}\beta}/\Gamma^{2\nu\beta\beta}$ do not depend on $M^{2\nu\beta\beta}$

$2\nu\mathrm{EP}\beta^-$ decay rate (g.s. $0^+ \longrightarrow 0^+$):

$$Q = M_i - M_f - 2m_e$$

$$\omega_2 = M_i - M_f - E_{1(\mathrm{b})} - E_{2(e)} - \omega_1$$

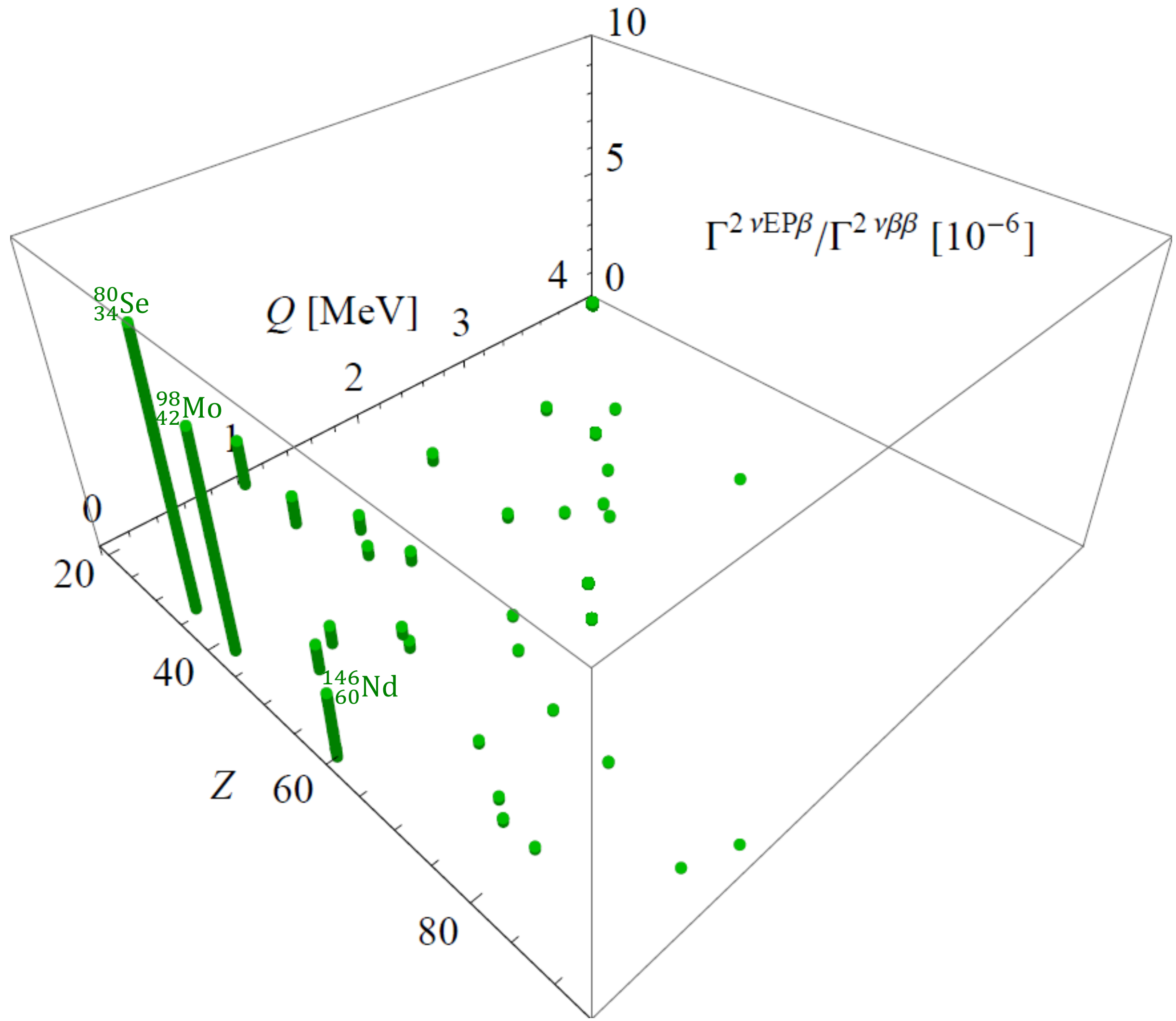
$$\Gamma^{2\nu\mathrm{EP}\beta} = g_A^4 \frac{G_\beta^4}{8\pi^7} \left|M^{2\nu\beta\beta}\right|^2 \times \pi \sum_{n=n_\mathrm{F}+1}^{\infty} B(Z_\mathrm{b}, E_\mathrm{b}) \int_{m_e}^{Q+2m_e-E_\mathrm{b}} \mathrm{d}E\, F(Z+2,E)\, E\, p \int_0^{Q+2m_e-E_\mathrm{b}-E} \mathrm{d}\omega_1\, \omega_1^2\, \omega_2^2$$

$2\nu\beta^-\beta^-$ decay rate:

$$\Gamma^{2\nu\beta\beta} = g_A^4 \frac{G_\beta^4}{8\pi^7} \left|M^{2\nu\beta\beta}\right|^2 \times \int_{m_e}^{Q+m_e} \mathrm{d}E_1\, F(Z+2,E_1)\, E_1\, p_1 \int_{m_e}^{Q+2m_e-E_1} \mathrm{d}E_2\, F(Z+2,E_2)\, E_2\, p_2 \int_0^{Q+2m_e-E_1-E_2} \mathrm{d}\omega_1\, \omega_1^2\, \omega_2^2$$

Ratios $\Gamma^{2\nu\mathrm{EP}\beta}/\Gamma^{2\nu\beta\beta}\,(Z,Q)$

- $2\nu\mathrm{EP}\beta^-$ leads to results analogous to $0\nu\mathrm{EP}\beta^-$; top 3: ${}^{80}_{34}\mathrm{Se}$, ${}^{98}_{42}\mathrm{Mo}$ and ${}^{146}_{60}\mathrm{Nd}$
- Ratio decreases rapidly with both Z and Q and ranges from 1.07×10^{-8} to 9.12×10^{-6}

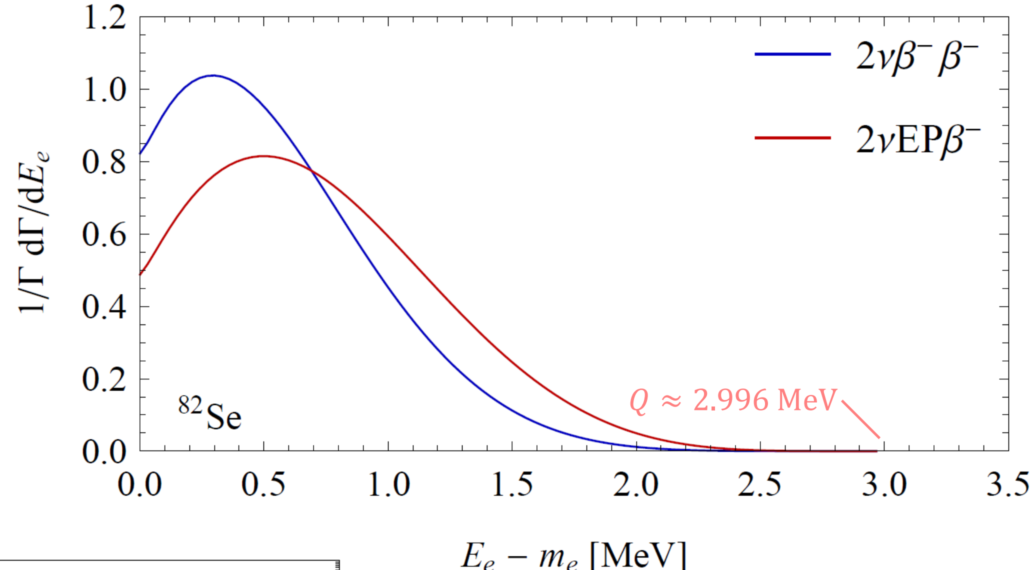


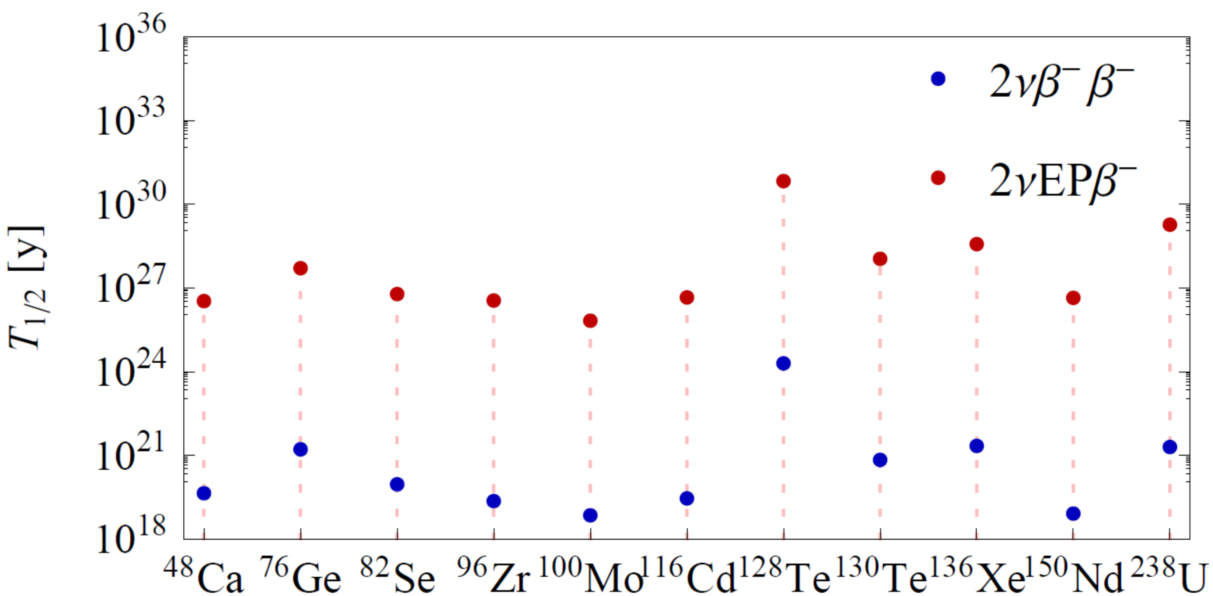
The relative significance of $\sim 10^{-6}$ is characteristic for a variety of nuclear processes involving atomic structure → a general property of the phase space of an electron in a bound state

Evaluation of Half-Lives $T_{1/2}^{2\nu\mathrm{EP}\beta}$

Single-electron $2\nu\beta^-\beta^-$ and $2\nu\mathrm{EP}\beta^-$ spectra (normalized to unity):

- In principle two distinct signatures
- However, for ^{82}Se the total decay rate $\Gamma^{2\nu\mathrm{EP}\beta}$ is suppressed by a factor of 1.51×10^{-7}
- Experiments with access to s.e.s. (SuperNEMO) could set limits on EP





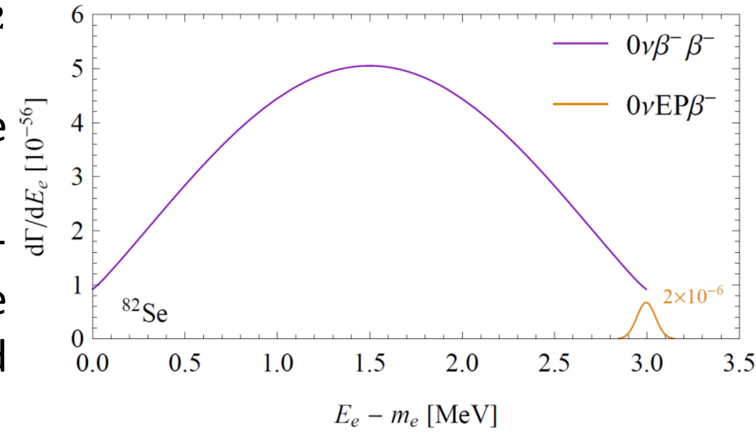
Half-lives $T_{1/2}^{2\nu\beta\beta}$:
[A. S. Barabash, Nucl. Phys. A, **935**, 52 (2015)]

- The half-lives $T_{1/2}^{2\nu\mathrm{EP}\beta}$ are approximately of the order of $T_{1/2}^{0\nu\beta\beta}$ (or even smaller if the effective Majorana neutrino mass $|m_{\beta\beta}| \ll 50$ meV)
- However, their $E_1 + E_2$ signatures are different (continuum vs. peak), so that the two modes are easily distinguished

Summary & Outlook

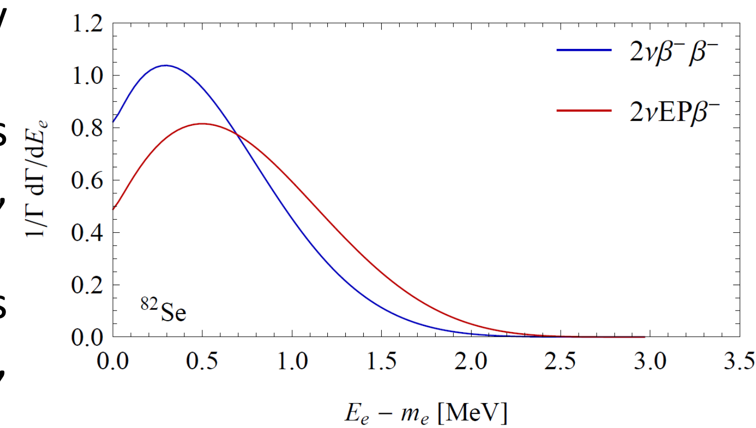
Summary:

- We studied single-electron modes of $0\nu\beta^-\beta^-$ and $2\nu\beta^-\beta^-$ in which one electron is produced in $s_{1/2}$ or $p_{1/2}$ bound state of daughter ion ${}_{Z+2}^{A}\mathrm{Y}^{2+}$
- We derived shapes of single-electron spectra to be inferred in next-gen $0\nu\beta\beta$ experiment SuperNEMO
- Overall suppression amounts for a factor of 10^{-9} – 10^{-6} due to shielding effect of nuclear charge (reduction of wave function on nuclear surface) and restricted phase space of bound electron



Outlook:

- Improved description of electron-shell structure by means of many-body Hartree–Fock approximation
- Generalization to various non-standard mechanisms of L violation (left-right symmetric interactions, heavy neutrino exchange, Majoron models, etc.)
- Other atomic modes of various rare processes (double-beta decay, double-electron capture, neutrino interactions, etc.)



Thank you for your attention!